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Department of Mathematics

NC STATE UNIVERSITY

MA 351 Intro Discrete Math Models, first mid-semester examination, Sep 19, 2002

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Your Name: SOLUTION
For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 4 problems, which are subdivided into 12 questions, where each question counts for the explicitly given number of points, adding to a total of **50 points**. Please write your answers in the spaces indicated, or below the questions, using the back of the sheets for scratch work and for completing the answers, if necessary. You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1	____
2	____
3	____
4	____
Total	____

Problem 1 (12 points)

(a, 8pts) Consider the linear recursion $h_{n+2} = 4h_n$ for $n \geq 0$ with $h_0 = 0, h_1 = 1$. Please list the first $h_0, \ldots, h_{10}$. Please compute a closed form solution (in the format like the one given in class for the Fibonacci numbers) for h_n.

$h_2 = h_4 = h_6 = h_8 = h_{10} = 0$, $h_3 = 4$, $h_5 = 16$, $h_7 = 128$, $h_9 = 512$.

Setting $h_n = q^n$ *we have* $q^2 = 4$: $\alpha = 2$, $\beta = -2$.

$h_n = a2^n + b(-2)^n$. *We use the initial conditions to find* a, b.

$$\left.\begin{array}{r} h_0 = a + b = 0, \\ h_1 = 2a - 2b = 1 \end{array}\right\} \implies a = \frac{1}{4}, b = -\frac{1}{4}$$

$h_n = 2^{n-2} + (-2)^{n-2}$.

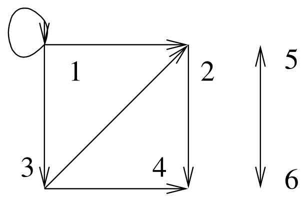
(b, 4pts) Is Galileo's famous model of the solar system a prescriptive or a descriptive mathematical model? Please explain.

Descriptive. Galileo asserts that the earth revolves around the sun based on his observations. He tries to model a real physical phenomenon.

Problem 2 (14 points): Consider the following digraph:

$$D = (\{1,2,3,4,5,6\},\{(1,1),(1,2),(1,3)(2,4),(3,2),(3,4),(5,6),(6,5)\}).$$

(a, 3pts) Please draw a picture of D.



(b, 3pts) Please write down the adjacency matrix M for D under the vertex order $(1,2,3,4,5,6)$.

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

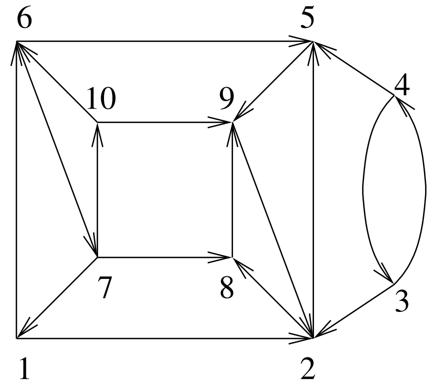
(c, 4pts) Please write down M^2.

$$\begin{bmatrix} 1 & 2 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(d, 4pts) Please write down the reachability matrix R for D under the vertex order $(1,2,3,4,5,6)$.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Problem 3 (14 points):
Consider the following digraph:



(a, 4pts) Please list the strong components of the above digraph.

$K_1 = \{1, 6, 7, 10\}$, $K_2 = \{2, 5, 8, 9\}$, $K_3 = \{3, 4\}$

(b, 4pts) Please draw the digraph that is the condensation of the above digraph.

$K_1 \longrightarrow K_2 \longleftarrow K_3$

(c, 4pts) Please list a vertex basis for the condensation and from it derive a vertex basis for the above digraph.

$B^* = \{K_1, K_3\}$, $B = \{1, 3\}$.

(d, 2pts) How many vertex bases does the above digraph have?

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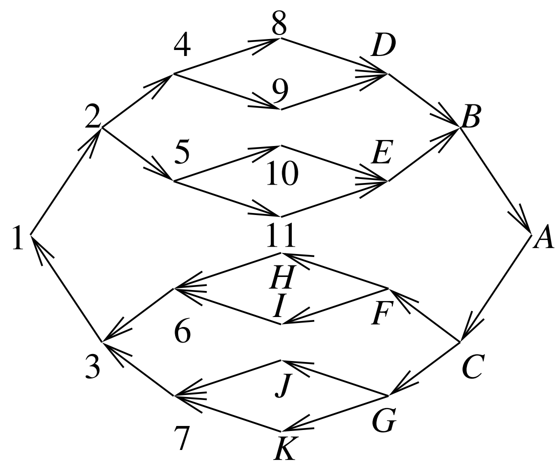
Problem 4 (10 points): Consider the following digraph with 22 vertices labelled $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, A, B, C, D, E, F, G, H, I, J, K\}$. according to the figure below.



(a, 2pts) Please select a pair of vertices such that the distance between the first to the second vertex is equal to the diameter of the above digraph.

$(4, E)$*; the diameter is* 14.

(b, 8pts) For the pair selected in (a), please list all distinct shortest paths as sequences of vertices from the first to the second vertex.

1.	4,8,	*D,B,A,C,*	*F,H*,6,	3,1,2,5,	10,*E*
2.	4,8,	*same*	*F,H*,6,	*same*	11,*E*
3.	4,8,		*F, I*, 6,		10,*E*
4.	4,8,		*F, I*, 6,		11,*E*
5.	4,8,		*G, J*, 7,		10,*E*
6.	4,8,		*G, J*, 7,		11,*E*
7.	4,8,		*G,K*,7,		10,*E*
8.	4,8,		*G,K*,7,		11,*E*
9.	4,9,		*F,H*,6,		10,*E*
10.	4,9,		*F,H*,6,		11,*E*
11.	4,9,		*F, I*, 6,		10,*E*
12.	4,9,		*F, I*, 6,		11,*E*
13.	4,9,		*G, J*, 7,		10,*E*
14.	4,9,		*G, J*, 7,		11,*E*
15.	4,9,		*G,K*,7,		10,*E*
16.	4,9,	*D,B,A,C,*	*G,K*,7,	3,1,2,5,	11,*E*