Find the Derivatives of inverse trig. functions

① $Y = \sin^{-1}(x)$

$\sin(Y) = \sin(\sin^{-1}(x)) = x$

$\therefore \sin Y = x$

$\Rightarrow \cos Y \frac{dY}{dx} = 1$ differentiated implicitly.

$\frac{dY}{dx} = \frac{1}{\cos Y}$ solving for $\frac{dY}{dx}$

$= \frac{1}{\sqrt{1-\sin^2 Y}}$ since $\sin^2 Y + \cos^2 Y = 1$

$= \frac{1}{\sqrt{1-x^2}}$ since $x = \sin Y$

$\therefore \boxed{\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}}$

② $Y = \cos^{-1}(x)$

$\cos(Y) = \cos(\cos^{-1}(x)) = x$

$\therefore \cos Y = x$

$\Rightarrow -\sin Y \frac{dY}{dx} = 1$: diff. implicitly.

$\Rightarrow \frac{dY}{dx} = \frac{-1}{\sin Y}$: solved for $\frac{dY}{dx}$

$= \frac{-1}{\sqrt{1-\cos^2 Y}}$: using $\sin^2 Y + \cos^2 Y = 1$

$= \frac{-1}{\sqrt{1-x^2}}$: since $\cos Y = x$

$\therefore \boxed{\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}}$

So the derivations for $\tan^{-1}(x)$, $\sec^{-1}(x)$, $\csc^{-1}(x)$, $\cot^{-1}(x)$ follow the same series of steps, although the algebra is a bit different.