NORTH CAROLINA STATE UNIVERSITY
MA 241 – 001 Test #4

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12 1.) Find the sum: $\sum_{n=1}^{\infty} \frac{10}{(n+2)(n+4)}$

12 2.) Find the sum: $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \ldots$

12 3.) Find the power series that represents the function: $f(x) = \frac{4x}{1+x^5}$

14 4.) For the series $\sum_{n=0}^{\infty} \frac{(x+3)^n}{2n+1}$, find the interval of convergence (be sure to check the endpoints).

14 5.) Find $T_4(x)$ for $f(x) = \sqrt{x}$ centered at a=4. Find an approximation for $\sqrt{5}$ by finding $T_4(5)$.

12 6.) Does $\sum_{n=1}^{\infty} (-1)^n \frac{1}{(2n-1)!}$ converge? If so, find the sum accurate to within 10^{-5}.

Choose two of the following - prove convergence or divergence.

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7.) $\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$ (comparison test)

8.) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ (integral test)

9.) $\sum_{n=2}^{\infty} \frac{2n-3}{7n+1}$

MA 241 VIDEO TEST #4

1.) $\sum_{n=1}^{\infty} \frac{10}{(n+2)(n+4)}$ telescoping; decompose using partial fractions

$$\frac{10}{(n+2)(n+4)} = \frac{A}{n+2} + \frac{B}{n+4} = \frac{A(n+4)}{(n+2)(n+4)} + \frac{B(n+2)}{(n+4)(n+2)}$$

$$10 = A(n+4) + B(n+2)$$

$n=-4$: $10 = A\cancel{(0)} + -2B$ $\quad B = -5$

$n=-2$: $10 = A(2) + B\cancel{(0)}$ $\quad A = 5$

$$\sum_{n=1}^{\infty} \frac{10}{(n+2)(n+4)} = \sum_{n=1}^{\infty} \left(\frac{5}{n+2} - \frac{5}{n+4}\right)$$

$$\sum_{n=1}^{\infty} \left(\frac{5}{n+2} - \frac{5}{n+4}\right) = \left(\frac{5}{3} - \cancel{\frac{5}{5}}\right) + \left(\frac{5}{4} - \cancel{\frac{5}{6}}\right) + \left(\cancel{\frac{5}{5}} - \cancel{\frac{5}{7}}\right)$$

$$+ \left(\cancel{\frac{5}{6}} - \frac{5}{8}\right) + \left(\cancel{\frac{5}{7}} - \frac{5}{9}\right) + \cdots$$

all terms eliminated except $\frac{5}{3} + \frac{5}{4} \to$ sum is $\frac{35}{12}$ SUM =

2.) $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \cdots$

$\left\{ \begin{array}{l} 5 + 5(-\tfrac{2}{3}) + 5(-\tfrac{2}{3})^2 + 5(-\tfrac{2}{3})^3 \cdots \\ 5 - \tfrac{10}{3} + \tfrac{20}{9} - \cdots \end{array} \right.$

$a = 5$ $\quad r = ?$ $\quad ar = \frac{-10}{3}$

$5r = \frac{-10}{3}$

infinite geometric series $\quad r = \frac{-10}{3} \cdot \frac{1}{5} = \frac{-2}{3}$

SUM =

$$S = \frac{a}{1-r} = \frac{5}{1-(-2/3)} = \frac{5}{1+2/3} = \frac{5}{5/3} = 5 \cdot 3/5 = 3$$

3.) $f(x) = \frac{4x}{1+x^5}$ $\quad \frac{a}{1-r}$ $\quad a = 4x$, $r = (-x^5)$

$$4x + 4x(-x^5) + 4x(-x^5)^2 + 4x(-x^5)^3 + 4x(-x^5)^4 + \cdots$$

$$4x - 4x^6 + 4x^{11} - 4x^{16} + 4x^{21} + \cdots$$

or $\sum_{n=0}^{\infty} (4x)(-x^5)^n$

4.) $\sum_{n=0}^{\infty} \frac{(x+3)^n}{2n+1}$ ratio test

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{\left|\frac{(x+3)^{n+1}}{2(n+1)+1}\right|}{\left|\frac{(x+3)^n}{2n+1}\right|} = \lim_{n\to\infty} \left|\frac{\cancel{(x+3)^{n+1}}}{\cancel{2n+3}} \cdot \frac{\cancel{2n+1}}{\cancel{(x+3)^n}}\right|$$

$= |x+3| < 1$ in order to converge

$|x+3| < 1 \longrightarrow$

$$-1 < x+3 < 1$$
$$-3 \quad -3 \quad -3$$
$$-4 < x < -2$$

check endpts:

① $x=-4$ $\quad \sum_{n=0}^{\infty} \frac{(-4+3)^n}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ alternating harmonic ∴ CONVERGENT

② $x=-2$ $\quad \sum_{n=0}^{\infty} \frac{(-2+3)^n}{2n+1} = \sum_{n=0}^{\infty} \frac{(1)^n}{2n+1}$ harmonic ∴ DIVERGENT

interval of convergence $\Rightarrow [-4, -2)$

5.) find $T_4(x)$ for $f(x) = \sqrt{x}$ $\quad a=4$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$f(x) = x^{1/2}$
$f'(x) = \frac{1}{2}x^{-1/2}$
$f''(x) = \frac{-1}{4}x^{-3/2}$
$f'''(x) = \frac{3}{8}x^{-5/2}$
$f^{(4)}(x) = \frac{-15}{16}x^{-7/2}$

$$\underbrace{\frac{f^0(4)}{0!}(x-4)^0}_{n=0} + \underbrace{\frac{f'(4)}{1!}(x-4)^1}_{n=1} + \underbrace{\frac{f''(4)}{2!}(x-4)^2}_{n=2}$$

$$\underbrace{\frac{f'''(4)}{3!}(x-4)^3}_{n=3} + \underbrace{\frac{f^{(4)}(4)}{4!}(x-4)^4}_{n=4} + \cdots$$

$$\frac{\sqrt{4}}{1} 1 + \frac{\frac{1}{2\sqrt{4}}}{1}(x-4)^1 + \frac{\frac{-1}{4\cdot(\sqrt{4})^3}}{2}(x-4)^2 + \frac{\frac{3}{8(\sqrt{4})^5}}{6}(x-4)^3$$

$$+ \frac{\frac{-15}{16(\sqrt{4})^7}}{24}(x-4)^4$$

$$T_4(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{4\cdot 8\cdot 2}(x-4)^2 + \frac{\cancel{3}\,1}{8\cdot 32\cdot \cancel{6}\,2}(x-4)^3$$

$$+ \frac{\cancel{-15}\,-5}{16\cdot 128\cdot \cancel{24}\,8}(x-4)^4$$

$$T_4(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3 - \frac{5}{16384}(x-4)^4$$

$$\sqrt{5} \approx T_4(5) = 2 + \frac{1}{4}(5-4) - \frac{1}{64}(5-4)^2 + \frac{1}{512}(5-4)^3$$

$$- \frac{5}{16384}(5-4)^4 = 2 + \frac{1}{4} - \frac{1}{64} + \frac{1}{512} - \frac{5}{16384} \approx 2.23602$$

(from calculator $\sqrt{5} \approx 2.23607$)

6) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{(2n-1)!}$ converge?

alt series test ① $\lim_{n\to\infty} |a_n| = 0$

$\lim_{n\to\infty} \frac{1}{(2n-1)!} = 0$, yes

ⓑ $a_{n+1} < a_n$

$$\frac{1}{(2(n+1)-1)!} < \frac{1}{(2n-1)!}$$

$$\frac{1}{(2n+1)!} < \frac{1}{(2n-1)!} \quad \text{yes (larger denom)}$$

$\therefore$ series CONV (by the alt series test)

$$\overset{n=1}{\frac{(-1)^1}{(2-1)!}} + \overset{n=2}{\frac{(-1)^2}{(3)!}} + \overset{n=3}{\frac{(-1)^3}{(5)!}} + \overset{n=4}{\frac{(-1)^4}{(7)!}} + \overset{n=5}{\frac{(-1)^5}{(9)!}}$$

$$\frac{-1}{1} + \frac{1}{6} - \frac{1}{120} + \frac{1}{5040} - \frac{1}{362880}$$

$$-1 + .16666666 - .0083333 + .000198412$$

$\boxed{-.0000027555}$ ← doesn't affect accuracy 10^{-5}

$-1 + .1666666 - .0083333 + .000198412$

$\approx -.841468222$

(2 of the following)

7) $\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$ comparison test

$$\frac{n!}{(n+2)!} = \frac{\cancel{n!}}{(n+2)(n+1)\cancel{n!}} = \frac{1}{(n+1)(n+2)} = \frac{1}{n^2+3n+2}$$

compare to $\sum \frac{1}{n^2}$ ← "p-series" $n>1$ ∴ CONVERGE

$$\frac{1}{n^2+3n+2} < \frac{1}{n^2}$$

smaller than convergent series, also CONV (by comparison test)

8.) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ integral test

$$\left(\frac{1}{2}\right)\int_1^{\infty} \frac{x(2)}{x^2+1}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u|\Big]$$

let $u = x^2+1$, $du = 2x\,dx$

$$= \frac{1}{2}\ln|x^2+1|\Big]_1^{\infty}$$

$$\lim_{A\to\infty}\left[\frac{1}{2}\ln|x^2+1|\right]_1^{A} = \frac{1}{2}\left[\ln|A^2+1| - \ln|1^2+1|\right]$$

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∴ INTEGRAL AND SERIES DIVERGE

9.) $\sum_{n=2}^{\infty} \frac{2n-3}{7n+1}$ $\lim_{n\to\infty} \frac{2n-3}{7n+1} = \frac{2}{7} \neq 0$

therefore $\sum_{n=2}^{\infty} \frac{2n-3}{7n+1}$ DIVERGES