NORTH CAROLINA STATE UNIVERSITY
MA 241 Video   Test # 3

Department of Mathematics
Dr. J. Griggs

1. Find the general solution of $y'' - 6y' + 5y = 8e^x$.

2. Solve: $y'' + 2y' + 2y = 0$ $y(0) = 0$ $y'(0) = 10$

3. Solve for the general solution: $y'' + 4y' + 4y = x^2 - 3x$

Determine whether the series is convergent – be thorough with justifications.

4. $\sum_{n=1}^{\infty} \frac{2+n^3}{1+2n^3}$

5. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+1}$

6. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

7. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

8. $\sum_{n=1}^{\infty} \frac{2}{5n^2-3}$

9. Find the interval of convergence and the radius of convergence: $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n(4^n)}$

10. Using a power series find
$\int_0^{.2} \frac{1}{1+x^5}dx$
accurate to five decimal places.

1.) $y'' - 6y' + 5y = 8e^x$

y_H: char eq: $r^2 - 6r + 5 = 0$

$(r-5)(r-1) = 0$

$r_1 = 5 \quad r_2 = 1$

$y_H = C_1 e^{5x} + C_2 e^{1 \cdot x}$

y_p: $y_p = \cancel{Ae^x}$ (no, part of y_H.)

$y_p = Axe^x$

$y_p' = Ax \cdot e^x + e^x(A)$

$y_p'' = Ax \cdot e^x + e^x(A) + Ae^x$

$(Axe^x + 2Ae^x) - 6(Axe^x + Ae^x) + 5(Axe^x) = 8e^x$

$Axe^x + 2Ae^x - 6Axe^x - 6Ae^x + 5Axe^x = 8e^x$

$xe^x(A - 6A + 5A) + e^x(2A - 6A) = 8e^x$

$-4Ae^x = 8e^x$

$-4A = 8 \qquad A = -2$

$y_p = -2xe^x$

$y = y_H + y_p$

$y = C_1 e^{5x} + C_2 e^x - 2xe^x$

2.) $y'' + 2y' + 2y = 0 \qquad y(0) = 0;\ y'(0) = 10$

char eq: $r^2 + 2r + 2 = 0$

$r = \frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2(1)} = \frac{-2 \pm \sqrt{-4}}{2}$

$r = \frac{-2 \pm 2i}{2} = -1 \pm 1i$

$\alpha \quad \beta$

$y = e^{-1x}[C_1 \cos x + C_2 \sin x]$

$y(0) = 0 \quad x = 0;\ y = 0$

$0 = e^{-1 \cdot 0}[C_1 \cos 0 + C_2 \sin 0]$

$0 = 1[C_1]$

$C_1 = 0$

$y = e^{-1x}[C_2 \sin x]$

$y' = e^{-x}\left[C_2 \cos x\right] + (C_2 \sin x)\cdot(e^{-x})(-1)$

$y' = e^{-x} C_2 \cos x + -e^{-x} C_2 \sin x$

$y'(0) = 10 \quad x=0 \quad y'=10$

$10 = e^{-0}\cdot C_2 \cdot \cos 0 - e^{-0}\cdot C_2 \sin 0$

$10 = C_2$

$y = e^{-x}\left[10 \sin x\right]$

3.) $y'' + 4y' + 4y = x^2 - 3x$

y_H: char eq: $r^2 + 4r + 4 = 0$

$(r+2)(r+2) = 0$

$r_1 = r_2 = -2$

$y_H = C_1 e^{-2x} + C_2 x e^{-2x}$

y_p:

$y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$(2A) + 4(2Ax+B) + 4(Ax^2+Bx+C) = x^2 - 3x$

$(4A)x^2 + (8A+4B)x + (2A+4B+4C) = x^2 - 3x$

$4A = 1 \quad A = 1/4$

$8A + 4B = -3$

$8(1/4) + 4B = -3$

$2 + 4B = -3$

$4B = -5$

$B = -5/4$

$2A + 4B + 4C = 0$

$2(1/4) + 4(-5/4) + 4C = 0$

$1/2 - 5/1 + 4C = 0$

$-\frac{9}{2} + 4C = 0$

$4C = 9/2$

$C = 9/8$

$y = y_H + y_p$

$y = C_1 e^{-2x} + C_2 x e^{-2x} + 1/4 x^2 - 5/4 x + 9/8$

4.) $\sum_{n=1}^{\infty} \frac{2+n^3}{1+2n^3}$

$\lim_{n\to\infty} \frac{2+n^3}{1+2n^3} = \frac{1}{2} \neq 0 \longrightarrow$ series DIV.

(if $\lim_{n\to\infty} a_n \neq 0$, then series diverges)

5.) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n+1}$ alternating series test

1.) $\lim_{n\to\infty} \frac{\sqrt{n}}{n+1} = \lim_{n\to\infty} \sqrt{\frac{n}{(n+1)^2}} = 0$ ✓

2.) $\frac{\sqrt{n+1}}{(n+1)+1} < \frac{\sqrt{n}}{n+1}$

$\sqrt{\frac{n+1}{(n+2)^2}} \quad 2 \quad \sqrt{\frac{n}{(n+1)^2}}$ ✓

∴ series converges

6.) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \Rightarrow$ use integral test.

$\int_2^{\infty} \frac{1}{x(\ln x)^2}dx = \int_2^{\infty} (\ln x)^{-2}\cdot\left(\frac{1}{x}\right)dx$

let $u = \ln x$

$du = \frac{1}{x}dx$

$= \int u^{-2}du = \left.\frac{u^{-1}}{-1}\right] = \left.\frac{-1}{u}\right]$

$= \left.\frac{-1}{\ln x}\right]_2^{\infty} = \left(\cancel{\frac{-1}{\ln\infty}}\right)^{0} - \left(\frac{-1}{\ln 2}\right) = \frac{1}{\ln 2}$

∴ series CONV.

7.) $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$ telescoping?

$\frac{1}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3} = \frac{A(n+3) + B(n)}{n(n+3)}$

$1 = A(n+3) + B(n) \quad n=0 \quad 1 = 3A \quad A = 1/3$

$n=-3 \quad 1 = -3B \quad B = -1/3$

$$\sum_{n=1}^{\infty}\left(\frac{1/3}{n}+\frac{-1/3}{n+3}\right)$$

$$=\sum_{n=1}^{\infty}\left[\frac{1}{3n}-\frac{1}{3(n+3)}\right]=\left(\frac{1}{3}-\cancel{\frac{1}{12}}\right)+\left(\frac{1}{6}-\cancel{\frac{1}{15}}\right)$$

$$+\left(\frac{1}{9}-\frac{1}{18}\right)+\left(\cancel{\frac{1}{12}}-\frac{1}{21}\right)+\left(\cancel{\frac{1}{15}}-\frac{1}{24}\right)$$

($1/3 + 1/6 + 1/9$ remain; all others will get knocked out as series progresses)

$\therefore$ telescoping series converges

8.) $$\sum_{n=1}^{\infty}\frac{2}{5n^2-3}$$

$\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges; p-series ($p=2$) $p>1$

$\frac{2}{5}\sum_{n=1}^{\infty}\frac{1}{n^2}$ also converges $\left(\sum_{n=1}^{\infty}\frac{2}{5n^2}\right)$

use limit comparison test:

$$\lim_{n\to\infty}\frac{\frac{2}{5n^2}}{\frac{2}{5n^2-3}}=\lim_{n\to\infty}\frac{\cancel{2}}{5n^2}\cdot\frac{5n^2-3}{\cancel{2}} \to 1$$

$\lim = 1$ $\therefore$ $\sum_{n=1}^{\infty}\frac{2}{5n^2-3}$ also CONV.

9.) $$\sum_{n=1}^{\infty}\frac{(x+2)^n}{n(4^n)}$$ use ratio test to determine interval of convergence.

$$\lim_{n\to\infty}\left|\frac{\frac{(x+2)^{n+1}}{(n+1)(4^{n+1})}}{\frac{(x+2)^n}{n(4^n)}}\right| = \lim_{n\to\infty}\left|\frac{(x+2)^{n+1}}{(n+1)4^{n+1}}\cdot\frac{n\cdot 4^n}{(x+2)^n}\right|$$

$$=\lim_{n\to\infty}\frac{|x+2|}{4}\cdot\frac{n}{n+1}\to 1 = \frac{|x+2|}{4}<1$$

$$-1<\frac{x+2}{4}<1 \quad -4<x+2<4$$
$$-6<x<2$$

radius of convergence $\Rightarrow R=4$

check endpts:

① $x=-6$ $$\sum_{n=1}^{\infty}\frac{(-6+2)^n}{n(4^n)}=\sum_{n=1}^{\infty}\frac{(-4)^n}{n(4)^n}$$
$$=\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$$ alt. harmonic $\therefore$ CONVERGES

② $x=2$ $$\sum\frac{(2+2)^n}{n\cdot 4^n}=\sum\frac{1}{n}$$
harmonic $\rightarrow$ DIVERGES

interval of convergence: $[-6, 2)$

10.) $$\frac{1}{1+x^5}\Rightarrow\frac{a}{1-r} \quad a=1 \quad r=-x^5$$

series $\Rightarrow 1-x^5+x^{10}-x^{15}+x^{20}-\dots$

$$\int_0^2\left(1-x^5+x^{10}-x^{15}+x^{20}-\dots\right)dx \quad \left\{\int(-1)^n x^{5n}dx = \int(-1)^n\frac{x^{5n+1}}{5n+1}\right.$$

$$\left[x-\frac{x^6}{6}+\frac{x^{11}}{11}-\frac{x^{16}}{16}+\frac{x^{21}}{21}-\dots\right]_0^{.2}$$

$$\frac{2}{10}-\frac{2^6}{10^6(6)}+\frac{2^{11}}{10^{11}(11)}-\frac{2^{16}}{10^{16}(16)}$$

$.200000 - .00001066 + .000000001$

won't affect 5 decimal places

sum $\approx .19999$