NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA 241 – 601 Video Class Summer II, 2004 Thursday, July 15, 2004 John Griggs

Please show all work and solutions on the paper provided (no work – no credit). Ten questions worth 10 points apiece. A portion of the table of integrals is provided for the test.

1.) Integrate and evaluate: $\int_1^2 \frac{4+u^2}{u^3}du$

2.) If $h(x)=\int_0^{x^2}\sqrt{1+r^3}\,dr$, find $h'(x)$.

3.) Integrate using substitution: $\int \frac{\sin x}{1+\cos^2 x}dx$

4.) Integrate using substitution: $\int_0^1 x^2(1+2x^3)^5 dx$

5.) Integrate by parts: $\int_0^{\pi/2} x \cos 2x \; dx$

6.) Decompose into partial fractions and integrate: $\int \frac{10}{(x-1)(x^2+9)}dx$

7.) Integrate: $\int \sin^3 x \cos^2 x \; dx$

8.) Integrate using the table of integrals: $\int \frac{e^{2x}}{\sqrt{2+e^x}}dx$

9.) Approximate using Simpson's Rule or the Trapezoidal Rule:

$$\int_0^{\pi} x^2 \sin x \; dx$$

10.) Evaluate; does the integral converge or diverge?

$$\int_2^{\infty} \frac{1}{(x+3)^{\frac{3}{2}}}dx$$

TABLE OF INTEGRALS

FORMS INVOLVING $a + bu$

47. $\int \frac{u\,du}{a+bu} = \frac{1}{b^2}(a+bu-a\ln|a+bu|)+C$

48. $\int \frac{u^2\,du}{a+bu} = \frac{1}{2b^3}\left[(a+bu)^2-4a(a+bu)+2a^2\ln|a+bu|\right]+C$

49. $\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right|+C$

50. $\int \frac{du}{u^2(a+bu)} = -\frac{1}{au}+\frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right|+C$

51. $\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)}+\frac{1}{b^2}\ln|a+bu|+C$

52. $\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)}-\frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right|+C$

53. $\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(a+bu-\frac{a^2}{a+bu}-2a\ln|a+bu|\right)+C$

54. $\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2}+C$

55. $\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}+C$

56. $\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(8a^2+3b^2u^2-4abu)\sqrt{a+bu}+C$

57. $\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right|+C, \quad \text{if } a>0$
$= \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}}+C, \quad \text{if } a<0$

58. $\int \frac{\sqrt{a+bu}}{u}du = 2\sqrt{a+bu}+a\int\frac{du}{u\sqrt{a+bu}}$

59. $\int \frac{\sqrt{a+bu}}{u^2}du = -\frac{\sqrt{a+bu}}{u}+\frac{b}{2}\int\frac{du}{u\sqrt{a+bu}}$

60. $\int u^n\sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2}-na\int u^{n-1}\sqrt{a+bu}\,du\right]$

61. $\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)}-\frac{2na}{b(2n+1)}\int\frac{u^{n-1}\,du}{\sqrt{a+bu}}$

62. $\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}}-\frac{b(2n-3)}{2a(n-1)}\int\frac{du}{u^{n-1}\sqrt{a+bu}}$

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(10 points each)

.) $\int_1^2 \frac{4+u^2}{u^3}\,du$

$= \int_1^2 \frac{4}{u^3}\,du + \int_1^2 \frac{u^2}{u^3}\,du = 4\int_1^2 u^{-3}\,du + \int_1^2 \frac{1}{u}\,du$

$= 4 \cdot \frac{u^{-2}}{-2}\Big]_1^2 + \ln|u|\Big]_1^2 = \frac{-2}{u^2} + \ln|u|\Big]_1^2$

$= \left(\frac{-2}{2^2} + \ln 2\right) - \left(\frac{-2}{1^2} + \ln 1\right)$

$= \left(\frac{-1}{2} + \ln 2\right) - (-2+0) = \frac{-1}{2} + \ln 2 + 2$

$\approx \boxed{2.1931}$

.) $h(x) = \int_0^{x^2} \sqrt{1+r^3}\,dr$

$h'(x) = \frac{d\left[\int_0^{x^2}\sqrt{1+r^3}\,dr\right]}{dx}$

$h'(x) = \sqrt{1+(x^2)^3} \cdot 2x$

$\boxed{h'(x) = 2x\sqrt{1+x^6}}$

.) $\int \frac{\sin x}{1+\cos^2 x}\,dx$ let $u = \cos x$

$du = -\sin x\,dx$

$-1\int \frac{-\sin x}{1+\cos^2 x}\,dx = -1\int \frac{du}{1+u^2} = -1 \cdot \tan^{-1} u$

$= \boxed{-\tan^{-1}(\cos x) + C}$

4.) $\int_0^1 x^2(1+2x^3)^5\,dx$ let $u = 1+2x^3$

$du = 6x^2\,dx$

$= \frac{1}{6}\int_0^1 6x^2(1+2x^3)^5\,dx = \frac{1}{6}\int_0^1 (1+2x^3)^5 \cdot 6x^2\,dx$

$= \frac{1}{6}\int u^5 \cdot du = \frac{1}{6}\frac{u^6}{6}\Big] = \frac{u^6}{36}\Big]$

$= \frac{(1+2x^3)^6}{36}\Big]_0^1 = \frac{(1+2\cdot 1^3)^6}{36} - \frac{(1+2\cdot 0^3)^6}{36}$

$= \frac{3^6}{36} - \frac{1^6}{36} = \frac{729-1}{36} = \frac{728}{36} = \boxed{\frac{182}{9} \approx 20.22}$

5.) $\int_0^{\pi/2} x \cdot \cos 2x\,dx$ let $u = x$ $v = \frac{1}{2}\sin 2x$

$du = 1 \cdot dx$ $dv = \cos 2x\,dx$

$= u \cdot v - \int v \cdot du$

$= (x)(\frac{1}{2}\sin 2x) - \int \frac{1}{2}\sin 2x \cdot dx$

$= \frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x\,dx$

$= \frac{1}{2}x\sin 2x - \frac{1}{2}\left[\frac{-1}{2}\cos 2x\right]$

$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\Big]_0^{\pi/2}$

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$$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x \Big]_0^{\pi/2}$$

$$= \left[\frac{1}{2}\left(\frac{\pi}{2}\right)\sin 2\left(\frac{\pi}{2}\right) + \frac{1}{4}\cos 2\left(\frac{\pi}{2}\right)\right] - \left[\frac{1}{2}\cdot 0\cdot\sin 2\cdot 0 + \frac{1}{4}\cos 2(0)\right]$$

$$= \left(\frac{\pi}{4}\sin\pi + \frac{1}{4}\cos\pi\right) - \left(0 + \frac{1}{4}\cos 0\right)$$

$$= \left(\cancel{\frac{\pi}{4}}\cancel{(0)} + \frac{1}{4}(-1)\right) - \left(\cancel{0} + \frac{1}{4}(1)\right)$$

$$= -\frac{1}{4} - \frac{1}{4} = \boxed{-1/2}$$

$$) \int \frac{10}{(x-1)(x^2+9)}\,dx \Rightarrow \frac{10}{(x-1)(x^2+9)} = \frac{A^{(x^2+9)}}{(x-1)_{(x^2+9)}} + \frac{(Bx+C)^{(x-1)}}{(x^2+9)_{(x-1)}}$$

$$10 = A(x^2+9) + (Bx+C)(x-1)$$

let $x=1$: $\quad 10 = A(10) + \cancel{(B+C)}(0)$

$$\boxed{A=1}$$

$$10 = Ax^2 + A(9) + Bx^2 + Cx - Bx - C$$

$$10 = x^2 + 9 + Bx^2 + Cx - Bx - C$$

$$10 = x^2(1+B) + x(C-B) + (9-C)$$

$1+B=0$
$\boxed{B=-1}$

$C-B=0$
$C-(-1)=0$
$C+1=0$
$\boxed{C=-1}$

$9-C=10$
$C=-1$

$$\int \frac{10}{(x-1)(x^2+9)}\,dx = \int \frac{1}{x-1}\,dx + \int \frac{-1\cdot x - 1}{x^2+9}\,dx$$

$$= \ln|x-1| + \int \frac{-x}{x^2+9}\,dx + \int \frac{-1}{x^2+9}\,dx$$

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$$= \ln|x-1| + \frac{-1}{2}\int \frac{-x\,(-2)}{x^2+9} + -1\int \frac{1}{x^2+9}\,dx$$

$$= \ln|x-1| - \frac{1}{2}\left[\ln|x^2+9|\right] - 1\left[\frac{1}{3}\tan^{-1}\frac{x}{3}\right]$$

$$= \boxed{\ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\tan^{-1}\frac{x}{3} + C}$$

$$7.)\ \int \sin^3 x\cos^2 x\,dx = \int \underbrace{\sin^2 x\cos^2 x}_{\substack{\text{change to}\\ \text{cos}}}\sin x\,dx$$

$$(\sin^2 x = 1-\cos^2 x)$$

$$\int\left[(1-\cos^2 x)\cos^2 x\right]\sin x\,dx$$

$$-1\int(\cos^2 x - \cos^4 x)\overset{(-1)}{\sin}x\,dx$$

let $u = \cos x$
$du = -\sin x\,dx$

$$-1\int(u^2-u^4)\,du$$

$$-1\left[\frac{u^3}{3} - \frac{u^5}{5}\right] = \boxed{\frac{-(\cos^3 x)}{3} + \frac{(\cos^5 x)}{5} + C}$$

$$8.)\ \int \frac{e^{2x}}{\sqrt{2+e^x}}\,dx = \int \frac{\overset{u}{\boxed{e^x}}\cdot\overset{du}{\boxed{e^x dx}}}{\sqrt{2+e^x}}$$

let $u = e^x$
$du = e^x dx$

$$\int \frac{u\cdot du}{\sqrt{2+u}} \xrightarrow[\substack{a=2\\ b=1}]{\text{use } \#55} = \frac{2}{3b^2}\left[(bu-2a)\sqrt{a+bu}\right] + C$$

#55: $\int \frac{u\,du}{\sqrt{a+bu}}$

$\overset{u}{=}$

#56: $\int \frac{u^2\,du}{\sqrt{a+bu}}$

241-6~~0~~01 t1 p5

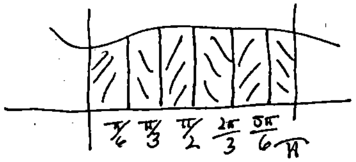
$$= \frac{2}{3(1)^2}\left|\left[1\cdot u - 2(2)\right]\sqrt{2+1\cdot u}\right] + C$$

$$= \frac{2}{3}\left[(e^x-4)\left(\sqrt{2+e^x}\right)\right] + C$$

1.) $\int_0^{\pi} x^2 \sin x\,dx$ $(n=6)$ ← omitted from test

$\Delta x = \frac{\pi}{6}$

OK to use $n=4$



$$A \approx Simp_6 = \frac{\pi/6}{3}\left[f(0) + 4f(\pi/6) + 2f(\pi/3) + 4f(\pi/2) + 2f(2\pi/3) + 4f(5\pi/6) + f(\pi)\right]$$

$$A \approx \frac{\pi}{18}\left[0^2\cdot\sin 0 + 4\left(\frac{\pi}{6}\right)^2\left(\sin\frac{\pi}{6}\right) + 2\left(\frac{\pi}{3}\right)^2\left(\sin\frac{\pi}{3}\right) + 4\left(\frac{\pi}{2}\right)^2 \sin\frac{\pi}{2} + 2\left(\frac{2\pi}{3}\right)^2 \sin\frac{2\pi}{3} + 4\left(\frac{5\pi}{6}\right)^2 \sin\frac{5\pi}{6} + (\pi)^2 \sin\pi\right]$$

$$A \approx \frac{\pi}{18}\left[\cancel{0} + 4\left(\frac{\pi^2}{36}\right)\left(\frac{1}{2}\right) + 2\left(\frac{\pi^2}{9}\right)\left(\frac{\sqrt{3}}{2}\right) + 4\left(\frac{\pi^2}{4}\right)(1) + 2\left(\frac{4\pi^2}{9}\right)\left(\frac{\sqrt{3}}{2}\right) + 4\left(\frac{25\pi^2}{36}\right)\left(\frac{1}{2}\right) + \cancel{\pi^2(0)}\right]$$

$$A \approx \left[\frac{\pi^2}{18} + \frac{\pi^2\cdot\sqrt{3}}{9} + \pi^2 + \frac{4\pi^2\sqrt{3}}{9} + \frac{25\pi^2}{18}\right]\cdot\frac{\pi}{18}$$

$$A \approx \left[\frac{9.87}{18} + \frac{9.87(1.732)}{9} + 9.87 + \frac{4(9.87)(1.732)}{9} + \frac{25(9.87)}{18}\right](.1745)$$

$$A \approx \left[.5483 + 1.8994 + 9.8696 + 7.5976 + 13.7078\right](.1745)$$

$$\boxed{A \approx 5.8683}$$

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10.) $$\int_2^{\infty} \frac{1}{(x+3)^{3/2}}\,dx = \lim_{A\to\infty}\int_2^{A}(x+3)^{-}\,dx$$

let $u = x+3$
$du = dx$

$$= \lim_{A\to\infty}\left[\frac{(x+3)^{-1/2}}{-1/2}\right]_2^A$$

$$= \lim_{A\to\infty}\left[\frac{-2}{\sqrt{x+3}}\right]_2^A = \lim_{A\to\infty}\left[\left(\frac{-2}{\sqrt{A+3}}\right) - \left(\frac{-2}{\sqrt{2+3}}\right)\right]$$

$$= \lim_{A\to\infty}\left(\cancel{\frac{-2}{\sqrt{A+3}}}\right)^{0} + \frac{2}{\sqrt{5}} = \boxed{\frac{2}{\sqrt{5}} \quad \therefore \text{CONVERGES}}$$