NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA141-601 Test #4 Wednesday, December 5, 2007 Dr. J. Griggs

Please show all work and answers on the plain white paper provided. Simplify completely unless directed otherwise.

1.) Using three iterations of Newton's Method, find the largest root of $x^3 - 3x + 1 = 0$. ($x_0 = 1.3$, *find* $x_1, x_2,$ *and* x_3)

2.) Find the general antiderivatives:

a.) $f(x) = 12x^5 + 7\sqrt{x} - \frac{5}{x} - 6$ b.) $g(x) = 5\sec^2 x + 3\cos x - 4e^x + 5$

3.) Evaluate the following definite integrals:

a.) $\int_{\ln 3}^{\ln 6} 8e^x dx$ b.) $\int_1^8 \frac{x-1}{\sqrt[3]{x^2}} dx$

4.) Evaluate using the Fundamental Theorem of Calculus:

Find $h'(x)$: $h(x) = \int_0^{x^2} \sqrt{1+r^3}\, dr$

5.) Find the area under the curve $y = 4x - x^2$ from x = 0 to x = 4 using Reimann Sums and limits: $\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
(check your answer by evaluating: $\int_0^4 (4x - x^2) dx$)

6.) Integrate using substitution:

a.) $\int \sec^3 x \tan x \, dx$ b.) $\int_0^1 x^2(1+5x^3)^4 \, dx$

7.) Integrate by parts: $\int x^4 \ln x \, dx$

(7 problems — 14 points each)

1.) 3 iterations of Newton's method:

$\boxed{y = x^3 - 3x + 1}$ largest root $\boxed{y' = 3x^2 - 3}$

$X_0 = 1.3$ (guess)

x	y
−2	−1
−1	4
0	1
1	−1
2	3 ← largest root
3	22

$X_1 = X_0 - \frac{f(X_0)}{f'(X_0)}$

$X_1 = 1.3 - \frac{f(1.3)}{f'(1.3)}$ $(1.3)^3 - 3(1.3) + 1 = -.703$; $3(1.3)^2 - 3 = 2.07$

$X_1 = 1.3 - \frac{(-.703)}{2.07} \approx 1.6396$

$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)}$

$X_2 = 1.6396 - \frac{f(1.6396)}{f'(1.6396)}$ $(1.6396)^3 - 3(1.6396) + 1 = .4889$; $3(1.6396)^2 - 3 = 5.0649$

$X_2 = 1.6396 - \frac{(.4889)}{5.0649}$

$X_2 = 1.6396 - .09653 \approx 1.54307$

$X_3 = X_2 - \frac{f(X_2)}{f'(X_2)}$ $(1.54307)^3 - 3(1.54307) + 1 = .044$

$X_3 = 1.54307 - \frac{f(1.54307)}{f'(1.54307)}$ $3(1.54307)^2 - 3 =$

$X_3 = 1.54307 - \frac{.04494}{4.1432}$

$X_3 = 1.54307 - .01085$

$X_3 \approx 1.53222$

2.) a.) $f(x) = 12x^5 + 7x^{1/2} - \frac{5}{x} - 6$

antideriv: $12\frac{x^6}{6} + 7\frac{x^{3/2}}{3/2} - 5\ln|x| - 6x + C$

$= 2x^6 + \frac{14}{3}x^{3/2} - 5\ln|x| - 6x + C$

b.) $g(x) = 5\sec^2 x + 3\cos x - 4e^x + 5$

antideriv: $5\tan x + 3\sin x - 4e^x + 5x + C$

3.) a.) $\int_{\ln 3}^{\ln 6} 8e^x\,dx = 8e^x\Big]_{\ln 3}^{\ln 6} = 8\left[e^{\ln 6} - e^{\ln 3}\right]$ $e^{\ln 6} = 6$, $e^{\ln 3} = 3$

$= 8[6 - 3] = 8[3] = 24$

b.) $\int_1^8 \frac{x-1}{x^{2/3}}\,dx = \int_1^8 \left(\frac{x}{x^{2/3}} - \frac{1}{x^{2/3}}\right)dx$

$= \int_1^8 \left(x^{1/3} - x^{-2/3}\right)dx = \frac{x^{4/3}}{4/3} - \frac{x^{1/3}}{1/3}\Big]_1^8$

$= \left[\frac{3}{4}x^{4/3} - 3x^{1/3}\right]_1^8 = \left(\frac{3}{4}8^{4/3} - 3\cdot 8^{1/3}\right) - \left(\frac{3}{4}1^{4/3} - 3\cdot 1^{1/3}\right)$

$= \left(\frac{3}{4}(16) - 3(2)\right) - \left(\frac{3}{4}(1) - 3(1)\right)$

$= (12 - 6) - \left(\frac{3}{4} - 3\right) = 9 - \frac{3}{4} = 8\frac{1}{4} = 8.25$

4.) $h(x) = \int_0^{x^2} \sqrt{1 + r^3}\,dr$

$h'(x) = \frac{d\left(\int_0^{x^2}\sqrt{1+r^3}\,dr\right)}{dx} = \sqrt{1 + (x^2)^3} \cdot (2x)$

$= 2x\sqrt{1 + x^6}$

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5.) $\text{area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\Delta x)\cdot \Delta x$

$a = 0 \quad \Delta x = \frac{b-a}{n} = \frac{4-0}{n} = \frac{4}{n} \qquad f(x) = 4x - x^2$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(0 + i\cdot\frac{4}{n}\right)\cdot\frac{4}{n}$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{4i}{n}\right)\cdot\frac{4}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left[4\left(\frac{4i}{n}\right) - \left(\frac{4i}{n}\right)^2\right]\frac{4}{n}$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{16i}{n} - \frac{16i^2}{n^2}\right)\cdot\frac{4}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{64i}{n^2} - \frac{64i^2}{n^3}\right)$

$A = \lim_{n\to\infty} \left[\frac{64}{n^2} \sum_{i=1}^{n} i - \frac{64}{n^3} \sum_{i=1}^{n} i^2\right] = \lim_{n\to\infty} \left(\frac{64}{n^2}\left[\frac{n(n+1)}{2}\right] - \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]\right)$

$\frac{n(n+1)}{2} \qquad \frac{n(n+1)(2n+1)}{6}$

$A = \lim_{n\to\infty} \left(\frac{64}{2}\cdot\cancel{\frac{n^2+n}{n^2}}^{\,1} - \frac{64}{6}\cancel{\frac{2n^3+\dots}{n^3}}^{\,2}\right)$

$A = \lim_{n\to\infty} \left(32(1) - \frac{32}{3}(2)\right) = 32 - \frac{64}{3} = \frac{96}{3} - \frac{64}{3} = \frac{32}{3}$

check:

$$\left\{\begin{aligned}&\int_0^4 (4x - x^2)\,dx = \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4 = \left(2(4)^2 - \frac{4^3}{3}\right) - (0)\\ &= 32 - \frac{64}{3} = \frac{96}{3} - \frac{64}{3} = \frac{32}{3}\end{aligned}\right\}$$

6.) a.) $\int \sec^3 x \tan x\,dx = \int \sec^2 x\,(\sec x \tan x)\,dx$

let $u = \sec x$

$du = \sec x \tan x\,dx$

$\Rightarrow \int u^2\,du = \frac{u^3}{3} + C$

$= \frac{(\sec x)^3}{3} + C$

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b.) $\int_0^1 x^2(1+5x^3)^4\,dx$

let $u = 1 + 5x^3$

$du = 15x^2\,dx$

$= \frac{1}{15}\int_0^1 \underbrace{(1+5x^3)^4}_{u^4}\ \underbrace{x^2\,dx\,(15)}_{du}$

$\Rightarrow \frac{1}{15}\int u^4\,du = \frac{1}{15}\cdot\frac{u^5}{5}\Big] = \frac{1}{75}(u)^5\Big]$

$\frac{1}{75}(1+5x^3)^5\Big]_0^1 = \frac{1}{75}\left[(1+5(1)^3)^5 - (1+5(0)^3)^5\right]$

$= \frac{1}{75}\left[6^5 - 1^5\right] = \frac{1}{75}\left[7776 - 1\right] = \frac{1}{75}\left[7775\right]$

$\approx 103.\overline{666}$

7.) $\int x^4 \cdot \ln x \cdot dx$ "by parts" $\left(\int u\,dv = uv - \int v\,du\right)$

let $u = \ln x \qquad v = x^5/5$

$du = \frac{1}{x}dx \qquad dv = x^4\,dx$

$\int u\,dv = (\ln x)(x^4) - \int \frac{x^5}{5}\cdot\frac{1}{x}\,dx$

$= x^4 \ln x - \frac{1}{5}\int x^4\,dx$

$= x^4 \ln x - \frac{1}{5}\frac{x^5}{5} + C$

$= x^4 \ln x - \frac{1}{25}x^5 + C$