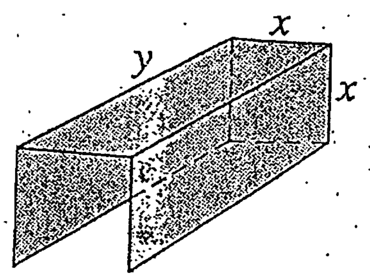
NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA 141 – 601 Test #3 Monday, November 5, 2007 Dr. J. Griggs

Show all work and answers on the plain white paper provided.

1.) Find $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$

2.) A net enclosure for golf practice is open at one end and has no floor (see figure). The volume of the enclosure is 83 1/3 cubic yards. Find the dimensions the require the least amount of netting (least surface area).



3.) Find the derivatives: a.) $y = e^{3x} \cdot \tan 5x$ b.) $y = \sqrt[4]{\frac{x^3+1}{x^3-1}}$

4.) Find $\frac{dy}{dx}$: $x^2 + 5xy - y^4 = 4$

5.) Find y' : $y = (\ln x)^{\cos x}$

6.) A plane flying horizontally at an altitude of 1 mile and a speed of 300 miles per hour passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is two miles away from the station.

7.) Find the linear approximation for the function $y = \ln x$ *at* $a = 1$. Use it to find (approximate) ln 1.3.

8.) Find the critical points (t and f(t)) for the function: $f(t) = 3t^4 + 4t^3 - 6t^2$, and use them to determine where the function is increasing and decreasing.

9.) Find the absolute maximum and absolute minimum values of the function on the given closed interval: $f(x) = 2x + \frac{72}{x}$ *on* $[1, 10]$

MA141-6.1 Test #3 11/5/07 J. Griggs

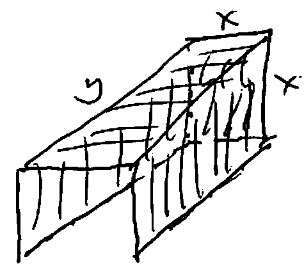
(11 points each)

1.) $\lim_{x\to 0} \dfrac{e^x + e^{-x} - 2}{1 - \cos 2x} = \dfrac{0}{0} \overset{\text{L'HOP}}{\Longrightarrow} \lim_{x\to 0} \dfrac{e^x - e^{-x}}{(\sin 2x)(2)}$

$\lim_{x\to 0} \dfrac{e^x - e^{-x}}{(\sin 2x)(2)} = \dfrac{0}{0} \overset{\text{L'HOP}}{\Longrightarrow} \lim_{x\to 0} \dfrac{e^x + e^{-x}}{2[(\cos 2x)(2)]} = \dfrac{2}{4}$

$= \dfrac{1}{2}$

2.) $V = \boxed{x^2 \cdot y = 83\tfrac{1}{3}} \Rightarrow y = \dfrac{83\frac{1}{3}}{x^2}$



S = surface area $= 3xy + x^2$

(no front; no bottom)

$S = 3xy + x^2 \qquad y = \dfrac{83\frac{1}{3}}{x^2}$

$S = 3x \cdot \dfrac{83\frac{1}{3}}{x^2} + x^2 = \dfrac{250}{x} + x^2 = 250x^{-1} + x^2$

$S' = -1 \cdot 250x^{-2} + 2x = \dfrac{-250}{x^2} + 2x = 0$

$2x = \dfrac{250}{x^2} \qquad 2x^3 = 250 \qquad x^3 = 125 \qquad \boxed{x = 5 \text{ yd}}$

$y = \dfrac{83\frac{1}{3}}{x^2} = \dfrac{83\frac{1}{3}}{25} \approx \boxed{3\tfrac{1}{3} \text{ yd}}$

$\boxed{S'' = \dfrac{500}{x^3} + 2 = +}$ (at $x=5$) $\therefore$ concave up

$\therefore$ <u>min</u> surface area

3.) a.) $y = e^{3x} \cdot \tan 5x$

$y' = e^{3x} \cdot [(\sec^2 5x) \cdot 5] + \tan 5x \cdot [(e^{3x}) \cdot 3]$

$y' = e^{3x} [5 \sec^2 5x + 3 \tan 5x]$

b.) $y = \sqrt[4]{\frac{x^3+1}{x^3-1}} = \left(\frac{x^3+1}{x^3-1}\right)^{1/4}$

$y' = \frac{1}{4}\left(\frac{x^3+1}{x^3-1}\right)^{-3/4} \cdot \left[\frac{(x^3-1) \cdot 3x^2 - (x^3+1) \cdot 3x^2}{(x^3-1)^2}\right]$

$y' = \frac{1}{4} \frac{(x^3+1)^{-3/4}}{(x^3-1)^{-3/4}} \left[\frac{\cancel{3x^5} - 3x^2 - \cancel{3x^5} - 3x^2}{(x^3-1)^2}\right]$

$\boxed{y' = \frac{1}{4} \frac{(x^3+1)^{-3/4}}{(x^3-1)^{-3/4}} \cdot \left[\frac{-6x^2}{(x^3-1)^2}\right]}$ this answer is sufficient

$y' = \frac{(-6x^2)(x^3+1)^{-3/4}}{4(x^3-1)^{-3/4}(x^3-1)^2} = \boxed{\frac{-3x^2(x^3+1)^{-3/4}}{2(x^3-1)^{5/4}}}$

4.) $x^2 + 5xy - y^4 = 4$ find dy/dx:

$2x + \left[(5x) \cdot \frac{dy}{dx} + y \cdot (5)\right] - 4y^3 \cdot \frac{dy}{dx} = 0$

$2x + 5y = 4y^3 \frac{dy}{dx} - 5x \frac{dy}{dx}$

$2x + 5y = \frac{dy}{dx}\left[4y^3 - 5x\right]$

$\boxed{\frac{2x+5y}{4y^3-5x} = \frac{dy}{dx}}$

5.) $y = (\ln x)^{\cos x}$

$\ln y = \ln (\ln x)^{\cos x}$ $\qquad d(\ln u) = \frac{1}{u} \cdot du$

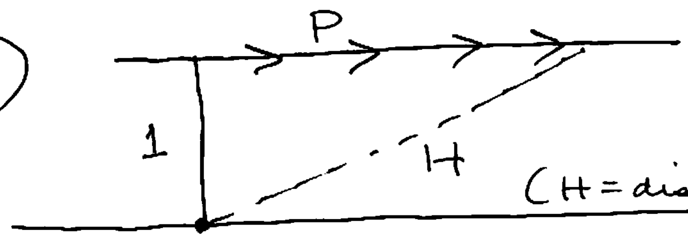
$\ln y = (\cos x)\left[\ln(\ln x)\right]$

$\frac{1}{y} \cdot \frac{dy}{dx} = (\cos x)\left[\frac{1}{\ln x} \cdot \frac{1}{x}\right] + \left[\ln(\ln x)\right] \cdot (-\sin x)$

$\frac{dy}{dx} = y\left[\frac{\cos x}{x \ln x} - (\sin x)\left[\ln(\ln x)\right]\right]$

$\frac{dy}{dx} = (\ln x)^{\cos x}\left[\frac{\cos x}{x \cdot \ln x} - (\sin x)\left[\ln(\ln x)\right]\right]$

6.)



(H = distance from plane to station)

$\frac{dP}{dt} = 300$ mi/hr

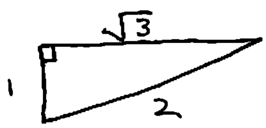
$\frac{dH}{dt} = ?$

$1^2 + P^2 = H^2$

$1 + P^2 = H^2$

$2P \cdot \frac{dP}{dt} = 2H \cdot \frac{dH}{dt}$

at this moment: (plane to station = 2 ⇒ H = 2)



$P = \sqrt{3}$

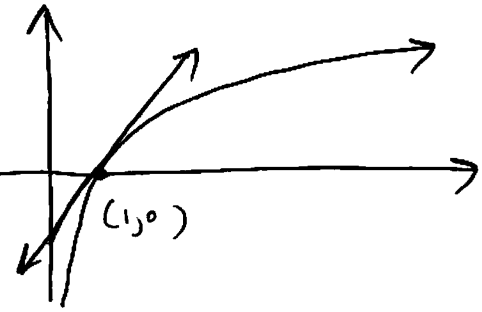
$H = 2$

$2P \cdot \frac{dP}{dt} = 2H \cdot \frac{dH}{dt}$

$2(\sqrt{3}) \cdot (300) = 2(2) \cdot \frac{dH}{dt}$

$\frac{2(\sqrt{3})(300)}{4} = \boxed{\frac{dH}{dt} \approx 259.8 \ \frac{mi}{hr}}$

7.) $y = \ln x$ at $a = 1$



$m = y' = \frac{1}{x}$ y' at 1: $\frac{1}{1} = 1$

$y - 0 = \frac{1}{1}(x-1)$

$[y - f(a) = f'(a)(x-a)]$

$y = x - 1$

$\ln 1.3 \approx 1.3 - 1 = .3$

8.) critical points for $f(t) = 3t^4 + 4t^3 - 6t^2$

$f'(t) = 3(4t^3) + 4(3t^2) - 6(2t) =$ m tan

$f'(t) = 12t^3 + 12t^2 - 12t = 12t(t^2 + t - 1) = 0$

$12t^2 = 0 \Rightarrow t = 0 \Rightarrow (0, f(0)) = (0, 0)$

$t^2 + t - 1 = 0$ $\qquad f(0) = 3(0)^4 + 4(0)^3 - 6(0)^2 = 0$

$t = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$ $\to \frac{-1-\sqrt{2}}{2} \approx -1.618$ $\to \frac{-1+\sqrt{3}}{2} \approx .618$

$f(.618) = 3(.618)^4 + 4(.618)^3 - 6(.618)^2 \approx -.9098$

$f(-1.618) = 3(-1.618)^4 + 4(-1.618)^3 - 6(-1.618)^2 \approx -12.090$

$(.618, -.9098)$ & $(-1.618, -12.090)$

?	?	?	?
DECR	INCR	DECR.	INCR

-1.618 $\qquad 0$ $\qquad .618$

$f'(-2) = -$ $\quad f'(-1) = +$ $\quad f'(.3) = -$ $\quad f'(1) = +$

INCR: $(-1.618, 0) \cup (.618, +\infty)$

DECR: $(-\infty, -1.618) \cup (0, .618)$

9.) absol max / min $f(x) = 2x + \frac{72}{x}$ on $[1, 10]$

$f'(x) = 2 + 72(-1x^{-2}) = 2 - \frac{72}{x^2} = 0$

$2 = \frac{72}{x^2} \quad 2x^2 = 72 \quad x^2 = 36 \quad x = \pm 6$

(−6 outside of interval)

$x = 6$

$f(6) = 2(6) + \frac{72}{6} = 12 + 12 = 24 \Rightarrow (6, 24)$

endpts: $(1, f(1))$ and $(10, f(10))$

$f(1) = 2(1) + \frac{72}{1} = 74 \Rightarrow (1, 74)$

$f(10) = 2(10) + \frac{72}{10} = 20 + 7.2 \Rightarrow 27.2 \Rightarrow (10, 27.2)$

$(6, \underline{\underline{24}})$ is absolute min.

$(1, \underline{\underline{74}})$ is absolute max