NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA141-601 Test #2 Wednesday, October ~~9~~, 2007 Dr. John Griggs
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Please put all work and answers on the plain white paper provided. Do not use the graphics mode of your calculator when the question asks YOU to graph.

1.) Give a full explanation as to why there is a root of the function $f(x) = x - \cos x$ in the interval (0,1).

2.) Find the equation of the tangent line to the curve at the given point:
$y = x^3 - 5x \qquad (-1,3)$

3.) Find y' using the **<u>definition of derivative</u>**: $y = \dfrac{3x-1}{x+4}$

4.) Find the vertical and horizontal asymptotes of $y = \dfrac{2x^2+x-1}{x^2+x-2}$

5.) Find the derivative and simplify:

a.) $y = 4x^3 + \dfrac{2}{x} - \sqrt{x} + 5x^{\frac{-1}{3}} - 12$

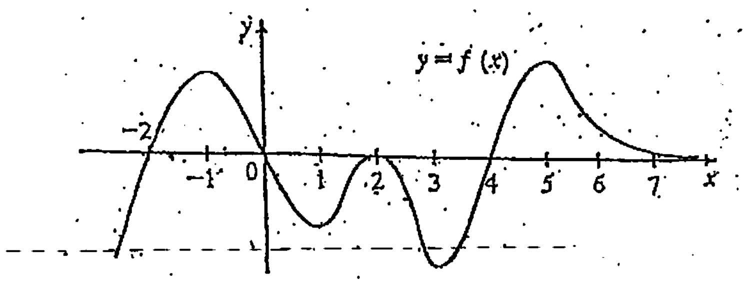
b.) $y = 8x^3 e^x$

6.) Find the derivative and simplify:

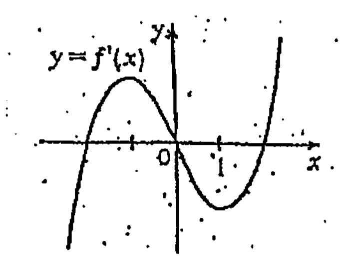
a.) $f(x) = \dfrac{1+\sin x}{x+\cos x}$

b.) $f(x) = \sec x \tan x$

7.) Use the graph of $y = f(x)$ to find the graph of $y = f'(x)$:



8.) Use the graph of $y = f'(x)$ to find the graph of $y = f(x)$: $f(0) = 2$



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SOLUTIONS (12.5 pts per problem)

1.) $f(x) = x - \cos x$

$f(0) = 0 - \cos 0$

$f(0) = 0 - 1 = -1$ (NEG)

$f(1) = 1 - \cos 1$ ← (1 radian)

$f(1) = 1 - .5403$

$f(1) = .46$ (POS)

$f(x)$ is CONTINUOUS, since $f(0)$ is NEG & $f(1)$ is POS, then by the Intermediate Value Theorem, $f(x)$ must be 0 in (0,1)

2.) $y' = 3x^2 - 5 = f'(x)$

$f'(-1) = 3(-1)^2 - 5 = 3 - 5 = -2$

$m = -2$

$y - y_1 = m(x - x_1)$

$y - 3 = -2(x - (-1))$

$y - 3 = -2(x+1)$

or

$y = -2x - 2 + 3$

$y = -2x + 1$

3.) DEF. OF DERIV. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{3(x+h)-1}{(x+h)+4} - \dfrac{3x-1}{x+4}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{(3x+3h-1)(x+4)}{(x+h+4)(x+4)} - \dfrac{(3x-1)}{(x+4)} \cdot \dfrac{(x+h+4)}{(x+h+4)}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{(3x+3h-1)(x+4) - (3x-1)(x+h+4)}{(x+h+4)(x+4)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2}+\cancel{3xh}-\cancel{x}+\cancel{12x}+12h-\cancel{4}-\cancel{3x^2}+\cancel{x}-\cancel{3xh}+h-\cancel{12x}+\cancel{4}}{(x+h+4)(x+4)\cdot h}$$

$$= \lim_{h\to 0} \frac{13\cancel{h}^{1}}{(x+h+4)(x+4)\cdot \cancel{h}_{1}} = \lim_{h\to 0} \frac{13}{(x+h+4)(x+4)}$$

$$= \frac{13}{(x+4)(x+4)} = \frac{13}{(x+4)^2}$$

[check: $\frac{d\left(\frac{3x-1}{x+4}\right)}{dx} = \frac{(x+4)(3)-(3x-1)(1)}{(x+4)^2} = \frac{\cancel{3x}+12-\cancel{3x}+1}{(x+4)^2} = \frac{13}{(x+4)^2}$]

) $y = \frac{2x^2+x-1}{x^2+x-2} = \frac{(2x-1)(x+1)}{(x+2)(x-1)}$

vertical asymptote: $x+2=0$ $\boxed{x=-2}$ $x-1=0$ $\boxed{x=1}$

(function undefined there; $f(x)\to\infty$)

horizontal asymptote:

$$\lim_{x\to\infty} \frac{2x^2+x-1}{x^2+x-2} = \lim_{x\to\infty} \frac{\frac{2x^2}{x^2}+\frac{x}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{x}{x^2}-\frac{2}{x^2}}$$

$$= \lim \frac{2+\cancel{\frac{1}{x}}^{\to 0}-\cancel{\frac{1}{x^2}}^{\to 0}}{1+\cancel{\frac{1}{x}}_{\to 0}-\cancel{\frac{2}{x^2}}_{\to 0}} = 2 \quad \boxed{y=2} \text{ is horiz. asymp.}$$

5.) a.) $y' = 4(3\cdot x^2) + 2(-1x^{-2}) - \frac{1}{2}(x^{-1/2}) + 5\left(\frac{-1}{3}x^{-4/3}\right)$

$$y' = 12x^2 - 2x^{-2} - \frac{1}{2}x^{-1/2} - \frac{5}{3}x^{-4/3}$$

b.) $y = 8x^3 \cdot e^x$

$y' = 8x^3 \cdot (e^x) + (e^x) \cdot 8(3x^2)$

$y' = e^x\left[8x^3 + 24x^2\right] = 8x^2 e^x (x+3) = y'$

6.) a.) $f(x) = \dfrac{1+\sin x}{x+\cos x}$

$f'(x) = \dfrac{(x+\cos x)(\cos x) - (1+\sin x)(1-\sin x)}{(x+\cos x)^2}$

$f'(x) = \dfrac{x\cos x + \cos^2 x - \left[1-\sin^2 x\right]}{(x+\cos x)^2}$

$f'(x) = \dfrac{x\cos x + \cancel{\cos^2 x} - \cancel{\cos^2 x}}{(x+\cos x)^2}$

$f'(x) = \dfrac{x\cos x}{(x+\cos x)^2}$

b.) $f(x) = (\sec x)(\tan x)$

$f'(x) = (\sec x)(\sec^2 x) + (\tan x)(\sec x \tan x)$

$f'(x) = \sec^3 x + \sec x \tan^2 x$

$1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$f'(x) = \sec x\left[\sec^2 x + \tan^2 x\right]$

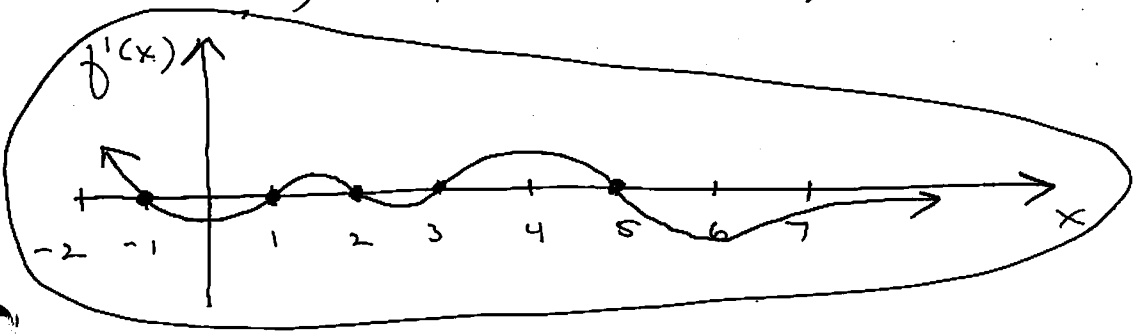
$f'(x) = \sec x\left[\sec^2 x + \sec^2 x - 1\right]$

$f'(x) = \sec x\left[2\sec^2 x - 1\right]$

7.) $\begin{cases} f(x) \text{ "flat"} \iff f'(x) = 0 \\ \text{at } x = -1, 1, 2, 3, 5 \end{cases}$

$\begin{cases} f(x) \text{ INCR} \iff f'(x) = + \\ (-\infty, -1) \text{ \& } (1, 2) \text{ \& } (3, 5) \end{cases}$

$\begin{cases} f(x) \text{ DECR} \iff f'(x) = - \\ (-1, 1) \text{ \& } (2, 3) \text{ \& } (5, +\infty) \end{cases}$



8.) $\begin{cases} f'(x) = 0 \iff f(x) \text{ "flat" there} \\ \text{at } x = -2, 0, 2 \end{cases}$

$\begin{cases} f'(x) = + \iff f(x) \text{ INCR} \\ (-2, 0) \text{ \& } (2, +\infty) \end{cases}$

$\begin{cases} f'(x) = - \iff f(x) \text{ DECR} \\ (-\infty, -2) \text{ \& } (0, 2) \end{cases}$

& $f(0) = 2$