NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA 141 – 601 Test #1 Fall '07 Monday, September 17, 2007 J. Griggs

Please put all work and answers on the white paper provided. Do not use a calculator that **does** calculus; no formula sheets or summary sheets allowed. Simplify **completely**.

1.) Find $f^{-1}(x)$, if $f(x) = \dfrac{x+1}{2x+1}$

Find the domain of $f(x)$ and the domain of $f^{-1}(x)$.

2. Graph $f(x) = \begin{cases} 2x-1 & x \geq 2 \\ \sqrt{4-x^2} & -2 < x < 2 \\ \dfrac{1}{x} & x \leq -2 \end{cases}$

3.) Solve for x:

a.) $x = \log_5 14$

b.) $e^{2x+1} = 31$

4.) Determine if the function is even or odd or neither - explain why:

a.) $y = x^3 - x^7$

b.) $y = 3\cos 2x$

5.) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases. Eliminate the parameter to find a Cartesian equation of the curve.

$x = t^2 \qquad y = t^3 \qquad (-3 \leq t \leq 3)$

6.) O^o Celsius (C) is equivalent to 32^0 Fahrenheit (F). 100^0 Celsius (C) is equivalent to 212^0 Fahrenheit (F) . The relationship between the two scales is linear ; find the linear function solved for F in terms of C. Use it to convert 37.1^0(C) to Fahrenheit.

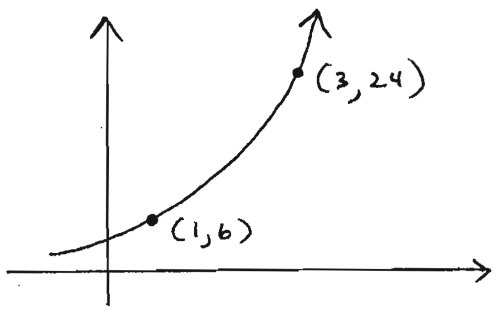
7.) For f(x), find $\dfrac{f(x+h)-f(x)}{h}$:

a.) $f(x)=\dfrac{x}{x+1}$

b.) $f(x)=3x^2-4x+11$

8.) If $f(x)=\sqrt{x-1}$ and $g(x)=x^2+2$, find $f\circ g,\ g\circ f,\ and\ \ g\circ g.$

9.) Find the exponential function $f(x)=Ca^x$ whose graph is given below:



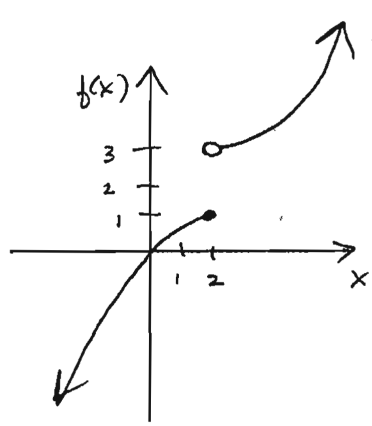
10.) Find the equation of the circle whose center is (-1,5) and passes through (-4,-6). (answer in $x^2+y^2+Dx+Ey+F=0$ form)

11.) Evaluate the following limits:

a.) $\lim\limits_{x\to 4}\dfrac{x^2-4x}{x^2-3x-4}$

b.) $\lim\limits_{h\to 0}\dfrac{(3+h)^{-1}-3^{-1}}{h}$

c.) $\lim\limits_{x\to 2^+} f(x)$



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SOLUTIONS: (9 points each)

1) $f(x) = \dfrac{x+1}{2x+1}$ find $f^{-1}(x)$: $y = \dfrac{x+1}{2x+1}$

$\dfrac{x}{1} = \dfrac{y+1}{2y+1}$

$x(2y+1) = y+1$

$2xy + x = y+1$

$2xy - y = 1-x$

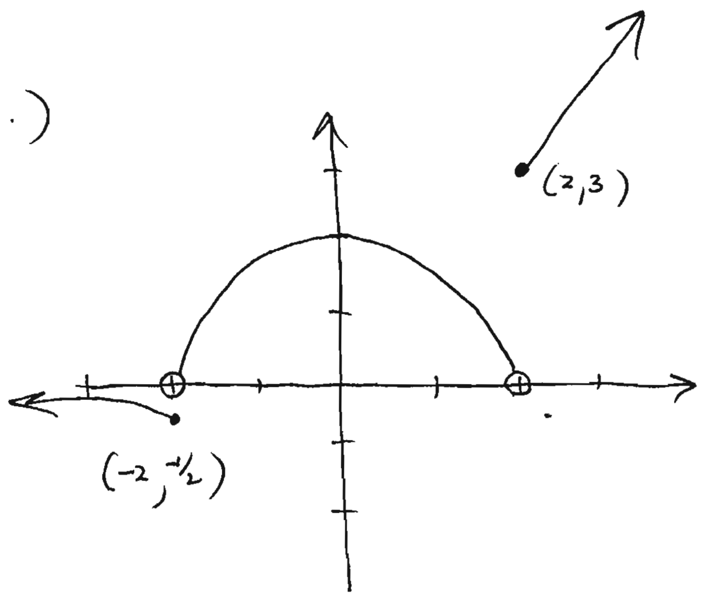
$y(2x-1) = 1-x$

$y = \dfrac{1-x}{2x-1} = f^{-1}(x)$

domain of $f(x)$: $2x+1 \neq 0$, $2x \neq -1$, $x \neq -\frac{1}{2}$ $\quad \mathbb{R} - \{-\frac{1}{2}\}$

domain of $f^{-1}(x)$: $2x-1 \neq 0$, $2x \neq 1$, $x \neq \frac{1}{2}$ $\quad \mathbb{R} - \{\frac{1}{2}\}$

2.)



$y = 2x-1$ $(x \geq 2)$

x	y
2	3
3	5
4	7

line

$y = \sqrt{4-x^2}$ $(-2 < x < 2)$

x	y	
-2	0	delete
-1	$\sqrt{3}$	
0	2	
1	$\sqrt{3}$	
2	0	delete

semi circle

$y = \frac{1}{x}$ $(x \leq -2)$

x	y
-2	$-\frac{1}{2}$
-3	$-\frac{1}{3}$
-4	$-\frac{1}{4}$

hyperbola

3.) Solve for x:

a.) $x = \log_5 14 \Rightarrow 5^x = 14$

$$\ln 5^x = \ln 14$$

$$x(\ln 5) = \ln 14$$

$$x = \frac{\ln 14}{\ln 5} \approx 1.6397$$

$\left(\text{check: } 5^{1.6397} \doteq 14\right)$

b.) $e^{2x+1} = 31$

$$\ln(e^{2x+1}) = \ln 31$$

$$(2x+1)\ln e = \ln 31 \qquad (\ln e = 1)$$

$$2x+1 = \ln 31$$

$$2x = \ln 31 - 1$$

$$x = \frac{\ln 31 - 1}{2} \approx 1.21699$$

$\left(\text{check: } e^{2(1.21699)+1} \doteq 31\right)$

4.) odd? even? neither?

a.) ODD because $f(-x) = -f(x)$

$$f(-x) = (-x)^3 - (-x)^7$$

$$= -x^3 + x^7$$

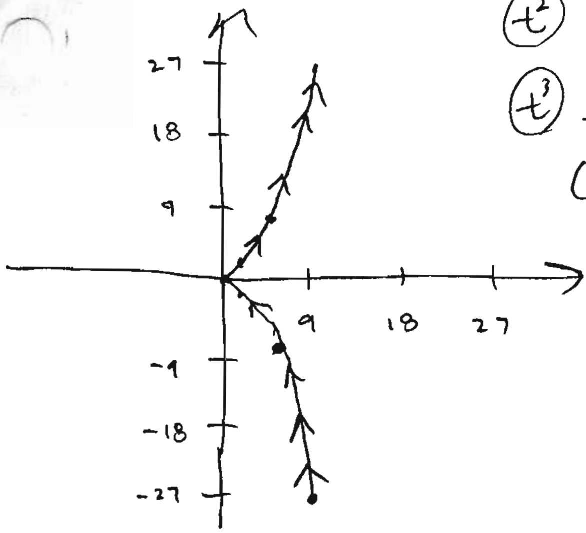
$$= -(x^3 + x^7) = -f(x)$$

b.) EVEN because $f(-x) = f(x)$

$$f(-x) = 3\cos 2(-x) = 3\cos(-2x)$$

$$= 3\cos 2x = f(x)$$

5.) $x = t^2 \quad y = t^3$



t	-3	-2	-1	0	1	2	3
(t^2) x	9	4	1	0	1	4	9
(t^3) y	-27	-8	-1	0	1	8	27

$(9,-27);\ (4,-8);\ (+1,-1);\ (0,0);\ \overset{(1,1)}{}\ (4,8);\ (9,27)$

$x = t^2 \quad \sqrt{x} = t \quad y = t^3 = (\sqrt{x})^3 = x^{3/2}$

$y = x^{3/2}$ or $x = y^{2/3}$

6.) (C, F): $(0, 32)$ & $(100, 212)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} \Rightarrow \dfrac{F_2 - F_1}{C_2 - C_1} = \dfrac{212-32}{100-0} = \dfrac{180}{100} = \dfrac{18}{10} = \dfrac{9}{5}$

$y - y_1 = m(x - x_1) \Rightarrow F - F_1 = m(C - C_1)$

$F - 32 = \frac{9}{5}(C - 0)$

$F = \frac{9}{5}C + 32$

$C = 37.1 \quad F = ?$

$F = \frac{9}{5}(37.1) + 32 = 98.78°F$

7.) a.) $f(x) = \dfrac{x}{x+1}$. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\left[\dfrac{(x+h)}{(x+h)+1}\right] - \left[\dfrac{x}{x+1}\right]}{h}$

$= \dfrac{\dfrac{(x+h)(x+1)}{(x+h+1)(x+1)} - \dfrac{(x)(x+h+1)}{(x+1)(x+h+1)}}{h} = \dfrac{(x+h)(x+1) - (x)(x+h+1)}{(x+h+1)(x+1)} \div$

$= \dfrac{(\cancel{x^2} + \cancel{xh} + h + \cancel{x}) - (\cancel{x^2} + \cancel{xh} + \cancel{x})}{(x+h+1)(x+1)\cdot h} = \dfrac{\cancel{h}}{(x+h+1)(x+1)\cancel{h}} \ (h \neq 0) = \dfrac{1}{(x+h+1)(x+1)}$

7. b.) $f(x) = 3x^2 - 4x + 11$ $\qquad \dfrac{f(x+h) - f(x)}{h}$

$$= \frac{[3(x+h)^2 - 4(x+h) + 11] - [3x^2 - 4x + 11]}{h}$$

$$= \frac{3(x^2 + 2xh + h^2) - 4x - 4h + 11 - 3x^2 + 4x - 11}{h}$$

$$= \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{-4x} - 4h \cancel{+11} \cancel{-3x^2} \cancel{+4x} \cancel{-11}}{h}$$

$$= \frac{\cancel{h}^{1}(6x + 3h - 4)}{\cancel{h}_{1}} = 6x + 3h - 4$$

$(h \neq 0)$

8.) $f(x) = \sqrt{x-1} \quad g(x) = x^2 + 2$

$\underline{(f \circ g)(x)} = f(g(x)) = f(x^2+2) = \sqrt{(x^2+2) - 1} = \sqrt{x^2+1}$

$\underline{(g \circ f)(x)} = g(f(x)) = g(\sqrt{x-1}) = (\sqrt{x-1})^2 + 2 = (x-1) + 2 = x+1$

$(g \circ g)(x) = g(g(x)) = g(x^2+2) = (x^2+2)^2 + 2 = x^4 + 4x^2 + 6$

9.) $f(x) = c \cdot a^x$

$(1,6): \; 6 = c \cdot a \qquad a = \dfrac{6}{c}$

$(3,24): \; 24 = c \cdot a^3$

$24 = c \cdot \left(\dfrac{6}{c}\right)^3 = c \cdot \dfrac{216}{c^3} = \dfrac{216}{c^2}$

$24 = \dfrac{216}{c^2} \qquad 24c^2 = 216 \qquad c^2 = \dfrac{216}{24} = 9$

$c = 3 \quad (c > 0)$

$a = \dfrac{6}{c} = \dfrac{6}{3} = 2 \implies f(x) = 3 \cdot 2^x$

10.) center $(-1,5)$ thru $(-4,-6)$

$r =$ dist from $(-1,5)$ to $(-4,-6)$

$$r = \sqrt{(-1-(-4))^2 + (5-(-6))^2}$$

$$r = \sqrt{9 + 121} = \sqrt{130}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x+1)^2 + (y-5)^2 = (\sqrt{130})^2$$

$$x^2+2x+1+y^2-10y+25 = 130$$

$$x^2+y^2+2x-10y-104 = 0$$

11.) a.) $\lim\limits_{x\to 4} \dfrac{x^2-4x}{x^2-3x-4} = \lim\limits_{x\to 4} \dfrac{x\cancel{(x-4)}}{\cancel{(x-4)}(x+1)}$ $(x \neq 4)$

$$= \lim_{x\to 4} \frac{x}{x+1} = \frac{4}{4+1} = \frac{4}{5}$$

b.) $\lim\limits_{h\to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h} = \lim\limits_{h\to 0} \dfrac{\frac{1}{3+h} - \frac{1}{3}}{h}$

$$= \lim_{h\to 0} \frac{\frac{1(3)}{(3+h)(3)} - \frac{1(3+h)}{3(3+h)}}{h} = \lim_{h\to 0} \frac{3-(3+h)}{(3+h)(3)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3}-\cancel{3}-h}{(3+h)(3)(h)} = \lim_{h\to 0} \frac{-\cancel{h}}{(3+h)(3)\cancel{h}} = \lim_{h\to 0} \frac{-1}{(3+h)(3)}$$

$$= \frac{-1}{9}$$

c.) $\lim\limits_{x\to 2^+} f(x) = 3$

(from the right)