NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA121-002 Test #4 Form B Thursday, April 17, 2008 Dr. J. Griggs

Please put all work and solutions in the stamped blue book provided. Begin each new problem on a new page (the back of the sheet is OK). Put your name, form of test (A or B), row number, and seat number on the outside of the blue book (preferably at the top). You may keep the test copy itself. Be sure to include proper units on the answers when appropriate.

1.) a.) Find the indefinite integral: $\int 8t^2(7t^3+3)^5\,dt$

b.) Solve for y: $\frac{dy}{dx} = 4x^3y$

2.) Find the area of the region bounded by the two curves:
$y = 8 - x^2 \text{ and } y = 5 - 2x$

3.) Find the equilibrium point, the consumer's surplus (labeled CS), and the producer's surplus (labeled PS) using the given supply and demand curves:
$D(x) = -.36x + 9 \quad \text{and} \quad S(x) = .14x + 2$

4.) In 2001, the demand for aluminum ore was 137.5 million tons and the demand was growing exponentially at the rate of 3.7% per year. If the demand continues to grow at this rate, how many tons of aluminum ore will the world use from 2001 to 2020?

5.) Evaluate the improper integral: $\int_0^\infty \frac{6}{x^3}\,dx$

Is the integral convergent or divergent?

6.) Find the volume of the solid generated when the region bounded by $y = \sqrt{2+x}$, $x = 3$, and $x = 9$ is revolved about the x-axis.

121-002 Test #4 form B 4/17/08 J. [illegible]

(4·6 pts each)

1) a.) $\int 8t^2(7t^3+3)^5\,dt = \frac{1}{21}\cdot 8\int (7t^3+3)^5\cdot \underbrace{t\cdot dt\,(21)}$

let $u = 7t^3+3$

$du = 21t^2\,dt$

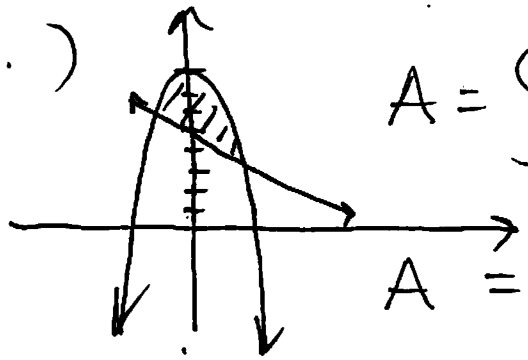
$= \frac{8}{21}\int u^5\,du = \frac{8}{21}\,\frac{u^6}{6}+C = \frac{4}{63}(7t^3+3)^6+C$

b.) $\frac{dy}{dx} = 4x^3y \qquad \frac{1}{y}\cancel{dx}\frac{dy}{\cancel{dx}} = 4x^3\cancel{y}\cdot dx\cdot\frac{1}{\cancel{y}}$

$\int \frac{1}{y}\,dy = \int 4x^3\,dx \qquad \ln y = x^4+C$

$e^{\ln y} = e^{x^4+C} \qquad y = e^{x^4}\cdot e^{c}$ or $y = B\cdot e^{x^4}$

2.)



$A = \int_{-1}^{3}\left[(8-x^2)-(5-2x)\right]dx$

$A = \int_{-1}^{3}(3-x^2+2x)\,dx$

pts of int:

$8-x^2 = 5-2x$

$0 = x^2-2x-3$

$0 = (x-3)(x+1)$

$x = 3, -1$

$= \left[3x - \frac{x^3}{3} + \cancel{2}\frac{x^2}{\cancel{2}}\right]_{-1}^{3}$

$= \left[\left(3(3) - \frac{3^3}{3} + (3)^2\right) - \left(3(-1) - \frac{(-1)^3}{3} + (-1)^2\right)\right]$

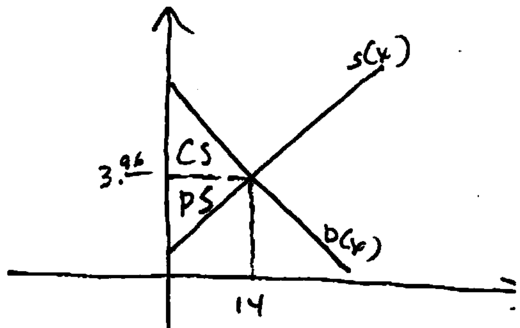
$= \left[(\cancel{9} - \cancel{9} + 9) - (-3 + \frac{1}{3} + 1)\right]$

$= 9+3-\frac{1}{3}-1 = 11-\frac{1}{3} = \frac{32}{3}$

(page 2)

3.) eq. pt: $(D(x)=S(x))$

$$
\begin{aligned}
-\cancel{.36x}+9 &= .14x+\cancel{2}\\
+\cancel{.36x}-2 &\quad +.36x-\cancel{2}\\
\hline
\frac{7}{.5} &= \frac{.50x}{.5}\\
14 &= x
\end{aligned}
$$



$$D(14)=S(14)=-.36(14)+9=.14(14)+2=\$3.96$$

CS:

$$
\begin{aligned}
CS &= \int_0^{14} D(x)\,dx - (14)(\$3.96)\\
&= \int_0^{14}(-.36x+9)\,dx - (14)(3.96)\\
&= \left[-\frac{.36x^2}{2}+9x\right]_0^{14} - (55.44)\\
&= \left[-.18x^2+9x\right]_0^{14} - (55.44)\\
&= \left[-.18(14)^2+9(14)-0\right] - (55.44)\\
CS &= 90.72-55.44 = \$35.28
\end{aligned}
$$

PS:

$$
\begin{aligned}
PS &= (14)(\$3.96) - \int_0^{14} s(x)\,dx\\
&= (14)(3.96) - \int_0^{14}(.14x+2)\,dx\\
&= (55.44) - \left[\frac{.14x^2}{2}+2x\right]_0^{14}\\
&= (55.44) - \left[.07x^2+2x\right]_0^{14}\\
&= (55.44) - \left[.07(14)^2+2(14)-0\right]\\
&= (55.44) - 41.72\\
PS &= \$13.72
\end{aligned}
$$

(page 3)

4.) $\int_0^{19} 137.5\, e^{.037t}\, dt = 137.5 \int_0^{19} e^{.037t}\, dt$

$= 137.5 \dfrac{1}{.037}\left[e^{.037t}\right]_0^{19} = \dfrac{137.5}{.037}\left[e^{.037(19)} - e^{.037(0)}\right]$

$= \dfrac{137.5}{.037}\left[2.0198 - 1\right] =$ 3789.81 million tons

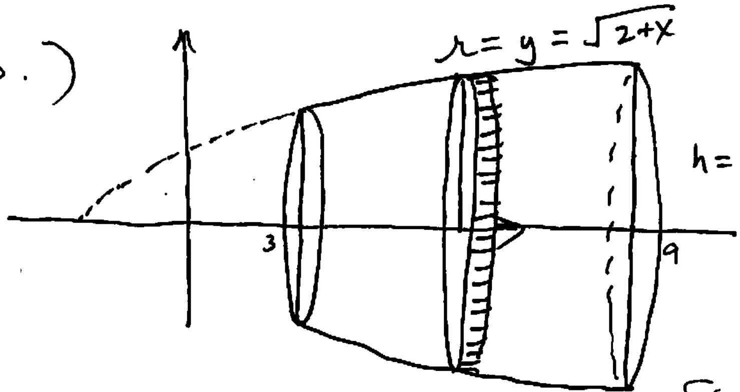
5.) $\int_1^{\infty} \dfrac{6}{x^3}\, dx = \lim_{A\to\infty} \int_1^{A} 6x^{-3}\, dx$

$= 6 \cdot \lim_{A\to\infty} \int_1^{A} x^{-3}\, dx = 6 \cdot \lim_{A\to\infty} \left[\dfrac{x^{-2}}{-2}\right]_1^{A}$

$= -3 \cdot \lim_{A\to\infty} \left[\dfrac{1}{x^2}\right]_1^{A} = -3 \cdot \lim_{A\to\infty} \left[\cancel{\dfrac{1}{A^2}}^{\,0} - \dfrac{1}{1^2}\right]$

$= -3(0-1) = 3$ $\therefore$ integral converges

6.)



$r = y = \sqrt{2+x}$

$h = \Delta x = dx$

$V = \int_3^9 \pi r^2 h$

$= \pi \int_3^9 (\sqrt{2+x})^2\, dx$

$= \pi \int_3^9 (2+x)\, dx$

$= \pi \left[2x + \dfrac{x^2}{2}\right]_3^9$

$= \pi \left[\left(2(9) + \dfrac{9^2}{2}\right) - \left(2(3) + \dfrac{3^2}{2}\right)\right]$

$= \pi \left[\left(18 + \dfrac{81}{2}\right) - \left(6 + \dfrac{9}{2}\right)\right]$

$= \pi \left[12 + \dfrac{72}{2}\right] = \pi\left[12 + 36\right] = 48\pi$

≈ 150.796