NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA121-004 Test #4 Form B Friday, April 17, 2009 Dr. J. Griggs

Please put all work and solutions in the stamped blue book provided. Begin each new problem on a new page (the back of the sheet is OK). Put your **name, form of test (A or B), row number, and seat number** on the outside of the blue book (preferably at the top). You may keep the test copy itself. Be sure to include proper units on the answers when appropriate.

1.) a.) Find the indefinite integral: $\int \frac{7x^2}{(5+2x^3)^4}\,dx$

b.) Solve for y: $\frac{dy}{dx} = \frac{4x}{y} \quad y = 1 \;\; when \;\; x = 3$

2.) Find the area of the region bounded by the two curves:

$$y = x^2 - 4x + 5 \quad and \quad y = -x^2 + 2x + 5$$

3.) Find the equilibrium point, the consumer surplus (labeled CS), and the producer surplus (labeled PS) using the given supply and demand curves:

$$D(x) = -.39x + 10 \quad and \quad S(x) = .11x + 4$$

4.) Find the future value of a continuous money flow in which \$3000 per year is being invested at 2.5% compounded continuously for 20 years.

5.) Evaluate the improper integral: $\int_0^\infty 3e^{-5x}\,dx$

Is the integral convergent or divergent?

6.) Find the volume of the solid generated when the region bounded by

$y = \frac{2}{\sqrt{x}}$, $x = 2$, and $x = 4$ is revolved about the x-axis.

Bonus (5 pts): A car with constant acceleration goes from 0 to 60 mph in ½ min. How far does the car travel during that time.

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(16 points per problem)

1.)
a.) $\int \frac{7x^2}{(5+2x^3)^4}\,dx = \int (5+2x^3)^{-4} \cdot 7x^2\,dx$

let $u = 5+2x^3$

$du = 6x^2\,dx$

$= 7 \cdot \frac{1}{6} \int \underbrace{(5+2x^3)^{-4}}_{u^{-4}} \cdot \underbrace{x^2 \cdot dx\, 6}_{du}$

$= \frac{7}{6} \int u^{-4}\,du = \frac{7}{6}\left[\frac{u^{-3}}{-3}\right] + C$

$= \frac{-7}{18}(5+2x^3)^{-3} + C$ or $\frac{-7}{18(5+2x^3)^3} + C$

b.) $\frac{dy}{dx} = \frac{4x}{y}$ $y=1$ when $x=3$

$y \cdot \cancel{dx}\,\frac{dy}{\cancel{dx}} = \frac{4x}{\cancel{y}} \cdot dx \cdot \cancel{y}$

$y\,dy = 4x\,dx$ $\int y\,dy = \int 4x\,dx$

$\Rightarrow \frac{y^2}{2} = 4\frac{x^2}{2} + C$ $\frac{y^2}{2} = 2x^2 + C$

$y^2 = 4x^2 + K$ $y = \pm\sqrt{4x^2+K}$ $x=3$, $y=1$

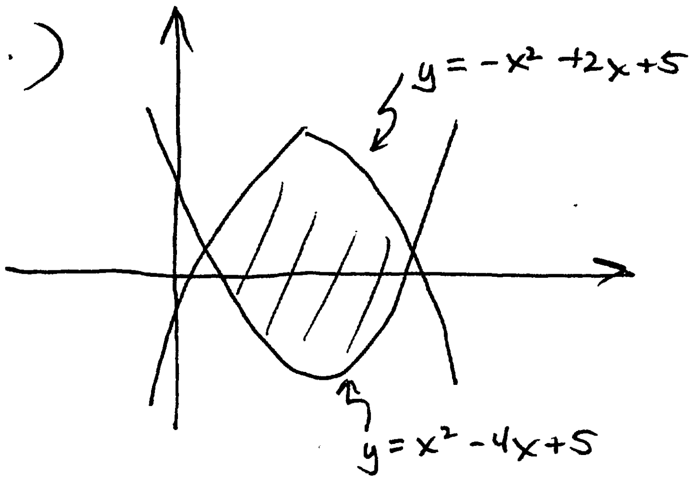
$1 = \sqrt{4(3)^2 + K}$

$1 = \sqrt{36 + K}$

$K = -35$

$y = \pm\sqrt{4x^2 - 35}$

2.)



pts of int:

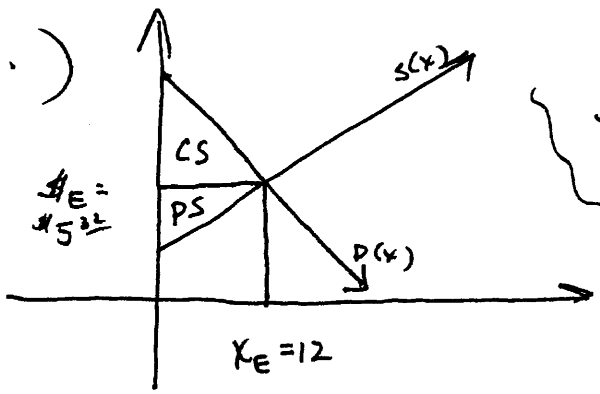
$$-\cancel{x^2}+\cancel{2x}+\cancel{5} = x^2-4x+\cancel{5}$$
$$+\cancel{x^2} -\cancel{2x} \qquad +x^2-2x$$
$$0 = 2x^2-6x$$
$$0 = 2x(x-3)$$
$$x=0;\ x=3$$

$$\int_0^3 \left[(-x^2+2x+5)-(x^2-4x+5)\right]dx = \int_0^3 (-2x^2+6x)\,dx$$

$$= \left[-\frac{2x^3}{3}+6\frac{x^2}{2}\right]_0^3 = \left[\left(\frac{-2}{3}(3)^3+3(3)^2\right)-(0)\right]$$

$$= -18+27 = 9$$

3.)



eq. pt:

$$-.39x+10 = .11x+\cancel{4}$$
$$+.39x \;-4 \quad +.39x\; -\cancel{4}$$
$$6 = .50x$$
$$\frac{6}{.50} = x = 12$$

$$D(12) = S(12) = -.39(12)+10$$
$$= .11(12)+4 = \$5.32$$

$(12, \$5.^{32})$ = EQ.PT

$$CS = \int_0^{12} D(x)\,dx - \left[(12)(5.32)\right]$$
$$= \int_0^{12}(-.39x+10)\,dx - [63.84]$$
$$= \left[-.39\frac{x^2}{2}+10x\right]_0^{12} - [63.84]$$
$$= \left[\left(-\frac{.39}{2}(12)^2+10(12)\right)-(0)\right] - [63.84] = -28.08+120-63.84$$

$CS = \$28.^{08}$

$$PS = \left[(12)(5.\underline{32})\right] - \int_0^{12} s(x)\,dx$$

$$= (63.84) - \left[\int_0^{12} (.11x+4)\,dx\right]$$

$$= 63.84 - \left[\frac{.11}{2}x^2 + 4x\right]_0^{12}$$

$$= 63.84 - \left[\left(\frac{.11}{2}(12)^2 + 4(12)\right) - (0)\right]$$

$$= 63.84 - 7.92 - 48$$

$$PS = \$7.\underline{92}$$

4.) $$\int_0^{20} 3000 \cdot e^{.025t}\,dt = 3000\int_0^{20} e^{.025t}\,dt$$

$$= 3000 \cdot \left[\frac{e^{.025t}}{.025}\right]_0^{20} = \frac{3000}{.025}\left[e^{.025t}\right]_0^{20}$$

$$= \frac{3000}{.025}\left[e^{.025(20)} - e^{.025(0)}\right] = \frac{3000}{.025}\left[1.64872 - 1\right]$$

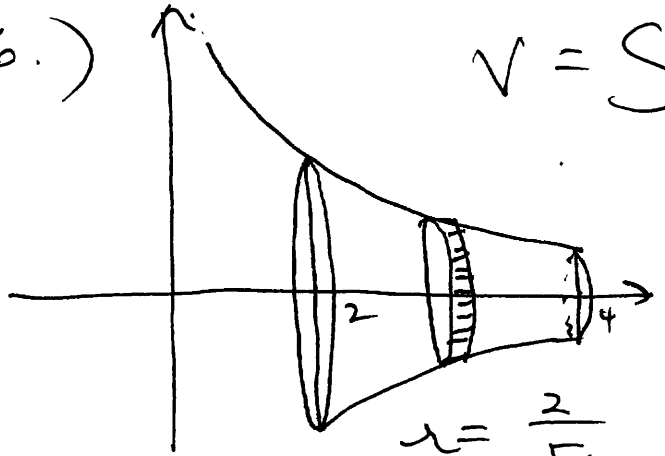
$$= \$77,846.\underline{55}$$

5.) $$\int_0^{\infty} 3e^{-sx}\,dx = \lim_{A\to\infty} 3\int_0^{A} e^{-sx}\,dx$$

$$= \lim_{A\to\infty} 3 \cdot \left[\frac{e^{-sx}}{-s}\right]_0^{A} = \lim_{A\to\infty} \frac{-3}{s}\left[e^{-sx}\right]_0^{A}$$

$$= \lim_{A\to\infty} \frac{-3}{s}\left[e^{-sA} - e^{-s(0)}\right] = \lim_{A\to\infty} \frac{-3}{s}\left[\cancel{\frac{1}{e^{sA}}}^{\,0} - 1\right] = \frac{3}{s}$$

6.)



$$V = \int \pi r^2 h$$

$$= \pi \int_2^4 \left(\frac{2}{\sqrt{x}}\right)^2 \cdot dx$$

$r = \frac{2}{\sqrt{x}}$

$h = \Delta x \rightarrow dx$

$$= \pi \int_2^4 \frac{4}{x} dx$$

$$= \pi \cdot 4 \int_2^4 \frac{1}{x} dx$$

$$= 4\pi \cdot \left[\ln x\right]_2^4 = 4\pi\left[\ln 4 - \ln 2\right]$$

$$= 4\pi\left[1.38629 - .693147\right]$$

$$\approx 4\pi\left[.69314\right] \approx 8.71$$

Bonus:

$a(t) = k$ $\quad \int a(t) = v(t) = \int k\,dt = kt + C_1 = v(t)$

$v(t) = kt + C_1$

$0 = k(0) + C_1$

$0 = C_1$

$v(0) = 0$

$v(\frac{1}{2}) = 1$ mi/min

$1 = k(\frac{1}{2})$

$k = 2$

$v(t) = 2t$

$\int v(t) = s(t) = \int 2t\,dt = t^2 + C_2 = s(t)$

$s(0) = 0$

$0 = 0^2 + C_2 \quad C_2 = 0$

$s(t) = t^2$

$t = \frac{1}{2} \quad s = ?$ $\quad s(\frac{1}{2}) = (\frac{1}{2})^2 = \frac{1}{4}$ mile