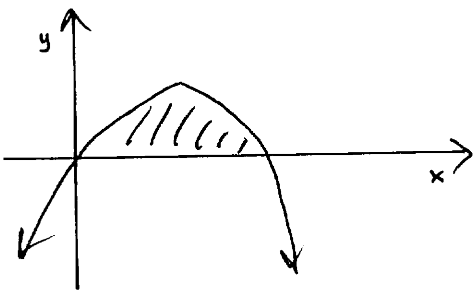
NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
Test #3 MA121-004 Form B Wednesday, March 25, 2009 Dr. J. Griggs

Please put all work and answers in the stamped blue book provided. Begin new problems on a new page; it is OK to use the back of the paper as a new page. Do not use the graphics mode of a graphing calculator on a problem that requires YOU to graph. Do not use a calculator that does calculus. You may keep the test copy itself.

1.) The average salary of a major league baseball player in 1971 was \$146,026. The average salary had grown to \$636,438 by 1987. Use the exponential growth model to predict the average salary for a player for 2009.

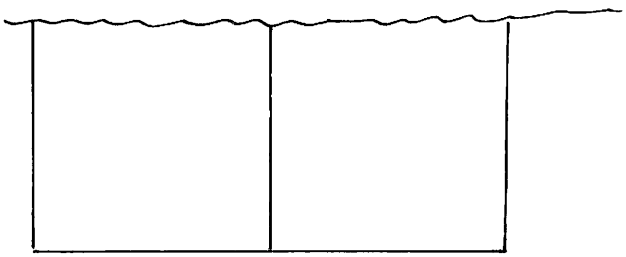
2.) Find the area bounded between the curve $f(x) = 5x - x^2$ and the x-axis.



3.) Find the derivative:

a.) $y = 5^x e^{3x}$ b.) $y = \ln(x^4 + 3x - 2)$

4.) A rancher wants to enclose two rectangular areas bordering a river, one for sheep and one for cattle. There is 240 yd of fencing available. What is the largest area that can be enclosed?



5.) Find the indefinite integrals:

a.) $\int (6x - 4\sqrt{x} + \frac{9}{x^3})dx$ b.) $\int (\frac{5}{x} + 7e^{3x})dx$

6.) After 38 days, a sample of radon-222 decayed to 27% of its original amount. What is the half-life of radon 222? How long will it take for the sample to decay to 10% of its original amount?

MA 121-002 TEST #3 3/25/09 J. GRIGGS

FORM B

TEST SOLUTIONS (16 POINTS EACH)

1.) $y = y_0 \cdot e^{kt}$

1972 $(t=0)$ \$146,026

1987 $(t=15)$ \$636,438

2009 $(t=37)$???

$$y = 146{,}026\, e^{kt}$$

$$636{,}438 = 146{,}026\, e^{k(15)}$$

$$\frac{636{,}438}{146{,}026} = e^{15k}$$

$$\ln\left(\frac{636{,}438}{146{,}026}\right) = 15k$$

$$k = \frac{\ln\left(\frac{636{,}438}{146{,}026}\right)}{15}$$

$$k \approx .09814$$

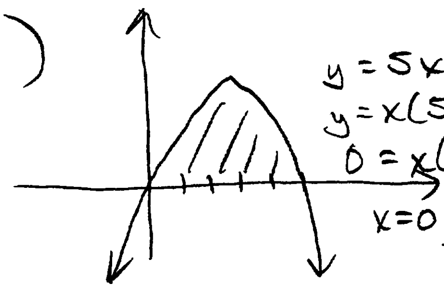
$$y = 146{,}026\, e^{.09814t}$$

$t = 37$ $y = ??$

$$y = 146{,}026\, e^{.09814(37)}$$

$$y = \$5{,}513{,}553.^{91}$$

2.)



$y = 5x - x^2$

$y = x(5-x)$

$0 = x(5-x)$

$x=0;\ x=5$

$$\int_0^5 (5x - x^2)\,dx$$

$$= \frac{5x^2}{2} - \frac{x^3}{3}\Big]_0^5$$

$$= \left(\frac{5}{2}(5)^2 - \frac{(5)^3}{3}\right) - (0-0) = \frac{125}{2} - \frac{125}{3}$$

$$= \frac{125 \cdot 3}{2 \cdot 3} - \frac{125 \cdot 2}{3 \cdot 2} = \frac{375}{6} - \frac{250}{6} = \frac{125}{6} \approx 20.83$$

3.) a.) $y = 5^x \cdot e^{3x}$

$$y' = 5^x \cdot d(e^{3x}) + e^{3x} \cdot d(5^x)$$

$$y' = 5^x \cdot e^{3x} \cdot 3 + e^{3x} \cdot 5^x \cdot \ln 5$$

$$y' = 5^x \cdot e^{3x}\,[3 + \ln 5]$$

3B.

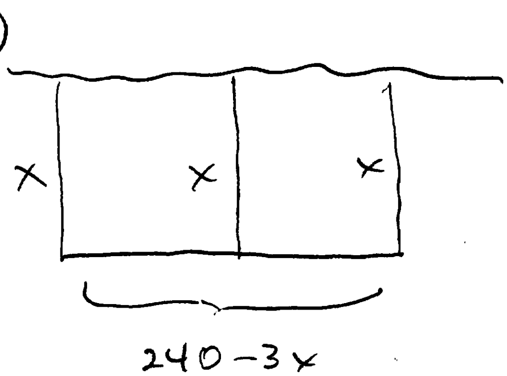
(page 2)

3.) b.) $y = \ln(x^4+3x-2)$

$$y' = \frac{1}{x^4+3x-2} \cdot d(x^4+3x-2)$$

$$y' = \frac{4x^3+3}{x^4+3x-2}$$

4.)



$A = x(240-3x)$

$A = 240x - 3x^2$

$A' = 240 - 6x$

$0 = 240 - 6x$

$6x = 240$

$x = 40$

$A'' = -6$

∴ CONC. DOWN

∴ MAX

$x = 40$ $\quad 240 - 3x = 240 - 3(40)$

$= 240 - 120$

$= 120$

40 yds by 120 yds

$A = 4800$ sq. yds

5.) a.) $\int \left(6x - 4\sqrt{x} + \frac{9}{x^3}\right) dx$

$\int (6x - 4x^{1/2} + 9x^{-3})\,dx$

$$= \frac{6x^2}{2} - \frac{4x^{3/2}}{3/2} + \frac{9x^{-2}}{-2} + C$$

$$= 3x^2 - \frac{8}{3}x^{3/2} - \frac{9}{2}x^{-2} + C$$

(page 3)

b.) $\int \left(\frac{5}{x} + 7e^{3x}\right) dx$

$= \int \left(5 \cdot \frac{1}{x} + 7e^{3x}\right) dx$

$= 5 \cdot \ln|x| + 7 \cdot \frac{1}{3} e^{3x} + C$

$= 5 \ln|x| + \frac{7}{3} e^{3x} + C$

6.) $t=0$ $y = 100\%$ $(y_0 = 100\%)$

$t = 38$ $y = 27\%$

$y = y_0 \cdot e^{kt}$

$27\% = 100\% \, e^{k(38)}$

$.27 = e^{38k}$

$\ln .27 = 38k$

$k = \frac{\ln .27}{38}$

$\boxed{k \approx -.034456}$

[ha]lf-life: $(t = ?)$

$50\% = 100\% \, e^{-.034456t}$

$.50 = e^{-.034456t}$

$\ln .50 = -.034456t$

$t = \frac{\ln .50}{-.034456} \approx \boxed{20.12 \text{ days}}$ ← half-life

$10\% = 100\% \, e^{-.034456t}$

$.10 = e^{-.034456t}$

$\ln .10 = -.034456t$

$t = \frac{\ln .10}{-.034456} \approx \boxed{66.83 \text{ days}}$ ↙ reduces to 10% of orig. amt