NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA 121-004 Test #2 Form B Monday, February 23, 2009 J. Griggs

Please put all work and answers in the stamped blue book provided. Graphing calculators can be used on this test; however, **the graphics mode is NOT to be used on ANY problem.** Simplify all derivatives **completely.** You may keep the test copy. Put your name, row number, seat number, and form of the test on the front of the blue book.

1) Find y' (using the quotient rule) and use it to find the equation of the tangent line to the function at the point (2, -7):

$$y = \frac{4x-1}{x^2-5}$$

2) Find and use $f'(x)$ and $f''(x)$ in order to find all critical points, points of inflection, areas where the function is increasing, decreasing, concave up and concave down (and graph it). Identify all relative maxima and minima.

$$f(x) = x^3 - 9x^2 + 15x - 6$$

3) Find y': a) $y = (4x-3)(3x^5 - 4x - 1)$
b) $y = \sqrt{x^6 - 3x^4 - x^2}$

4) An object is projected upward from the top of a building. Its distance above the ground
($s(t)$ in feet) after t seconds is given by $s(t) = -16t^2 + 128t + 105$. Find the height of the object at $t = 2.5$ sec., and the velocity of the object at $t = 2.5$ sec., and the acceleration of the object at $t = 2.5$ sec. (Units are important in each answer.)

5) Open Air Waste Management is designing a rectangular construction dumpster that will be twice as long as it wide and must hold 12 cubic yards of debris. Find the dimensions of the dumpster that will minimize the surface area.

6) Find the absolute maximum and absolute minimum values of the function from the polynomial in problem #2 above on the interval [0,8].

121-00 TEST #2B 2/23/09 SOLUTIONS

16 POINTS EACH:

1.) $y' = \dfrac{(x^2-5)(4)-(4x-1)(2x)}{(x^2-5)^2} = \dfrac{4x^2-20-(8x^2-2x)}{(x^2-5)^2}$

$y' = \dfrac{4x^2-20-8x^2+2x}{(x^2-5)^2} = \dfrac{-4x^2+2x-20}{(x^2-5)^2} = f'(x)$

$f'(2) = \dfrac{-4(2)^2+2(2)-20}{(2^2-5)^2} = \dfrac{-16+4-20}{1} = -36+4 = -32$

(pt $(2,-7)$) $\quad -32 = m_{TAN}$

$y - y_1 = m(x - x_1)$

$m = -32$; pt $(2,-7)$

$y-(-7) = -32(x-2)$

$y+7 = -32(x-2)$

or $y = -32x+64-7$

or $y = -32x+57$

2.) $y = x^3-9x^2+15x-6$

$y' = 3x^2-18x+15 = 3(x^2-6x+5) = 3(x-5)(x-1)$

$y' = 0 = 3(x-5)(x-1)$ at $x=5$; $x=1$

$(5, f(5)) \Rightarrow f(5) = 5^3 - 9\cdot 5^2 + 15\cdot 5 - 6$

$f(5) = 125-225+75-6 = -31$

$\underline{\underline{(5,-31)}}$

$(1, f(1)) \Rightarrow f(1) = 1^3 - 9\cdot 1^2 + 15(1) - 6$

$f(1) = 1-9+15-6 = 1$

$\underline{\underline{(1,1)}}$

$f'(x)$:

$f'(0) = +$ $\quad$ 1 $\quad$ $f'(3) = -$ $\quad$ 5 $\quad$ $f'(6) = +$

$f(x)$ INCR $\quad$ $f(x)$ DECR $\quad$ $f(x)$ INCR

$y'' = 6x - 18$ $\qquad y'' = 0 = 6x - 18 \Rightarrow x = 3$

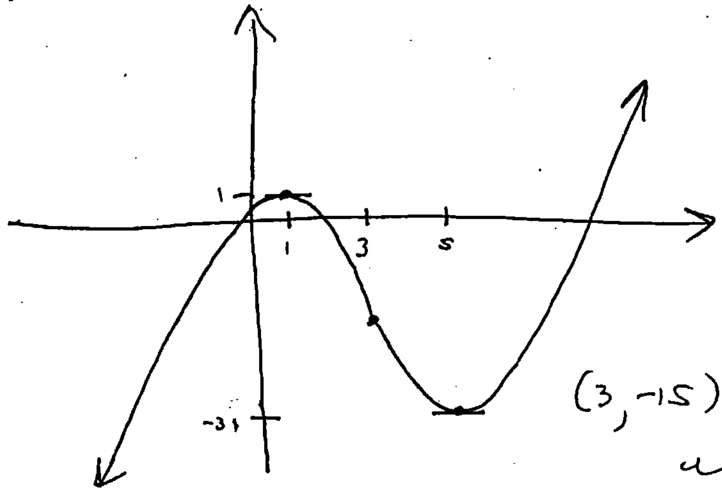
$(3, f(3)) \Rightarrow f(3) = 3^3 - 9 \cdot 3^2 + 15(3) - 6$

$(3, -15)$ $\qquad f(3) = 27 - 81 + 45 - 6 = -15$

$f''(x)$:

$f''(0) = -$ $\qquad$ 3 $\qquad f''(4) = +$

$f(x)$ C. DOWN $\qquad$ C. UP



$(3, -15)$ pt of infl.

3 a.) $y' = (4x-3)(15x^4 - 4) + (3x^5 - 4x - 1)(4)$

$y' = 60x^5 - 45x^4 - 16x + 12 + 12x^5 - 16x - 4$

$y' = 72x^5 - 45x^4 - 32x + 8$

b.) $y = (x^6 - 3x^4 - x^2)^{1/2}$

$y' = \frac{1}{2}(x^6 - 3x^4 - x^2)^{-1/2} \cdot d(x^6 - 3x^4 - x^2)$

$y' = \frac{1}{2}(x^6 - 3x^4 - x^2)^{-1/2} \cdot (6x^5 - 12x^3 - 2x)$

4.) $s(t) = -16t^2 + 128t + 105$ (s in ft; t in sec)

$s'(t) = v(t) = -32t + 128$

$s''(t) = v'(t) = a(t) = -32$

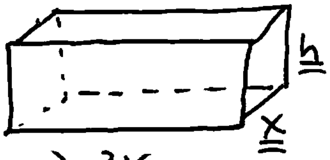
$s(2.5) = -16(2.5)^2 + 128(2.5) + 105$

$= -16(6.25) + 320 + 105$

$= -100 + 425 = 325$ ft $= s(2.5)$

$v(2.5) = -32(2.5) + 128 = -80 + 128 = 48$ ft/sec $= v(2.5)$

$a(2.5) = -32$ ft/sec/sec

5.) 

(no top) $2x$

$(2x)(x)(h) = 12$

$2x^2 h = 12$

$h = \frac{12}{2x^2} = \frac{6}{x^2}$

$SA = \underbrace{(2x \cdot x)}_{\text{bottom}} + \underbrace{(2x \cdot h) \cdot 2}_{\text{front \& back}} + \underbrace{2(xh)}_{\text{right side \& left side}}$

$SA = 2x^2 + 4xh + 2xh$

$SA = 2x^2 + 6xh = 2x^2 + 6x\left(\frac{6}{x^2}\right) = 2x^2 + \frac{36}{x}$

$SA' = 4x + 36(-1x^{-2}) = 4x - \frac{36}{x^2} = 0$

$4x = \frac{36}{x^2} \qquad 36 = 4x^3 \qquad x^3 = 9$

$x = \sqrt[3]{9} \approx 2.08\text{yd}$ $\quad 2x \approx 4.16\text{yd}$ $\quad h = \frac{6}{x^2} \approx 1.38\text{yd}$

$SA'' = 4 - 36(-2x^{-3}) = 4 + \frac{72}{x^3} = +$ (when $x = \sqrt[3]{9}$)

$\therefore$ CONCAVE UP $\therefore$ MIN

imensions:

2.08 yds by 4.16 yds by 1.38 yds

6.) $y = x^3 - 9x^2 + 15x - 6 \quad [0, 8]$

critical points: $(1, 1)$ & $(5, -31)$

endpoints:

$(0, ?)$: $f(0) = 0^3 - 9 \cdot 0^2 + 15 \cdot 0 - 6 = -6$

$(0, -6)$

$(8, ?)$: $f(8) = 8^3 - 9 \cdot 8^2 + 15 \cdot 8 - 6$

$= 512 - 576 + 120 - 6$

$= 50$

$(8, 50)$

$(1, 1)$; $(5, -31)$; $(0, -6)$; $(8, 50)$

ABSOLUTE $\qquad$ ABSOLUTE MAX