MA121-004 TEST 1 FORM B 1/30/09 J. GRIGGS

16 POINTS EACH

1.) a.) $\lim_{x \to 3} \frac{x^2+2x-15}{x-3} = \lim_{x \to 3} \frac{(x-3)(x+5)}{(x-3)}$

$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+5)}{\cancel{(x-3)}} = \lim_{x \to 3} (x+5) = 8$

$(x \neq 3)$

b.) $\lim_{x \to \infty} \frac{x^2-3x+7}{2x-1} = \lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{7}{x^2}}{\frac{2x}{x^2} - \frac{1}{x^2}}$

$= \lim_{x \to \infty} \frac{1 - \cancel{\frac{3}{x}}^{\to 0} + \cancel{\frac{7}{x^2}}^{\to 0}}{\cancel{\frac{2}{x}}_{\to 0} - \cancel{\frac{1}{x^2}}_{\to 0}} = \frac{1}{0} =$ DOES NOT EXIST (or ∞)

2.) a.) $f(x) = \begin{cases} x+3, & x \leq -1 \\ x^2+2, & x > -1 \end{cases}$

$y = x+3 \ (x \leq -1)$

x	y
-1	2
-2	1
-3	0

$y = x^2+2 \ (x > -1)$

x	y
-1	3
0	2
1	3
2	6

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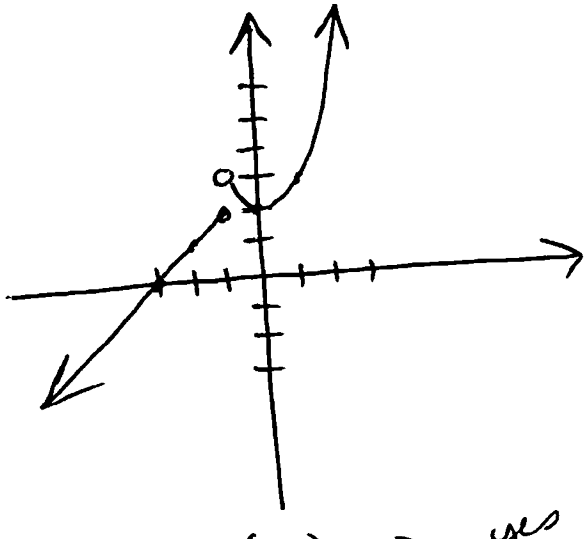

b.) contin?

1.) $f(-1)$ exists? $f(-1) = 2$ yes

2.) $\lim_{x \to -1} f(x)$ exists? no

$\lim_{x \to -1^+} f(x) = 3$

$\lim_{x \to -1^-} f(x) = 2$

∴ DISCONTINUOUS

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3.) $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[3(x+h)^2 - 5(x+h)+7] - [3x^2-5x+7]}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2) - 5x - 5h + 7 - 3x^2 + 5x - 7}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{-5x} - 5h \cancel{+7} \cancel{-3x^2} \cancel{+5x} \cancel{-7}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(6x+3h-5)}{\cancel{h}} \quad (h \neq 0)$$

$$= \lim_{h\to 0} (6x + \underbrace{3h}_{0} - 5) = 6x-5$$

(short cut: $3(2x^1) - 5(1) + 0 = 6x - 5$)

4.) $f(x) = x^2 + \sqrt{x} = x^2 + x^{1/2}$ (4, 18)

$f'(x) = 2x + \frac{1}{2} x^{-1/2} = 2x + \frac{1}{2\sqrt{x}} = m_{TAN}$

$f'(4) = 2(4) + \frac{1}{2\sqrt{4}} = 8 + \frac{1}{4} = \frac{33}{4}$

$y - y_1 = m(x - x_1)$

$y - 18 = \frac{33}{4}(x-4)$

or $33x - 4y - 60 = 0$

or $y = \frac{33}{4}x - 15$

5.) $f(x) = x^2 - 6x + 3$

vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ $b=-6$, $a=1$ $V\left(\frac{6}{2(1)}, f\left(\frac{6}{2}\right)\right)$

$V(3, -6)$ $f(3) = 3^2 - 6\cdot 3 + 3 = 9 - 18 + 3$

y-int (set $x=0$)

$y = 0^2 - 6\cdot 0 + 3 = 3$ (0, 3)

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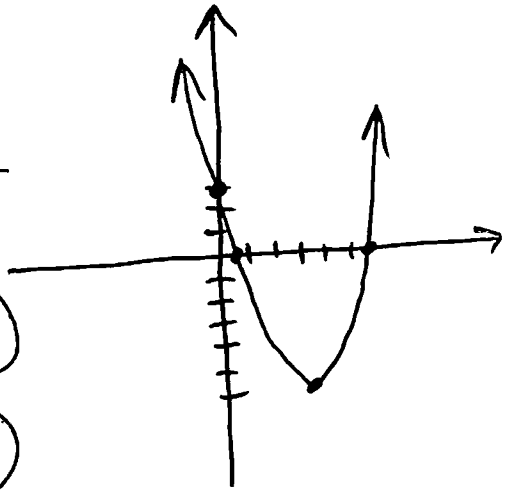
x-int (set y = 0)

$$0 = x^2 - 6x + 3$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)} = \frac{6 \pm \sqrt{24}}{2}$$

$$x_1 = \frac{6 + \sqrt{24}}{2} \approx 5.45 \qquad (5.45, 0)$$

$$x_2 = \frac{6 - \sqrt{24}}{2} \approx .55 \qquad (.55, 0)$$



6.) a.)

$$\frac{\frac{3}{x+h} - \frac{3}{x}}{h} = \frac{\frac{3(x)}{(x+h)(x)} - \frac{3(x+h)}{x(x+h)}}{h}$$

$$= \frac{3x - 3(x+h)}{(x+h)(x)} \cdot \frac{1}{h} = \frac{\cancel{3x} - \cancel{3x} - 3h}{(x+h)(x)(h)}$$

$$= \frac{-3\cancel{h}^{1}}{(x+h)(x)(\cancel{h})_{1}} \quad (h \neq 0) = \frac{-3}{(x+h)x}$$

b.)

$$f(x) = 12x^3 - 3x^{1/4} + 3x^{1/2} - 5x^{-3} - 3$$

$$f'(x) = 12(3x^2) - 3\left(\tfrac{1}{4}x^{-3/4}\right) + 3\left(\tfrac{1}{2}x^{-1/2}\right) - 5(-3x^{-4}) - 0$$

$$f'(x) = 36x^2 - \frac{3}{4}x^{-3/4} + \frac{3}{2}x^{-1/2} + 15x^{-4}$$