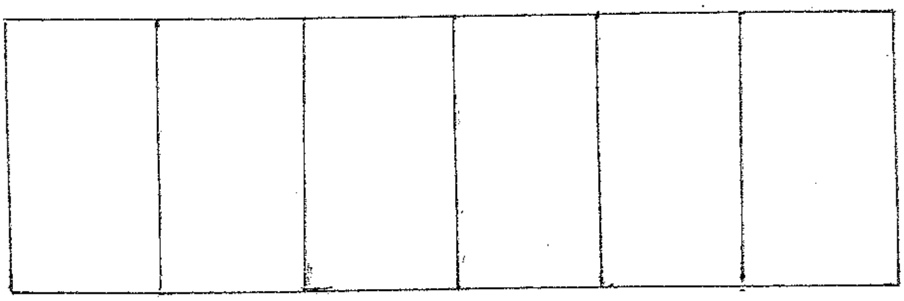
NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121 Test #3 Form B Thursday, October 29, 2009 Dr. J. Griggs

Please put all work and answers in the stamped blue book provided. Do not use a calculator that does calculus - show your work to justify your answers. Simplify all derivatives completely. Put your name, form of test (A, B, or C) , row letter, and seat number on the front of the blue book.

1.) Find y': $y = (e^{x^2} - 3)^5$

2.) A veterinarian has 120 feet of fencing and wishes to construct six dog kennels by first building a fence around a rectangular region, and then subdividing that region into six smaller rectangles by placing five fences parallel to one of the sides. What dimensions of the region will maximize the total area?



3.) Evaluate the indefinite integral: $\int (4x^2 - 9x^{\frac{2}{3}} + \frac{8}{x} + 8x^{-3} - 6e^{5x})dx$

4.) Find y': $y = \ln(7x^2 - 4)$

5.) The number of women graduating from 4-year colleges in the United States has grown exponentially from 1930, when 48,700 women earned such a degree, to 2005, when 832,400 women received such a degree. Use this data to predict the number of degrees that women will receive in 2013.

6.) Find the exact area (using a definite integral) of the region under the curve $y = 9 - x^2$ from x=-3 to x=3.

7.) Find y': a.) $y = 5^{x^3+4}$ b.) $y = \log_7(x^3 - x)$

8.) The temperature in a hot tub is 104°, and the room temperature is 74°. The water cools to 95° in 15 minutes. Using Newton's Law of Cooling ($T = ae^{kt} + M$), determine the amount of time it would take for the water to cool to 81°.

121. Test #3 FALL '09 10/29/09 GRIGGS

FORM B

(8 problems; 12 points each)

1.) $y = (e^{x^2} - 3)^5$

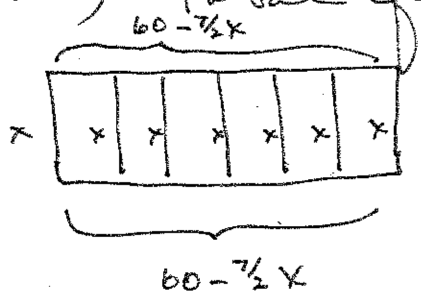
$$y' = 5 \cdot (e^{x^2} - 3)^4 \cdot d(e^{x^2} - 3)$$

$$y' = 5 \cdot (e^{x^2} - 3)^4 \cdot e^{x^2} \cdot d(x^2)$$

$$y' = 5 \cdot (e^{x^2} - 3)^4 \cdot e^{x^2} \cdot 2x$$

$$y' = 10x \cdot e^{x^2} \cdot (e^{x^2} - 3)^4$$

2.) ~~total fence~~: 120 ft.



$$\frac{120 - 7x}{2} = 60 - \frac{7}{2}x$$

$$A = x\left(60 - \frac{7}{2}x\right)$$

$$A = 60x - \frac{7}{2}x^2$$

$$A' = 60 - \frac{7}{2}(2x) = 60 - 7x = 0$$

$$60 = 7x \qquad x = \frac{60}{7} \approx 8.57$$

other side: $60 - \frac{7}{2}\left(\frac{60}{7}\right) = 60 - 30 = 30$

$A'' = -7$ (always NEG, ∴ C. DOWN, ∴ MAX)

dimensions: $\frac{60}{7}$ ft by 30 ft

≈ 8.57

T3B (page 2)

3.) $\int (4x^2 - 9x^{2/3} + \frac{8}{x} + 8x^{-3} - 6e^{5x})\,dx$

$= 4\left(\frac{x^3}{3}\right) - 9\left(\frac{x^{5/3}}{5/3}\right) + 8\int \frac{1}{x}dx + 8\left(\frac{x^{-2}}{-2}\right) - 6\left(\frac{1}{5}e^{5x}\right) + C$

$= \frac{4}{3}x^3 - \frac{27}{5}x^{5/3} + 8 \cdot \ln|x| - 4x^{-2} - \frac{6}{5}e^{5x} + C$

4.) $y = \ln(7x^2 - 4)$

$y' = \frac{1}{7x^2-4} \cdot d(7x^2-4) = \frac{14x}{7x^2-4} = y'$

5.) $y = y_0 \cdot e^{kt}$

$t = 0$ (1930), $y = 48{,}700$; $t = 75$ (2005), $y = 832{,}400$; $t = 83$ (2013), $y = ??$

$(y_0 = 48{,}700)$

$y = 48{,}700\, e^{kt}$

$t = 75$ $y = 832{,}400$

$832{,}400 = 48{,}700 \cdot e^{k(75)}$ $\frac{832{,}400}{48{,}700} = e^{75k}$

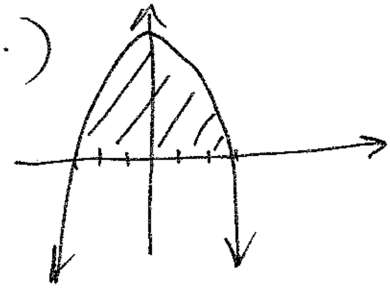
$\ln\left(\frac{832{,}400}{48{,}700}\right) = 75k$ $k = \frac{\ln\left(\frac{832{,}400}{48{,}700}\right)}{75} \approx .03785$

$y = 48{,}700 \cdot e^{.03785t}$

$t = 83$ $y = ??$

$y = 48{,}700 \cdot e^{.03785(83)} \approx 1{,}126{,}759$

135 (page 3)

6.) 

(graph not necessary)

$$\int_{-3}^{3}(9-x^2)\,dx = \left[9x - \frac{x^3}{3}\right]_{-3}^{3}$$

$$= \left[9(3) - \frac{3^3}{3}\right] - \left[9(-3) - \frac{(-3)^3}{3}\right]$$

$$= \left[27 - \frac{27}{3}\right] - \left[-27 + \frac{27}{3}\right]$$

$$A = 27 + 27 - \frac{27}{3} - \frac{27}{3} = 54 - 18 = 36$$

7.) a.) $y = 5^{x^3+4}$ $\quad y' = 5^{x^3+4}(\ln 5)\cdot(3x^2)$

b.) $y = \log_7(x^3 - x)$ $\quad y' = \frac{1}{(x^3-x)\ln 7}\cdot(3x^2-1)$

$$y' = \frac{3x^2-1}{(x^3-x)\cdot\ln 7}$$

8.) $T = a\cdot e^{kt} + m$ $\quad (m = 74^\circ)$

$t = 0$, $T = 104^\circ$ $\quad \left\{ \begin{matrix} t = 15 \\ (t = .25\text{ hr.}) \\ T = 95^\circ \end{matrix} \right\}$ $\quad t = ??$, $T = 81^\circ$

$104 = a\cdot e^{k(0)} + 74$ $\qquad 104 - 74 = a$ $\qquad a = 30$

$T = 30\cdot e^{kt} + 74$

$t = 15 \quad T = 95$

$95 = 30\cdot e^{k(15)} + 74$

$21 = 30\cdot e^{15k}$

$\frac{21}{30} = e^{15k}$ $\qquad \ln\left(\frac{21}{30}\right) = 15k$

$k = \frac{\ln(21/30)}{15} \approx -.023778$

$t = ? \quad T = 81^\circ$

$81 = 30e^{-.023778t} + 74$

$7 = 30e^{-.023778t}$

$\frac{7}{30} = e^{-.023778t}$ $\qquad \ln(7/30) = -.023778t$ $\qquad t = \frac{\ln(7/30)}{-.023778} \approx 61.2$ min

(1.02 hr.)