NORTH CAROLINA STATE UNIVERSITY Department of Mathematics
MA121 Test #2 Form C Thursday, October 1, 2009 Dr. J. Griggs

Please put all work and answers in the stamped blue book provided; one problem per page (the back of a page constitutes a new page). Graphing calculators can be used; however, **the graphics mode is NOT to be used on any problem.** Please put your **name** and **form of test** on the front of the blue book; put your **row letter** and **seat number** on the upper right hand corner of the blue book. Simplify all derivatives completely.

1.) Find all intercepts and asymptotes; use $f'(x)$ to determine where the function is increasing and decreasing; graph it: $f(x)=\dfrac{3x+4}{x+5}$

2.) a.) Find y' using the product rule: $y=(2x+7)(5x^2-x+6)$

b.) Find y' using the quotient rule: $y=\dfrac{4x^2+x-9}{x^2+1}$

3.) Find all intercepts and asymptotes for the function: $y=\dfrac{x^2-x-12}{x+2}$

(you do not need to graph it)

4.) Find and use $f'(x)$ *and* $f''(x)$ in order to find all critical points, points of inflection, areas where the function is increasing, decreasing, concave up and concave down, and all relative maximum and minimum values of the function. Graph the function: $f(x)=5-6x^2-2x^3$

5.) Find the equation of the tangent line to the given curve at the indicated point:
$y=(4x^2-7x+5)^3$ (1,8)

6.) A stone is projected upward from a platform 35 feet above the ground with an initial velocity of 80 feet per second. The height *s(t)* above the ground (in feet) at time *t* (in seconds) is given by $s(t)=-16t^2+80t+35$. Find the height of the stone at t=1 sec, the velocity of the stone at t=1 sec, and the acceleration of the stone at t=1 sec. (appropriate units are important)

7.) Find the absolute maximum and absolute minimum values of the function on the given closed interval: $f(x)=x^3-x^2-x+4$ *on* $[0,3]$

8.) Find the critical point(s); determine where the function is increasing and decreasing; and graph: $y=(x+2)^{2/3}-4$

Bonus: (5 points) Write out all of the words to NCSU's Alma Mater.

121 . TEST #2 FORM C FALL '09

SOLUTIONS: (12 pts each)

1. $f(x) = \dfrac{3x+4}{x+5}$

intercepts:

(1) y-int: $(x=0)$

$y = \dfrac{0+4}{0+5} = 4/5 \Rightarrow (0, 4/5)$

(2) x-int: $(y=0)$

$0 = \dfrac{3x+4}{x+5}$ $\quad 0 = 3x+4$ $\quad x = -4/3$ $\quad (-4/3, 0)$

$f'(x) = \dfrac{(x+5)(3)-(3x+4)(1)}{(x+5)^2}$

$f'(x) = \dfrac{\cancel{3x}+15-\cancel{3x}-4}{(x+5)^2}$

$f'(x) = \dfrac{11}{(x+5)^2}$

$f'(x) \neq 0$

$f'(x)$ undef at $x=-5$

$f'(-6) = +$, $f'(0) = +$

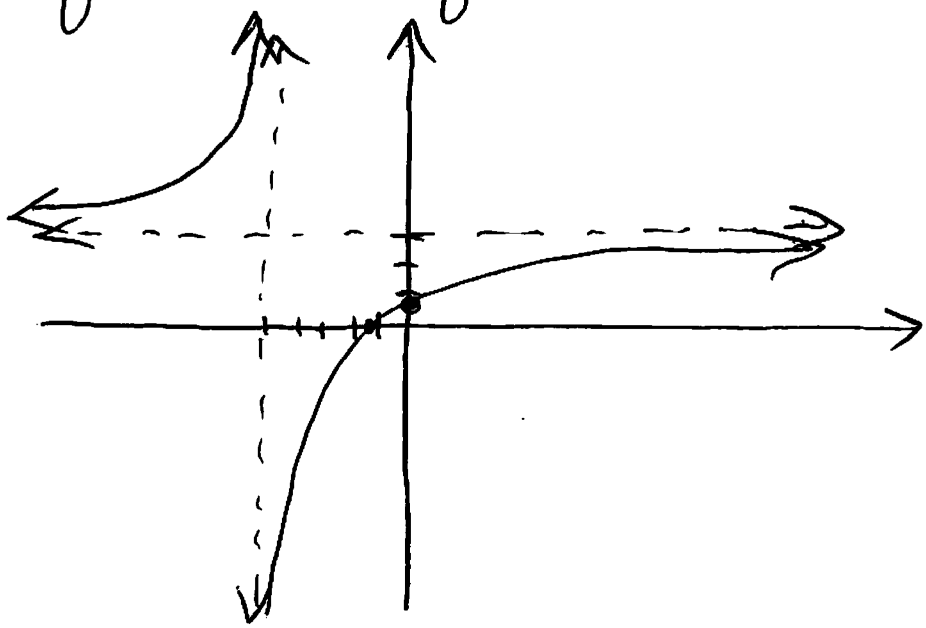
f: INCR $\quad -5 \quad$ f: INCR

asymp:

(1) vert: $x = -5$

(2) horiz: $y = 3$

$\lim\limits_{x\to\infty} \dfrac{3x+4}{x+5} = 3$



2. a.) $y = (2x+7)(5x^2-x+6)$

$y' = (2x+7)(10x-1)+(5x^2-x+6)(2)$

$y' = 20x^2+70x-2x-7+10x^2-2x+12$

$y' = 30x^2+66x+5$

T2CP2

b.) $y = \dfrac{4x^2+x-9}{x^2+1}$

$$y' = \frac{(x^2+1)(8x+1) - (4x^2+x-9)(2x)}{(x^2+1)^2}$$

$$y' = \frac{(8x^3+8x+x^2+1) - (8x^3+2x^2-18x)}{(x^2+1)^2}$$

$$y' = \frac{\cancel{8x^3}+8x+x^2+1-\cancel{8x^3}-2x^2+18x}{(x^2+1)^2}$$

$$y' = \frac{-x^2+26x+1}{(x^2+1)^2}$$

3.) $y = \dfrac{x^2-x-12}{x+2}$

asymp:

(1) vert: $x = -2$

(2) horiz: none

$$\lim_{x\to\infty} \frac{x^2-x-12}{x+2} = \text{D.N.E.}$$

(3) oblique: $y = x-3$

$$\begin{array}{r}
x-3 \\
x+2\,\overline{)\,x^2-x-12} \\
-(x^2+2x) \\
\hline
-3x-12 \\
-(-3x-6) \\
\hline
-6
\end{array}$$

intercepts:

(1) y-int: $(x=0)$

$$y = \frac{0\ \ 0-12}{0+2} = -6$$

$(0,-6)$

(2) x-int: $(y=0)$

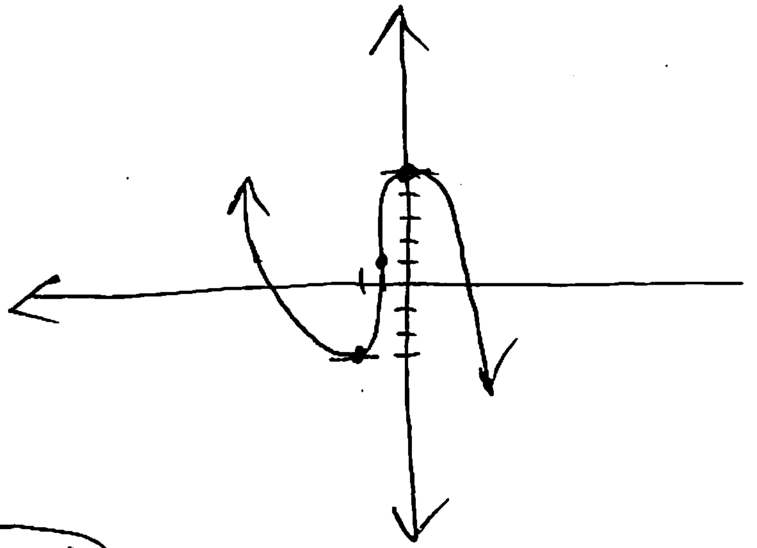
$$0 = \frac{x^2-x-12}{x+2} = \frac{(x-4)(x+3)}{x+2}$$

$$0 = (x-4)(x+3)$$

$x = 4, -3$

$(4,0)$ & $(-3,0)$

T2C P3



(4) $y = 5 - 6x^2 - 2x^3$

$y' = -12x - 6x^2$

$y' = -6x(2 + x) = 0$

$x = 0 \quad x = -2$

$(0, f(0)) = (0, 5)$

$(-2, f(-2)) = (-2, -3)$

$5 - 6(-2)^2 - 2(-2)^3 = f(-2)$

$5 - 24 + 16 = f(-2)$

$-3 = f(-2)$

$f'(x)$:

$f'(-3) = -$, $f'(-1) = +$, $f'(1) = -$

f: DECR. -2 f: INCR 0 f: DECR

$y'' = -12 - 12x$

$y'' = -12(1 + x) = 0$

$x = -1$

$(-1, f(-1)) = (-1, 1)$

$f(-1) = 5 - 6(-1)^2 - 2(-1)^3$

$f(-1) = 5 - 6 + 2$

$f(-1) = 1$

$f''(x)$:

$f''(-2) = +$, $f''(0) = -$

C. UP -1 C. DOWN

rel min: $(-2, -3)$

rel max: $(0, 5)$

point of INFL: $(-1, 1)$

incr: $(-2, 0)$

decr: $(-\infty, -2) \cup (0, +\infty)$

c. up: $(-\infty, -1)$

c. down: $(-1, +\infty)$

INTERVALS

T2CP4

(5) $y = (4x^2 - 7x + 5)^3$ eq. of tangent line at $(1, 8)$

$y' = 3(4x^2 - 7x + 5)^2 (8x - 7)$

y' at $x = 1$: $3(4 \cdot 1^2 - 7 \cdot 1 + 5)^2 (8 \cdot 1 - 7)$

$f'(1) = 3(2)^2(1) = 12$

$y - 8 = 12(x - 1)$ or $y = 12x - 4$

(6.) $s(t) = -16t^2 + 80t + 35$

$s(1) = -16(1)^2 + 80(1) + 35 = -16 + 80 + 35$

$s(1) = 99$ ft

$v(t) = s'(t) = -32t + 80$

$v(1) = -32(1) + 80 = -32 + 80$

$v(1) = 48$ ft/sec

$a(t) = v'(t) = s''(t) = -32$

$a(1) = -32$ ft/sec/sec

(7) $f(x) = x^3 - x^2 - x + 4$ on $[0, 3]$

endpts: $(0, f(0))$ & $(3, f(3))$

$f(0) = 0^3 - 0^2 - 0 + 4 = 4$

$f(3) = 3^3 - 3^2 - 3 + 4 = 27 - 9 - 3 + 4 = 19$

$(0, 4)$ & $(3, 19)$

$f'(x) = 3x^2 - 2x - 1$ $\quad 0 = (3x + 1)(x - 1)$

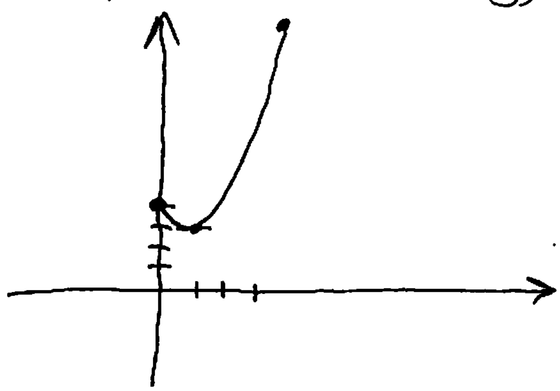
not in interval $\rightarrow$ $\cancel{x = -1/3}$ $\quad x = 1$

T2CPS:

$f(1) = 1^3 - 1^2 - 1 + 4 = 1 - 1 - 1 + 4 = 3$

$(1, 3)$

graph (not necessary):



absolute $\underline{\text{min}}$: 3 or $(1,3)$

absolute $\underline{\text{max}}$: 19 or $(3,19)$

8.) $y = (x+2)^{2/3} - 4$

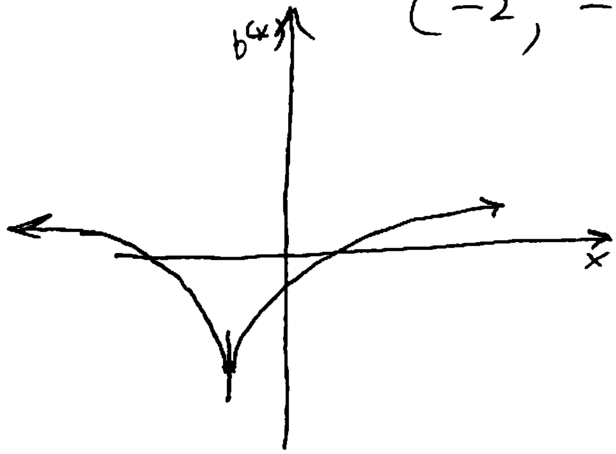
$y' = \frac{2}{3}(x+2)^{-1/3} = \dfrac{2}{3\sqrt[3]{x+2}}$

① $\dfrac{2}{3\sqrt[3]{x+2}} \overset{?}{=} 0$ (no, never $= 0$)

② $\dfrac{2}{3\sqrt[3]{x+2}}$ undef? yes, when $x = -2$

$(-2, f(-2))$ $f(-2) = (-2+2)^{2/3} - 4 = -4$

$(-2, -4)$ ← vertical tangent line there



$f'(x)$:

$f'(-3) = -$ | $f'(0) = +$

$f(x)$ decr. -2 $f(x)$ incr

<u>BONUS</u>: (5 pts) NCSU'S ALMA MATER:

Where the winds of Dixie softly blow
O'er the fields of Caroline,

There stands ever cherished, N.C. State,
As thy honored shrine
So lift your voices! Loudly sing,
From hill to ocean side!

Our hearts ever hold you, N.C. State --
In the folds
Of our love and pride.