NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121 Test #1 Form C Thursday, September 10, 2009 Dr. J. Griggs

Put all work and answers in the stamped blue book provided. Make sure your **name, row letter, seat number, and form of test (A, B, or C)** are on the outside of your blue book – one problem per page please. The back of a page is considered a new page. Graphing calculators **(that do not do calculus)** can be used; however, you are not to use the graphics mode on this test when the question asks **YOU** to graph.

1.) a.) Simplify the difference quotient completely: $\dfrac{\dfrac{2}{5(x+h)}-\dfrac{2}{5x}}{h}$; provided $h \neq 0$.

b.) Find $f'(x)$: $f(x)=13x^4-\dfrac{3}{\sqrt{x}}+10\sqrt[3]{x}+6x-2$

2.) Find $f'(x)$ using the DEFINITION OF DERIVATIVE: $f(x)=-2x^2+5x-8$

3.) a.) Graph the following: $f(x)=\begin{cases} x^2-3\ , & x \geq -2 \\ 2x+1\ , & x<-2 \end{cases}$

b.) Is the function above (problem 3a) continuous at x = -2? (verify;3 possible steps)

4.) Find the equation of the tangent line to the graph of $f(x)=2x^2-\dfrac{4}{x}$ at the point (4,31).

5.) Find the vertex, **all** intercepts, and graph (no graphing calculator):
$f(x)=2x^2+3x-2$

6.) a.) Evaluate: $\lim\limits_{x \to 4} \dfrac{x^2-2x-8}{x-4}$

b.) Evaluate: $\lim\limits_{x \to \infty} \dfrac{4x^2-3}{x^2+3x-5}$

7.) Find the domain of f(x) and write your answer in interval notation: $f(x)=\dfrac{\sqrt{x+4}}{3x}$

8.) Find the average rate of change of f(x) from x = 1 to x = 2; find the instantaneous rate of change at x = 2. $f(x)=-16x^2+96x+20$

Bonus (5 points): For $f(x)=\sqrt{x}$, find $f'(x)$ using the DEFINITION OF DERIVATIVE.

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(12 POINTS EACH)

1.) a.) $\left[\frac{2}{5(x+h)} - \frac{2}{5x}\right] \cdot \frac{1}{h} = \left[\frac{2 \cdot x}{5(x+h) \cdot x} - \frac{2 \cdot (x+h)}{5x \cdot (x+h)}\right] \cdot \frac{1}{h}$

$= \left[\frac{2x - 2(x+h)}{5(x+h) \cdot x}\right] \cdot \frac{1}{h} = \frac{\cancel{2x} - \cancel{2x} - 2h}{5(x+h) \cdot x \cdot h} = \frac{-2\cancel{h}^{1}}{5(x+h) \cdot x \cdot \cancel{h}_{1}}$ $(h \neq 0)$

$= \frac{-2}{5(x+h) \cdot x}$

b.) $f(x) = 13x^4 - 3x^{-1/2} + 10x^{1/3} + 6x - 2$

$f'(x) = 13(4x^3) - 3(-\frac{1}{2}x^{-3/2}) + 10(\frac{1}{3}x^{-2/3}) + 6(1) - 0$

$f'(x) = 52x^3 + \frac{3}{2}x^{-3/2} + \frac{10}{3}x^{-2/3} + 6$

2.) $f(x) = -2x^2 + 5x - 8$ $f'(x)$ by def of deriv.

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[-2(x+h)^2 + 5(x+h) - 8] - [-2x^2 + 5x - 8]}{h}$

$= \lim_{h \to 0} \frac{-2(x^2 + 2xh + h^2) + 5x + 5h - 8 + 2x^2 - 5x + 8}{h}$

$= \lim_{h \to 0} \frac{\cancel{-2x^2} - 4xh - 2h^2 + \cancel{5x} + 5h - \cancel{8} + \cancel{2x^2} - \cancel{5x} + \cancel{8}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}^{1}(-4x - 2h + 5)}{\cancel{h}_{1}}$ $(h \neq 0)$ $= \lim_{h \to 0} (-4x - \underset{0}{\cancel{2h}} + 5) = -4x + 5$

3.) a.)

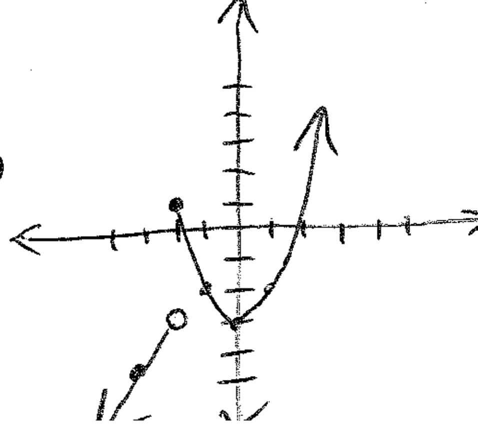
$y = x^2 - 3$ $(x \geq -2)$

x	y
-2	1
-1	-2
0	-3
1	

$y = 2x + 1$ $(x < -2)$

x	y
-2	-3
-3	-5
-4	-7

delete



TEST 1, FORM C (page 2)

3.) b.) ¿ contin. at $x=-2$?

1.) $f(-2)$ exists? yes $\quad f(-2)=1$, $(-2,1)$

2.) $\lim\limits_{x\to -2} f(x)$ exists?

$\lim\limits_{x\to -2^+} f(x) = 1 \qquad \lim\limits_{x\to -2^-} f(x) = -3$

$\lim\limits_{x\to -2} f(x) =$ does not exist

$\therefore$ DISCONTINUOUS

4.) $f(x) = 2x^2 - 4x^{-1} \quad f'(x) = 4x + 4x^{-2} = 4x + \frac{4}{x^2}$

$f'(4) = 4\cdot(4) + \frac{4}{4^2} = 16 + \frac{1}{4} = \frac{64}{4} + \frac{1}{4} = \frac{65}{4}$

$y - 31 = \frac{65}{4}(x-4)$

$4(y-31) = 65(x-4)$
$4y - 124 = 65x - 260$
$4y = 65x - 260 + 124$
$4y = 65x - 136$
$y = \frac{65}{4}x - 34$

5.) $y = 2x^2 + 3x - 2$

vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \quad \frac{-b}{2a} = \frac{-3}{4}$

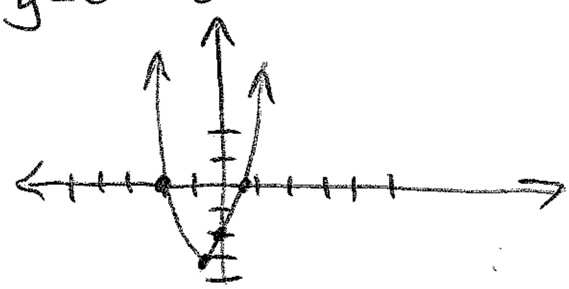
$f\left(\frac{-3}{4}\right) = 2\left(\frac{-3}{4}\right)^2 + 3\left(\frac{-3}{4}\right) - 2 = 2\left(\frac{9}{16}\right) - \frac{9}{4} - 2 = \frac{9}{8} - \frac{9}{4} - 2$

$= \frac{9}{8} - \frac{18}{8} - \frac{16}{8} = \frac{9-34}{8} = \frac{-25}{8} \approx -3.125 \quad \left(-\frac{3}{4}, -3.125\right)$ V

int: $x=0 \quad y = 2\cdot 0^2 + 3\cdot 0 - 2 = -2 \quad (0,-2)$

$y=0 \quad 0 = 2x^2 + 3x - 2 \quad 0 = (2x-1)(x+2)$

$x = \frac{1}{2}, -2 \quad \left(\frac{1}{2}, 0\right)$ \& $(-2,0)$



TEST 1, FORM C (page 3)

6.) a.) $\lim_{x\to 4} \frac{(x^2-2x-8)}{x-4} = \lim_{x\to 4} \frac{\cancel{(x-4)}(x+2)}{\cancel{(x-4)}}$ $(x \neq 4)$

$= \lim_{x\to 4} (x+2) = 6$

b.) $\lim_{x\to\infty} \frac{4x^2-3}{x^2+3x-5} = \lim_{x\to\infty} \frac{\frac{4x^2}{x^2} - \frac{3}{x^2}}{\frac{x^2}{x^2} + \frac{3x}{x^2} - \frac{5}{x^2}}$

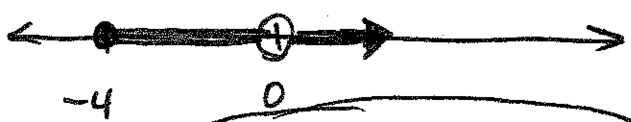
$= \lim_{x\to\infty} \frac{4 - \frac{3}{x^2} \to 0}{1 + \underbrace{\frac{3}{x}}_{0} - \underbrace{\frac{5}{x^2}}_{0}} = \frac{4}{1} = 4$

7.) $f(x) = \frac{\sqrt{x+4}}{3x}$ domain: ① $x+4 \geq 0$

$x \geq -4$

② $3x \neq 0$

$x \neq 0$



$[-4, 0) \cup (0, \infty)$

8.) $f(x) = -16x^2 + 96x + 20$

average rate of change, $x = 1$ to $x = 2$

AVE: $\frac{f(2) - f(1)}{2-1} = \frac{[-16(2)^2+96(2)+20] - [-16(1)^2+96(1)+20]}{1}$

$= \frac{[-64 + 192 + 20] - [-16 + 96 + 20]}{1}$

$= -64 + 192 + \cancel{20} + 16 - 96 - \cancel{20} = 48$

instantaneous rate of change at $x = 2$:

INST: $f'(x) = -32x + 96$ $f'(2) = -32(2) + 96 = 32$

Test 1; FORM C (page 4)

Bonus (5 points): $f(x) = \sqrt{x}$ find $f'(x)$

by <u>DEF. OF DERIVATIVE</u>:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{\cancel{h}^{\,1}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} \quad (h \neq 0)$$

$$= \lim_{h \to 0} \frac{1}{(\sqrt{x+\underset{0}{h}} + \sqrt{x})} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$