NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121 Test #1 Form B Thursday, September 10, 2009 Dr. J. Griggs

Put all work and answers in the stamped blue book provided. Make sure your **name, row letter, seat number, and form of test (A, B, or C)** are on the outside of your blue book – one problem per page please. The back of a page is considered a new page. Graphing calculators **(that do not do calculus)** can be used; however, you are not to use the graphics mode on this test when the question asks **YOU** to graph.

1.) a.) Simplify the difference quotient completely: $\dfrac{\dfrac{3}{2(x+h)}-\dfrac{3}{2x}}{h}$; provided $h \neq 0$.

b.) Find $f'(x)$: $f(x)=12x^4-\dfrac{5}{\sqrt{x}}+12\sqrt[3]{x}+7x-8$

2.) Find $f'(x)$ using the DEFINITION OF DERIVATIVE: $f(x)=-3x^2+8x-5$

3.) a.) Graph the following: $f(x)=\begin{cases} x^2-4 \ , & x \geq -2 \\ 2x+7 \ , & x<-2 \end{cases}$

b.) Is the function above (problem 3a) continuous at x = -2? (verify;3 possible steps)

4.) Find the equation of the tangent line to the graph of $f(x)=2x^2-\dfrac{4}{x}$ at the point (1,-2).

5.) Find the vertex, **all** intercepts, and graph (no graphing calculator):
$f(x)=2x^2-x-3$

6.) a.) Evaluate: $\lim\limits_{x \to 4} \dfrac{x^2-5x+4}{x-4}$

b.) Evaluate: $\lim\limits_{x \to \infty} \dfrac{3x^2+2}{x^2+4x-3}$

7.) Find the domain of f(x) and write your answer in interval notation: $f(x)=\dfrac{\sqrt{x+3}}{2x}$

8.) Find the average rate of change of f(x) from x = 1 to x = 4; find the instantaneous rate of change at x = 4. $f(x)=-16x^2+64x+50$

Bonus (5 points): For $f(x)=\sqrt{x}$, find $f'(x)$ using the DEFINITION OF DERIVATIVE.

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(12 POINTS EACH)

1.) a.) $\left[\frac{3}{2(x+h)} - \frac{3}{2x}\right]\cdot\frac{1}{h} = \left[\frac{3\cdot x}{2(x+h)\cdot x} - \frac{3\cdot(x+h)}{2x\cdot(x+h)}\right]\cdot\frac{1}{h}$

$= \left[\frac{3x-3(x+h)}{2(x+h)\cdot x}\right]\cdot\frac{1}{h} = \left[\frac{\cancel{3x}-\cancel{3x}-3h}{2(x+h)\cdot x\cdot h}\right] = \frac{-3\cancel{h}}{2(x+h)\cdot x\cdot \cancel{h}},\ (h\neq 0)$

$= \frac{-3}{2(x+h)\cdot x}$

b.) $f(x) = 12x^4 - 5x^{-1/2} + 12x^{1/3} + 7x - 8$

$f'(x) = 12(4\cdot x^3) - 5\left(-\frac{1}{2}\cdot x^{-3/2}\right) + 12\left(\frac{1}{3}\cdot x^{-2/3}\right) + 7(1) - 0$

$f'(x) = 48x^3 + \frac{5}{2}x^{-3/2} + 4x^{-2/3} + 7$

2.) $f(x) = -3x^2+8x-5$ find $f'(x)$ by def of derivative

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{[-3(x+h)^2+8(x+h)-5]-[-3x^2+8x-5]}{h}$

$= \lim_{h\to 0}\frac{-3(x^2+2xh+h^2)+8x+8h-5+3x^2-8x+5}{h}$

$= \lim_{h\to 0}\frac{\cancel{-3x^2}-6xh-3h^2\cancel{+8x}+8h\cancel{-5}\cancel{+3x^2}\cancel{-8x}\cancel{+5}}{h}$

$= \lim_{h\to 0}\frac{\cancel{h}(-6x-3h+8)}{\cancel{h}},\ (h\neq 0) = \lim_{h\to 0}(-6x-\underbrace{3h}_{0}+8) = -6x+8$

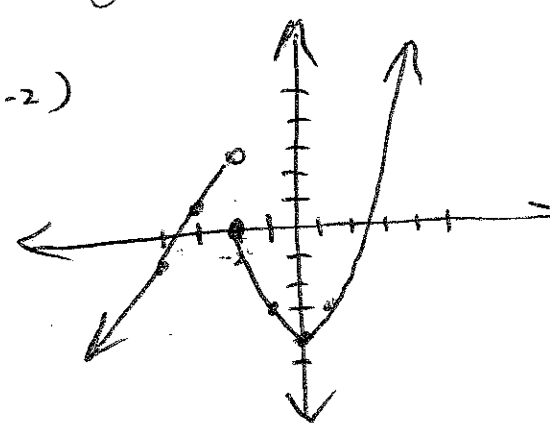
3.) $y = x^2-4\ (x\geq -2)$, $y = 2x+7\ (x<-2)$

a.)

x	y
−2	0
−1	−3
0	−4
1	−3

x	y
−2	3
−3	1
−4	−1

delete



Test 1, FORM B, (page 2)

3.) b.) ¿contin at $x=-2$?

1.) $f(-2)$ exists? yes, $f(-2)=0$, $(-2,0)$

2.) $\lim\limits_{x\to -2} f(x)$ exists?

$\lim\limits_{x\to -2^+} f(x)=0 \qquad \lim\limits_{x\to -2^-} f(x)=3$

$\lim\limits_{x\to -2} f(x) =$ does not exist

$\therefore$ DISCONTINUOUS

4.) $f(x)=2x^2-4x^{-1} \qquad f'(x)=4x+4x^{-2}=4x+\frac{4}{x^2}$

$f'(1)=4(1)+\frac{4}{1^2}=8=m_{TAN}$

$y-(-2)=8(x-1)$ $\qquad y=8x-8-2$

$y+2=8(x-1)$ $\qquad y=8x-10$

5.) $y=2x^2-x-3$

vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \qquad \frac{-b}{2a}=\frac{1}{4}$

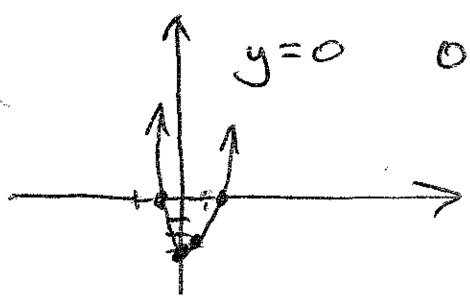
$f\left(\frac{1}{4}\right)=2\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)-3=2\left(\frac{1}{16}\right)-\frac{1}{4}-3=\frac{1}{8}-\frac{1}{4}-3$

$f\left(\frac{1}{4}\right)=\frac{1}{8}-\frac{2}{8}-\frac{24}{8}=\frac{1-26}{8}=\frac{-25}{8}\approx -3.125 \qquad V\left(\frac{1}{4},-3.125\right)$

int: $x=0 \quad y=2\cdot 0^2-0-3=-3 \quad (0,-3)$

$y=0 \quad 0=2x^2-x-3 \quad 0=(2x-3)(x+1)$

$x=3/2, -1 \quad (3/2,0)$ & $(-1,0)$



Test 1, FORM B, (page 3)

6.) a.) $\lim_{x \to 4} \frac{x^2-5x+4}{x-4} = \lim_{x \to 4} \frac{\cancel{(x-4)}(x-1)}{\cancel{(x-4)}}$ $(x \neq 4)$

$= \lim_{x \to 4} (x-1) = 4-1 = 3$

b.) $\lim_{x \to \infty} \frac{3x^2+2}{x^2+4x-3} = \lim_{x \to \infty} \frac{\frac{3x^2}{x^2} + \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{4x}{x^2} - \frac{3}{x^2}}$

$= \lim_{x \to \infty} \frac{3 + \frac{2}{x^2} \to 0}{1 + \underbrace{\frac{4}{x}}_{0} - \underbrace{\frac{3}{x^2}}_{0}} = \frac{3}{1} = 3$

7.) $f(x) = \frac{\sqrt{x+3}}{2x}$ domain: ① $x+3 \geq 0$
$x \geq -3$
② $2x \neq 0$
$x \neq 0$

$[-3,0) \cup (0,\infty)$

8.) $f(x) = -16x^2 + 64x + 50$
average rate of change, $x=1$ to $x=4$

AVE: $\frac{f(4)-f(1)}{4-1} = \frac{[-16(4)^2+64(4)+50] - [-16(1)^2+64(1)+50]}{3}$

$= \frac{[-\cancel{256} + \cancel{256} + 50] - [-16+64+50]}{3}$

$= \frac{\cancel{50} + 16 - 64 - \cancel{50}}{3} = \frac{-48}{3} = -16$

instantaneous rate of change at $x=4$:

INST: $f'(x) = -32x + 64$ $f'(4) = -32(4)+64 = -64$

Test 1; FORM B (page 4)

Bonus (5 points): $f(x) = \sqrt{x}$ find $f'(x)$

by DEF. OF DERIVATIVE:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{\cancel{h}^{1}}{\cancel{h}_{1}(\sqrt{x+h} + \sqrt{x})} \quad (h \neq 0)$$

$$= \lim_{h\to 0} \frac{1}{(\sqrt{x+\underset{0}{h}} + \sqrt{x})} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$