NORTH CAROLINA STATE UNIVERSITY Department of Mathematics

MA121 Test #1 Form A Thursday, September 10, 2009 Dr. J. Griggs

Put all work and answers in the stamped blue book provided. Make sure your **name, row letter, seat number, and form of test (A, B, or C)** are on the outside of your blue book – one problem per page please. The back of a page is considered a new page. Graphing calculators **(that do not do calculus)** can be used; however, you are not to use the graphics mode on this test when the question asks **YOU** to graph.

1.) a.) Simplify the difference quotient completely: $\dfrac{\dfrac{2}{3(x+h)}-\dfrac{2}{3x}}{h}$; provided $h \neq 0$.

b.) Find $f'(x)$: $f(x)=11x^5-\dfrac{4}{\sqrt{x}}+15\sqrt[3]{x}+8x-7$

2.) Find $f'(x)$ using the DEFINITION OF DERIVATIVE: $f(x)=-3x^2+5x-8$

3.) a.) Graph the following: $f(x)=\begin{cases} x^2-5 \ , & x \geq -2 \\ 2x+5 \ , & x<-2 \end{cases}$

b.) Is the function above (problem 3a) continuous at x = -2? (verify;3 possible steps)

4.) Find the equation of the tangent line to the graph of $f(x)=2x^2-\dfrac{4}{x}$ at the point (2,6).

5.) Find the vertex, **all** intercepts, and graph (no graphing calculator):
$f(x)=2x^2-3x-5$

6.) a.) Evaluate: $\lim\limits_{x \to 4} \dfrac{x^2-3x-4}{x-4}$

b.) Evaluate: $\lim\limits_{x \to \infty} \dfrac{2x^2+1}{x^2+4x-3}$

7.) Find the domain of f(x) and write your answer in interval notation: $f(x)=\dfrac{\sqrt{x+2}}{5x}$

8.) Find the average rate of change of f(x) from x = 1 to x = 3; find the instantaneous rate of change at x = 3. $f(x)=-16x^2+48x+100$

Bonus (5 points): For $f(x)=\sqrt{x}$, find $f'(x)$ using the DEFINITION OF DERIVATIVE.

MA121 TEST #1 FORM A 9/10/09

(12 POINTS EACH)

1.) a.) $\left[\dfrac{2}{3(x+h)} - \dfrac{2}{3x}\right]\cdot\dfrac{1}{h} = \left[\dfrac{2\cdot x}{3(x+h)\cdot x} - \dfrac{2\cdot(x+h)}{3x\cdot(x+h)}\right]\cdot\dfrac{1}{h}$

$$\frac{2x - 2(x+h)}{3(x+h)\cdot x\cdot h} = \frac{\cancel{2x} - \cancel{2x} - 2h}{3(x+h)\cdot x\cdot h} = \frac{-2\cancel{h}}{3(x+h)\cdot x\cdot \cancel{h}} = \frac{-2}{3(x+h)\cdot x}$$

b.) $f(x) = 11x^5 - 4x^{-1/2} + 15\cdot x^{1/3} + 8x - 7$

$f'(x) = 11(5x^4) - 4\left(-\tfrac{1}{2}x^{-3/2}\right) + 15\left(\tfrac{1}{3}\cdot x^{-2/3}\right) + 8(1) - 0$

$f'(x) = 55x^4 + 2x^{-3/2} + 5x^{-2/3} + 8$

2.) $f(x) = -3x^2+5x-8$ [$f'(x)$ by def. of deriv.]

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{[-3(x+h)^2+5(x+h)-8]-[-3x^2+5x-8]}{h}$$

$$= \lim_{h\to 0}\frac{-3(x^2+2xh+h^2)+5x+5h-8+3x^2-5x+8}{h}$$

$$= \lim_{h\to 0}\frac{-\cancel{3x^2}-6xh-3h^2+\cancel{5x}+5h-\cancel{8}+\cancel{3x^2}-\cancel{5x}+\cancel{8}}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(-6x-3h+5)}{\cancel{h}},\ (h\neq 0) = \lim_{h\to 0}(-6x-\underbrace{3h}_{0}+5) = -6x+5$$

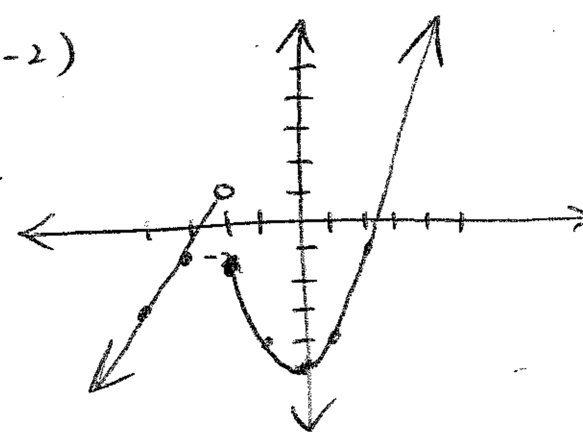
3.) $y = x^2-5\ (x \geq -2)$ | $y = 2x+5\ (x<-2)$

a.)

x	y
−2	−1
−1	−4
0	−5
1	−4
2	−1

x	y
−2	1
−3	−1
−4	−3

delete



Test 1; Form A (page 2)

3.) b.) ¿contin at $x = -2$?

1.) $f(-2)$ exists? yes, $f(-2) = -1$, $(-2,-1)$

2.) ¿ $\lim\limits_{x\to -2} f(x)$ exists?

$\lim\limits_{x\to -2^+} f(x) = -1$ $\lim\limits_{x\to -2^-} f(x) = 1$

$\lim\limits_{x\to -2} f(x) =$ does not exist

∴ DISCONTINUOUS

4.) $f(x) = 2x^2 - 4x^{-1}$ $f'(x) = 4x + 4x^{-2} = 4x + \frac{4}{x^2}$

$f'(2) = 4(2) + \frac{4}{2^2} = 8 + 1 = 9 = m_{TAN}$

$y - 6 = 9(x-2)$ $y = 9x - 18 + 6$

$y = 9x - 12$

5.) $f(x) = 2x^2 - 3x - 5$

vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ $-\frac{b}{2a} = \frac{3}{4}$

$f\left(\frac{3}{4}\right) = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) - 5 = 2\left(\frac{9}{16}\right) - \frac{9}{4} - 5$

$f(3/4) = \frac{9}{8} - \frac{9}{4} - 5 = \frac{9}{8} - \frac{18}{8} - \frac{40}{8} = \frac{9-58}{8} = \frac{-49}{8} \approx -6.125$

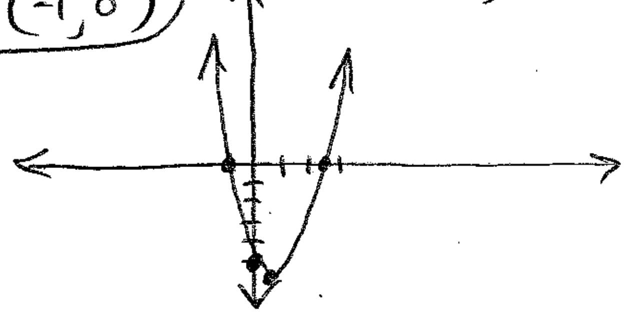
$V\left(\frac{3}{4}, -6.125\right)$

int: $x = 0$ $y = -5$ $(0,-5)$

$y = 0$ $0 = 2x^2 - 3x - 5$ $0 = (2x-5)(x+1)$

$x = \frac{5}{2}, -1$

$\left(\frac{5}{2}, 0\right)$ & $(-1, 0)$



Test 1; FORM A (page 3)

6.) a.) $\lim_{x\to 4} \dfrac{x^2-3x-4}{x-4} = \lim_{x\to 4} \dfrac{\cancel{(x-4)}(x+1)}{\cancel{(x-4)}} \quad (x \neq 4)$

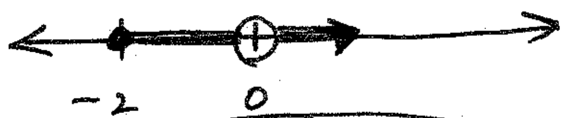
$= \lim_{x\to 4} (x+1) = 5$

b.) $\lim_{x\to\infty} \dfrac{2x^2+1}{x^2+4x-3} = \lim_{x\to\infty} \dfrac{\frac{2x^2}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{4x}{x^2}-\frac{3}{x^2}}$

$= \lim_{x\to\infty} \dfrac{2+\underbrace{\frac{1}{x^2}}_{\to 0}}{1+\underbrace{\frac{4}{x}}_{0}-\underbrace{\frac{3}{x^2}}_{0}} = \dfrac{2}{1} = 2$

7.) $f(x) = \dfrac{\sqrt{x+2}}{5x}$ domain: ① $x+2 \geq 0$

$x \geq -2$



② $5x \neq 0$

$x \neq 0$

$[-2,0) \cup (0,\infty)$

8.) $f(x) = -16x^2+48x+100$

average rate of change, $x=1$ to $x=3$:

AVE: $\dfrac{f(3)-f(1)}{3-1} = \dfrac{[-16(3)^2+48(3)+100]-[-16(1)^2+48(1)+100]}{2}$

$= \dfrac{[-144+144+100]-[-16+48+100]}{2}$

$= \dfrac{100+16-48-100}{2} = \dfrac{-32}{2} = -16$

instantaneous rate of change at $x=3$:

INST: $f'(x) = -32x+48 \qquad f'(3) = -32(3)+48 = -48$

Test 1; FORM A (page 4)

Bonus (5 points): $f(x) = \sqrt{x}$ find $f'(x)$

by DEF. OF DERIVATIVE:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h} - \sqrt{x})}{h} \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{\cancel{h}^{1}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} \quad (h \neq 0)$$

$$= \lim_{h \to 0} \frac{1}{(\sqrt{x+\underset{0}{h}} + \sqrt{x})} = \frac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$