MA 141-005 (White) Exam 4 100 Points 11/21/05

Show all your work. Name: Key

1. (10 points) Find the most general antiderivative of the function

$$f(x) = \frac{1+3x}{\sqrt{x}}.$$

$$f(x) = \frac{1}{\sqrt{x}} + 3\sqrt{x} = x^{-1/2} + 3x^{1/2}$$

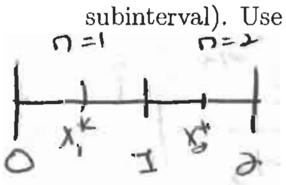
$$\Rightarrow F(x) = \frac{x^{1/2}}{1/2} + \frac{3x^{3/2}}{3/2} + c$$

$$= 2x^{1/2} + 2x^{3/2} + c$$

2. (15 points) Approximate the integral

$$\int_0^2 x^3\,dx$$

using the Midpoint rule (i.e., the Riemann sum $\sum_{i=1}^{n} f(x_i^*)\Delta x_i$ where each x_i^* is the midpoint of its subinterval). Use two subintervals (i.e., $n = 2$).



$x_1^* = \frac{1}{2}$

$x_2^* = \frac{3}{2}$

$f(x) = x^3$

$\Delta x = \frac{2-0}{2} = 1$

$$\int_0^2 x^3\,dx \approx \sum_{i=1}^{2} (x_i^*)^3\,\Delta x = \left(\tfrac{1}{2}\right)^3 \cdot 1 + \left(\tfrac{3}{2}\right)^3 \cdot 1$$

$$= \frac{1}{8} + \frac{27}{8} = \frac{28}{8} = \frac{14}{4} = \frac{7}{2}$$

3. (20 points) Given functions f and g, use the facts that $\int_0^4 f(x)dx = 4$, $\int_4^6 f(x)dx = 3$, $\int_0^6 g(x)dx = -5$, and $\int_2^6 g(x)dx = -4$ to evaluate the following integrals:
   (a) $\int_0^4 (2 \cdot f(x) + 1)dx$
   (b) $\int_0^6 f(x)dx$
   (c) $\int_0^6 [f(x) - 4 \cdot g(x)]\, dx$
   (d) $\int_0^2 g(x)dx$.

a. $\int_0^4 (2 \cdot f(x) + 1)dx = 2\int_0^4 f(x)dx + \int_0^4 dx$

$= 2 \cdot 4 + 4 = 12$

b. $\int_0^6 f(x)dx = \int_0^4 f(x)dx + \int_4^6 f(x)dx = 4+3 = 7$

c. $\int_0^6 [f(x) - 4 \cdot g(x)]dx = \int_0^6 f(x)dx - 4\int_0^6 g(x)dx$

$= 7 - 4(-5) = 27$

d. $\int_0^2 g(x)dx = \int_0^6 g(x)dx - \int_2^6 g(x)dx = -5 - (-4) = -5 + 1 = -1$

4. (15 points) Evaluate the following integrals:
   (a) $\int_1^7 \frac{1}{\sqrt[3]{t}}dt$
   (b) $\int e^x + \frac{5}{\sqrt[3]{x^2}}dx$

a. $\int_1^7 \frac{1}{t^{1/3}}dt = \int_1^7 t^{-1/3}dt = \left.\frac{t^{2/3}}{2/3}\right|_1^7$

$= \left.\frac{3}{2} t^{2/3}\right|_1^7 = \frac{3}{2}\left[7^{2/3} - 1^{2/3}\right]$

$= \frac{3}{2}\left[(49)^{1/3} - 1\right]$

b. $\int e^x dx + \int 5x^{-2/3}dx = e^x + \frac{5x^{1/3}}{1/3} + c$

$= e^x + 15x^{1/3} + c$

5. (10 points) Compute

$$\frac{d}{dx}\int_x^4 \sin(t^3)dt.$$

$$= -\frac{d}{dx}\int_4^x \sin(t^3)\,dt \quad = \quad -\sin(x^3)$$

6. (15 points) Using some kind of substitution, find the value of the following integral:

$$\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}dx.$$

Let $u = \ln x \quad \Rightarrow \quad du = \frac{1}{x}dx$

$X = e \quad \Rightarrow \quad u = \ln(e) = 1$

$X = e^4 \quad \Rightarrow \quad u = \ln(e^4) = 4$

$$\Rightarrow \int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\,dx = \int_1^4 \frac{1}{u^{1/2}}\,du = \int_1^4 u^{-1/2}\,du$$

$$= 2u^{1/2}\Big|_1^4 = 2\,[2-1] = 2$$

7. (15 points) Using integration by parts, find the value of

$$\int_0^{\pi} x\cos(x)dx.$$

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$\Rightarrow$ pick $u = x$, $dv = \cos(x)dx$

then $du = dx$, $V = \sin(x)$

$$\int_0^{\pi} x\cos(x)dx = x\sin(x)\Big|_0^{\pi} - \int_0^{\pi} \sin(x)dx$$

$$= [\pi\sin(\pi) - 0] + \cos(x)\Big|_0^{\pi}$$

$$= 0 + \cos(\pi) - \cos(0)$$

$$= -1 - 1$$

$$= -2$$