MA 141-005 Exam 3 100 Points 10/28/05

Show all your work. Name: Key

1. (12 points) Let $f(x) = x^{1/x}$. find the value of $f'(e)$.

$\ln[f(x)] = \ln(x^{1/x}) = \frac{1}{x}\ln x$

$[\ln f(x)]' = [\frac{1}{x}\ln(x)]'$

$\frac{1}{f(x)} f'(x) = \frac{-1}{x^2}\ln(x) + \frac{1}{x^2}$

$f'(x) = \frac{x^{1/x}}{x^2}[1 - \ln(x)]$

$f'(e) = \frac{x^{1/x}}{x^2}[1-1] = 0$

2. (12 points) Find a linear approximation for

$$f(x) = \frac{1}{(2+x)^3} = (2+x)^{-3}$$

at $a = 0$.

$f'(x) = -3(2+x)^{-4} = \frac{-3}{(2+x)^4}$

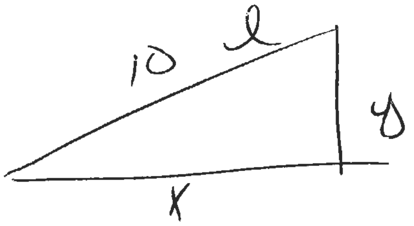
$f'(0) = \frac{-3}{2^4} = \frac{-3}{16}$

$y - f(0) = \frac{-3}{16}(x-0)$

$y - \frac{1}{8} = \frac{-3}{16}x$

$y = \frac{-3}{16}x + \frac{1}{8}$

3. (13 points) A ladder 10 feet long is leaning against a wall. If the foot of the ladder is being pulled away from the wall at 3 feet per second, how fast is the top of the ladder sliding down the wall when the foot of the ladder is 8 feet from the wall?



wh $x = 8$

$y = 6$

$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(l^2)$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = \cancel{2l\frac{dl}{dt}}\ 0$

$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -\frac{8}{6}(3) = \frac{-24}{6} = -4\frac{ft}{sec}$

It is slid at 4ft/sec.

4. (13 points) Using the 1st derivative find a local maximum and minimum of $x^3 - 27x + 8$. Using the 2nd derivative test determine whether these values are local maximums or minimums.

$f(x) = x^3 - 27x + 8$, $f'(x) = 3x^2 - 27$

$f'(x) = 0 = 3(x^2-9)$

$= 3(x+3)(x-3)$

$\boxed{x = \pm 3 \quad c.p.}$

$f''(x) = 6x$

$f''(3) = 18 > 0 \Rightarrow x = 3$, f has a min

$f''(-3) = -18 < 0 \Rightarrow x = 3$ is a max

5. (12 points) On what interval(s) is

$$f(x) = x^3 - 3x^2 - 9x$$

increasing? Decreasing? (Hint: Make a table.) What are the points of inflection for $f(x)$?

$f'(x) = 3x^2 - 6x - 9$
$= 3(x^2 - 2x - 3)$
$= 3(x-3)(x+1)$
$\Rightarrow x = 3, -1$ are c.p.

	$x-3$	$x+1$	$f'(x)$
$(-\infty, -1)$	$-$	$-$	$+$
$(-1, 3)$	$-$	$+$	$-$
$(3, \infty)$	$+$	$+$	$+$

f Increas.? $(-\infty, -1) \cup (3, \infty)$

Decr. $(-1, 3)$

$f''(x) = 6x - 6 \Rightarrow x = 1$ is a POI

6. (12 points) Two positive numbers have a product 200. Find the minimum value of the sum of one number plus twice the other.

$xy = 200 \Rightarrow y = \frac{200}{x}$

$S = x + 2y \Rightarrow$ ~~xxxx~~ $S = x + 2\left(\frac{200}{x}\right)$

$\Rightarrow S = x + \frac{400}{x}$

$S'(x) = 1 - \frac{400}{x^2}$

$\Rightarrow x = \pm 20$ or 20 $(x > 0)$

$\Rightarrow y = 10$

7. Answer the following questions concerning L'Hopital's (L'Hospital's) Rule concerning the limit

$$\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right)$$

(a) (7 points) How might you rewrite this to use L'Hopital's Rule? What indeterminate form does this rewritten version have?

i.) $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}$

ii) $\frac{0}{0}$ form

(b) (7 points) Can we use L'Hopital's Rule on this problem? How do we know? What is the above limit?

i. yes. $\sin\left(\frac{1}{x}\right)$ is always diff'able. $\frac{1}{x}$ is diff'able as $x\to\infty$. ~~at~~ f' is near ~~cont~~ "zero" but not quite.

$$\lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)\left(\cancel{-\frac{1}{x^2}}\right)}{\cancel{-\frac{1}{x^2}}} = \lim_{x\to\infty} \cos\left(\frac{1}{x}\right) = 1$$

8. Do the following requested components concerning

$$\sqrt[5]{34}.$$

(a) (4 points) Rewrite this in the form $f(x) = 0$ so that you can use Newton's Method to find $\sqrt[5]{34}$.

$x = \sqrt[5]{34}$ $x^5 - 34 = 0$

$f(x) = x^5 - 34$

$f'(x) = 5x^4$

(b) (4 points) If the initial iterate $x_1 = 2$ is chosen, what is the second approximation x_2?

$$x_2 = 2 - \frac{(2)^5 - 34}{5(2)^4}$$

$$= 2 - \left(\frac{32-34}{5(16)}\right)$$

$$= 2 - \left(\frac{-2}{80}\right) = 2 + \frac{1}{40} = \boxed{\frac{81}{80}}$$

(c) (4 points) What problem might we have with Newton's Method based upon values of $f(x)$ or $f'(x)$?

if $f'(x) = 0$ ~~th~~

Newton step ~~the~~ fails