

Show all your work.

Name: _____

1. (12 points) Let $f(x) = x^{1/x}$. find the value of $f'(e)$.

2. (12 points) Find a linear approximation for

$$f(x) = \frac{1}{(2+x)^3}$$

at $a = 0$.

3. (13 points) A ladder 10 feet long is leaning against a wall. If the foot of the ladder is being pulled away from the wall at 3 feet per second, how fast is the top of the ladder sliding down the wall when the foot of the ladder is 8 feet from the wall?

4. (13 points) Using the 1st derivative find a local maximum and minimum of $x^3 - 27x + 8$. Using the 2nd derivative test determine whether these values are local maximums or minimums.

5. (12 points) On what interval(s) is

$$f(x) = x^3 - 3x^2 - 9x$$

increasing? Decreasing? (Hint: Make a table.) What are the points of inflection for $f(x)$?

6. (12 points) Two positive numbers have a product 200. Find the minimum value of the sum of one number plus twice the other.

7. Answer the following questions concerning L'Hopital's (L'Hospital's) Rule concerning the limit

$$\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right)$$

- (a) (7 points) How might you rewrite this to use L'Hopital's Rule? What indeterminate form does this rewritten version have?

- (b) (7 points) Can we use L'Hopital's Rule on this problem? How do we know? What is the above limit?

8. Do the following requested components concerning

$$\sqrt[5]{34}.$$

- (a) (4 points) Rewrite this in the form $f(x) = 0$ so that you can use Newton's Method to find $\sqrt[5]{34}$.
- (b) (4 points) If the initial iterate $x_1 = 2$ is chosen, what is the second approximation x_2 ?
- (c) (4 points) What problem might we have with Newton's Method based upon values of $f(x)$ or $f'(x)$?