Share-Altering Technical Progress

by

John J. Seater

Economics Department
North Carolina State University
Raleigh, NC 27612

john_seater@ncsu.edu

January 2004

*I thank Francesco Caselli, John Dawson, Adrian Fleissig, Oded Galor, Claudiney Pereira, and Paul Segerstrom for helpful comments and criticisms.

Abstract

The fact that factor shares change over time was well-established by a substantial literature thirty to forty years ago, yet the systematic temporal variation in factor shares has received little attention. The chapter explores the macroeconomic behavior and implications of share-altering technical change. For the sake of analytical tractability, the analysis is restricted to the simple case of Cobb-Douglas production, but even in such a restricted framework a number of intriguing results are obtained. Share-altering technical change offers explanations for several seemingly unrelated phenomena: the Luddite uprisings in early 19th England, the widening gap between skilled and unskilled wages in the U.S., and the failure by many underdeveloped countries to adopt recent technological advances. Also, share-altering change can make total factor productivity a misleading indicator of both the direction and magnitude of technical progress. In the short run, share-altering technical change can produce a non-monotonic adjustment path of output. In the long run, share-altering technical change tends to be self-perpetuating. Also, although there can be no balanced growth in the presence of share-altering technical progress, growth nevertheless can be asymptotically balanced. Finally, share-altering technical change raises doubts about the plausibility of some endogenous growth models.

I. Introduction

This chapter examines some implications of technical change that alters the relative shares of output accruing to the factors of production. This type of technical change was studied extensively in the first "golden age" of growth theory (after Solow-Swan to about 1980) but until very recently has been largely ignored in the theoretical literature of the second golden age (the post-Romer literature on endogenous growth), which generally treats technical change exclusively as an increase in total factor productivity. Similarly, in the real business cycle literature, productivity shocks always take the form of changes in the total factor productivity coefficient in front of the production function; factor shares are assumed unchanged.

There are at least two reasons why share-neutral technical change seems likely to be the exception rather than the rule. The first is the nature of the production function itself. A production function is merely a mathematically convenient way to express the relation between output on the one hand and factors of production on the other. Technical change by definition alters that relation. There seems no reason whatsoever to assume *a priori* that such a change in the production function is confined to only one of the function's parameters, and changes in parameters other than the neutral TFP coefficient generally alter factor shares.[1]

The second and more important reason for doubting share-neutrality of technical change is that the data show systematic changes in factor shares over time, a fact that has been known for several

---

[1]For example, in the CES production function

$$Y = A[aK^{\psi} + (1-a)L^{\psi}]^{1/\psi}$$

changes in either a or ψ alter factor shares.

decades[2]. An example of this "old knowledge" is given in Table 1, which is taken from Sato (1970) and reports capital's share in the private non-farm sector of the US economy over the period 1909-1960. Figure 1 plots the data and shows quite clearly that physical capital's share of output fluctuated considerably from year to year and followed an overall downward trend over the entire period. Capital's share has a maximum of 0.397 and a minimum of 0.301, nearly a 33 percent increase from the minimum to the maximum, a large swing for a number that constitutes the exponent of the Cobb-Douglas production function. The time trend is statistically significant. Regressing the log of capital's share on time yields

$$\log \alpha_t = \underset{(0.96785)}{1.56048} - \underset{(0.00050)}{0.00137}\, t$$

where α is capital's share and numbers in parentheses are standard errors. The p-value of time's coefficient is 0.0085. In addition, there is a great deal of fluctuation about the trend, as indicated by the adjusted R-squared value of only 0.113. These data belie the oft-heard assertion that "capital's share is roughly constant at about thirty percent"; capital's share quite clearly is not constant over either the short run or the long run. Sato and Hoffman (1968) present data for Japan over the period 1930-1960; the regression coefficient of capital's share on the log of time is -0.039 with a standard error of 0.014 (p-value of 0.0094) and an adjusted R-squared of 0.189. Goldberg's (1964) data for Canada over the period 1926-1958 give a regression coefficient on time of -0.0039 with a standard error of 0.0017 (p-value of 0.025) and an adjusted R-squared of 0.124. Schultze (1964) provides further evidence of

[2]As far as I can tell, Solow (1958) was the first to point out the temporal variability of factor shares.

considerable short-run variation in U. S. factor shares over the period 1922-1959. More recent data confirm that intertemporal variability in factor shares continues to the present. Kahn and Lim (1998) show that the shares of equipment, production workers, and non-production workers in the U.S. all have trends and variation about those trends over the period 1959-91. Krueger (1999) shows that raw (or unskilled) labor's share in the U.S. has fallen by about half over the past sixty years. Blanchard (1997, 1998) finds that capital's share in Europe has been similarly variable, falling before the mid-1980s and then rising sharply thereafter in many European countries. We have more than just simple statistics showing changes in capital's share. A wide range of formal econometric investigations finds capital and labor shares changing over time. Sato and Kendrick (1963), David and Klundert (1965), Sato (1970), Sato and Hoffman (1968), to name a few, do so by estimating production functions; Binswanger (1974), Stevenson (1980), and Kumbhakar and Lozano-Vivas (2002) reach the same conclusions by estimating cost functions. Finally, the past thirty years or so in the US have seen a large increase in skilled labor's output share at the expense of unskilled labor's share (Bound and Johnson, 1995)[3]. Factor shares clearly are not constant, even approximately.

The present chapter takes share-altering technical change as exogenous and analyzes some of its macroeconomic implications. The discussion begins with an examination of the effects of share-altering technical change on the economy's steady state and the properties of the dynamic adjustment path. The conditions under which a share-altering technical change will be adopted are discussed.

---

[3]Katz and Murphy (1992), among others, document that the wages of skilled workers have increased relative to those of unskilled workers. Bound and Johnson (1995) show that the supply of skilled workers also has increased relative to that of unskilled workers, thus establishing that skilled workers' income share has risen relative to that of unskilled workers.

Share-altering change can produce adjustment paths for output that are not monotonic. In particular, share-altering technical change that is beneficial in a present value sense nonetheless can cause output to fall initially. This behavior contrasts sharply with that implied by technical progress that raises only total factor productivity. That kind of progress necessarily produces monotonic adjustment paths for output, at least in the simple kinds of models considered here. It also is shown that, when technical progress includes elements that alter factor shares, total factor productivity can fall (rise) as the result of technical change that raises (lowers) aggregate output, suggesting that total factor productivity may be a rather poor indicator of technical progress. Share-altering technical progress also tends to be self-perpetuating. Technical change that raises a factor's income share leads to an increase in that factor relative to other factors. That increase in turn makes further increases in the same factor's share more valuable. Finally, the growth implications of share-altering technical progress are examined. It is shown that such progress is capable of generating sustained growth of output that is asymptotically balanced[4]. The limiting growth rate may be self-perpetuating because the limiting production function appears to be of AK form. The growth discussion raises interesting questions about the structure of the R&D sector of the economy.

[4]An enormous early literature examined the origins and extent of biased technical progress, beginning with Kennedy (1964). The recent shift in relative wages for skilled and unskilled labor in the U. S. has revived research into biased change. Caselli (1999), Kiley (1999), and Acemoglu (2003) exemplify recent theoretical investigations. Some investigators, such as Borjas and Ramey (1994) and Dinopoulos, Syropoulos, and Xu (1999), argue that changes in the pattern of trade explain the relative wage shift between skilled and unskilled workers. Although possibly true, that explanation begs the question of why trade patterns have shifted. Apparently, the ultimate explanation must be changes in technology, government intervention, or tastes. Baldwin and Cain (2000), Harrigan and Balaban (1999), and Bartel and Sicherman (1999) are among those providing evidence that technical change explains the relative wage shift.

An interesting aspect of the analysis suggesting the importance of share-altering technical change in economic life is that the theory offers a unified explanation for several quiet unrelated events: the Luddite uprisings of early 19th century England, the changes in relative wages of U.S. skilled and unskilled workers in the late 20th century, and the failure of some countries, notably underdeveloped ones, to adopt the latest technological breakthroughs and instead to continue using outmoded production methods.

II. General Model Specification

We will confine attention to a one-sector model of final output Y produced by three factors of production: labor L, capital K, and human capital H. With human capital entering as a separate factor of production, we should interpret L as (man hours of) raw or unskilled labor. In all that we do, production will be Cobb-Douglas:

$$\begin{aligned} Y &= F(K, L, H) \\ &= AK^{\alpha}H^{\beta}L^{1-\alpha-\beta} \end{aligned} \tag{1}$$

where α, β, and $1-\alpha-\beta$ are the income shares of K, H, and L, respectively. We assume constant returns to scale, so $0 < \alpha, \beta, \alpha+\beta < 1$. The use of Cobb-Douglas production to discuss variable factor shares may seem surprising; the usual assumption with Cobb-Douglas is that shares are constant. Samuelson (1965), for example, begins his discussion of induced innovation with the Cobb-Douglas case and imposes without discussion the restriction that factor shares are constant, and Duffy and Papageorgiou (2000) do the same thing. However, the Cobb-Douglas specification does not require constancy of shares. Indeed, Sato and Beckmann (1968) study production functions in Germany, Japan, and the United States and find that a Cobb-Douglas function with time-varying factor shares is acceptable for all three countries. For Japan, it even offers the best fit among the fourteen types of production function considered. Given these results, the use of Cobb-Douglas production with changing factor shares in the following theoretical discussion seems not at all unreasonable[5]. Those skeptical of its validity can take its use here as a mathematical convenience; many of the results derived below apparently would carry over to more complex specifications, such as the CES function.

The accumulation equations for K and H are

(2) $$dK/dt = I_K - \delta K$$

(3) $$dH/dt = I_H - \delta H$$

where I_K and I_H are investment in K and H and must satisfy the usual constraints

(4) $$0 \leq I_K \leq Y$$

(5) $$0 \leq I_H \leq Y$$

(6) $$0 \leq I_K + I_H \leq Y$$

The representative household seeks to maximize its utility subject to the appropriate constraints. In general, utility would depend on both consumption and labor and also possibly human capital.

---

[5]A number of early studies (e.g., Sato, 1970) present evidence that the elasticity of substitution is less than one, which would rule out the Cobb-Douglas function. However, those studies all fail to distinguish between raw labor time and human capital, attributing all labor income to man-hours. If human capital were lumped with physical capital rather than with man-hours, the estimated elasticity of substitution would rise substantially. Duffy and Papageorgiou (2000) recently have presented evidence rejecting the Cobb-Douglas function for many countries; however, they consider only a specification with fixed factor shares. It thus is not clear that the Cobb-Douglas function is ruled out by existing evidence.

Allowing variable labor complicates the analysis without adding any insight, so to keep things as simple as possible, we suppose that labor is fixed (no variation in household hours and also no population growth). Similarly, including human capital in the utility function adds analytical complications but not insight. We therefore treat utility as a function of consumption alone. The household's maximization problem then is

(7) $$\max_{C, I_K, I_H} \int_0^{\infty} U(C_t) e^{-\rho t} dt$$

subject to (2)-(6) and to the overall budget constraint

(8) $$0 = Y - C - I_K - I_H$$

and the bounds on consumption

(9) $$0 \leq C \leq Y$$

Although (9) permits the corner solutions of C = 0 and C = Y, all interesting behavior that we shall analyze occurs in the interior, so henceforth we will ignore the possibility of corner solutions in C. Corner solutions in the two investment quantities I_K and I_H, however, remain important, and we will consider them where necessary.

III. Two Factors of Production

The general model has two state variables, K and H, so its behavior is rather complex. We therefore begin by restricting attention to the special case where K and H are lumped together into a single aggregate (which will be denoted by K). Mathematically, we can achieve this reduction in dimension simply by setting β equal to zero. The model then reduces to the standard Cass growth model with growth restricted to zero because there is no population growth or exogenous technical progress. This simplification will allow us to derive some sharper results concerning the system's dynamics than in the general model, and in any case this simpler model is widely used in much of macroeconomic analysis, so studying it in some detail is worthwhile. We will examine the three-factor model later.

The production function now is:

(10) $$Y = AK^{\alpha}L^{1-\alpha}$$

We can use equation (8) to solve for I_K in terms of everything else and rewrite equation (2) as

(11) $$dK/dt = F(K,L) - C - \delta K$$

The current value Hamiltonian is

(12) $$V = U(C) + \psi[F(K,L) - C - \delta K]$$

where ψ is the costate variable. The necessary conditions are

(13) $$dK/dt = \partial V/\partial\psi = F(K,L) - C - \delta K$$

(14) $$d\psi/dt = -\partial V/\partial K + \rho\psi = -\psi[\alpha AK^{\alpha-1}L^{1-\alpha} - (\rho+\delta)]$$

(15) $$\partial V/\partial C = 0 = U_C - \psi$$

(16) $$K_0 \text{ given}$$

(17) $$\lim_{t\to\infty} \psi_t K_t e^{-\rho t} = 0$$

We obtain the equilibrium loci from these conditions in the usual way and so do not dwell on the details of their derivation. The phase diagram is shown in Figure 2. The equilibrium locus for capital (dK/dt = 0) is given by

(18) $$0 = AK^{\alpha}L^{1-\alpha} - C(\psi) - \delta K$$

where C(ψ), obtained from the first-order condition (15), is the function relating consumption to the costate variable ψ and having a negative first derivative: C'(ψ) < 0. The equilibrium locus for the costate variable (dψ/dt = 0) is given by

(19)
$$0 = \alpha A K^{\alpha-1} L^{1-\alpha} - (\rho + \delta)$$

from which we obtain the steady state value of K:

(20)
$$K^{*} = \left[\left(\frac{\alpha A}{\rho+\delta}\right)\right]^{\frac{1}{1-\alpha}} L$$

As always, approach to the steady state is monotonic.

III.A. *Pure Share-Altering Technical Change.*

Let us now consider a technical invention alters only the factor shares, leaving total factor productivity *A* the same. In particular, suppose an invention is discovered that raises α[6]. Will the invention be adopted? To decide, we examine the effect of an increase in α on the path of output. The impact effect on Y of an increase in α is the change in Y induced by a change in α while keeping K at its initial level K_0:

(21)
$$\begin{aligned} \partial Y/\partial\alpha &= AK^{\alpha}L^{1-\alpha}\ln(K/L) \\ &= Y\ln(K/L) \\ &\gtrless 0 \quad \text{as} \quad \ln(K/L) \gtrless 0 \end{aligned}$$

The dynamic effect on Y takes into account the change in the steady state value of Y brought about by the change in K:

---

[6]Results for the case where the invention reduces α are completely symmetric to those discussed here.

(22)

$$dY/d\alpha = (1-\alpha)^{-1}AK^{\alpha}L^{1-\alpha}[1+\ln(K/L)]$$
$$= (1-\alpha)^{-1}Y[1+\ln(K/L)]$$
$$\gtrless 0 \quad \text{as} \quad 1+\ln(K/L) \gtrless 0$$

Clearly, $\partial Y/\partial\alpha < dY/d\alpha$, so there are three cases to consider: (i) 0 < $\partial Y/\partial\alpha$, $dY/d\alpha$, (ii) $\partial Y/\partial\alpha$ < 0 < $dY/d\alpha$, and (iii) $\partial Y/\partial\alpha$, $dY/d\alpha$ < 0.

In case (i), the invention always is adopted. An increase in α raises K*, causing the dψ/dt=0 locus to shift to the right[7]. Because ∂Y/∂α > 0 in case (i), it is straightforward to see from (18) that the dK/dt=0 locus shifts down. We have the situation shown in Figure 3. The steady state moves from point E_0 to point E_1, at which the capital stock is higher and ψ is lower (and therefore consumption C is higher) than originally. The exact dynamic adjustment path that will be taken to the new steady state is unclear. It could be that, immediately after the invention is adopted, ψ jumps up (C jumps down) and the upper path P_U is followed, or it could be that ψ jumps down (C jumps up) and the lower path P_L is followed. Which path is taken depends on the magnitudes of the production and utility function derivatives and of the other system parameters, but in any case adoption of the invention is optimal[8]. Also, output goes monotonically to its new steady state value.

---

[7]The derivative of K* with respect to α, obtained from (20), is:

$$\frac{\partial K*}{\partial\alpha} = \left(\frac{1}{1-\alpha}\right)K\left[\frac{1}{\alpha}+\ln\left(\frac{K}{L}\right)\right]$$

The term in brackets exceeds 1+ ln(K/L) because α > 1. We are assuming that ∂Y/∂α > 0, which requires 1+ ln(K/L). Consequently, ∂K*/∂α > 0.

[8]See the Appendix for the proof.

The invention is never adopted in case (iii). Output and consumption are lower along the entire adjustment path, so utility is unambiguously reduced by adoption.

More interesting is case (ii), where adoption initially reduces output but ultimately raises it. In this case, an increase in α shifts the dK/dt=0 locus up, and we have the situation shown in Figures 4 and 5. Figure 4 is drawn with the new steady state value ψ_1 of the costate variable above the initial value ψ_0. The value of ψ is above ψ_0 along any possible dynamic adjustment path, meaning that consumption and thus utility are always lower than they were before adoption. Adoption is unambiguously suboptimal. Another possibility, however, is shown in Figure 5, where ψ_1 is below ψ_0, implying that consumption and utility eventually exceed their pre-adoption levels. If the eventual increase in utility is great enough to dominate the early decline, adoption will be optimal. For adoption to be optimal, adoption must increase lifetime utility:

(23)
$$\int_0^\infty U[C(\psi_t)]e^{-\rho t}dt > \int_0^\infty U[C(\psi_0)]e^{-\rho t}dt = \frac{U[C(\psi_0)]}{\rho}$$

where ψ_0 on the right side is the pre-invention steady state value of ψ and ψ_t on the left side is the current value of ψ as it moves along the dynamic adjustment path to the new steady state. On the right side, we can use (15) to solve for ψ_0 in terms of A, ρ, δ, L, and α. However, on the left side, we must know the entire path of ψ, which requires an explicit forms for the utility function U. Even then, an analytic solution usually is not possible. Nonetheless, there generally will be situations in which adoption is optimal. For such a situation to arise, the parameters A, ρ, δ and α must satisfy the two inequalities that define case (ii):

$$\ln\left(\frac{K}{L}\right) < 0 < 1+\ln\left(\frac{K}{L}\right)$$

which are equivalent to

$$-1 < \ln\left(\frac{\alpha A}{\delta+\rho}\right)^{\frac{1}{1-\alpha}} = \ln\left(\frac{K}{L}\right) < 0$$

where we have used (20) to substitute for K/L. In addition, the parameters A, ρ, δ, L, α, and the parameters of the utility function must satisfy (23). We thus have a total of three inequalities to satisfy and many more than three parameters to vary. In general, we should be able to find combinations that satisfy all three inequalities, thus leading to the conclusion that adoption of the invention can be optimal even though it initially reduces output, consumption, and utility.

An interesting aspect of this case is that, in those situations when adoption is optimal, output's path to the new steady state is not monotonic; Y first declines and subsequently rises. This behavior contrasts sharply with increases in total factor productivity (the coefficient A in our model). In response to technical progress that raises only TFP, output's path to the new steady state must be monotonic.

Another interesting aspect of share-altering technical change is that it tends to be self-perpetuating. For both cases (i) and (ii), the capital stock rises if adoption occurs. The change in K induced by an increase in α is:

(24)
$$\frac{dK}{d\alpha} = \left(\frac{1}{1-\alpha}\right)K\left[\frac{1}{\alpha}+\ln\left(\frac{K}{L}\right)\right]$$
$$> 0 \quad \text{if and only if} \quad \left[\frac{1}{\alpha}+\ln\left(\frac{K}{L}\right)\right] > 0$$

However,

$$\left[\frac{1}{\alpha}+\ln\left(\frac{K}{L}\right)\right] > \left[1+\ln\left(\frac{K}{L}\right)\right] \quad \text{because} \quad 0 < \alpha < 1$$

Adoption requires that the expression on the right side of the foregoing inequality (which determines the sign of $dY/d\alpha$) be positive, so if adoption occurs, the expression on the left side of the inequality also will be positive and K will increase. Because labor is constant, the increase in K guarantees that the economy's capital/labor ratio also rises. This rise in K/L makes ln(K/L) more positive and so makes future adoption of the same kind of technical change more desirable. In this sense, technical change that alters shares in a given direction tends to perpetuate itself.

III.B. *An Application to Economic History: Luddism.*

It is possible for an increase in α to raise total income but reduce labor's income. Labor income Y^L is total income multiplied by labor's share, $(1 - \alpha)Y$. The derivative of labor income with respect to α is

$$\frac{dY^L}{d\alpha} = -Y+(1-\alpha)\frac{\partial Y}{\partial \alpha}$$

$$= Y\ln(K/L)$$

Comparing this expression with (22) shows that $dY/d\alpha$ can be positive at the same time that $dY^L/d\alpha$ is negative, so that an increase in α can simultaneously raise output and lower labor income. Intuitively, the pie is increased but labor's share is decreased by more than an offsetting amount, leaving labor with less income even though total income is higher.

This possibility may have characterized England's situation during the famous episode of the Luddites. The Luddites were textile workers who sporadically destroyed textile machinery in various English towns over the period 1811-1816, blaming the machines for the prevailing unemployment and low wages. The Luddites usually are written off in the history books as misguided fanatics, but in fact they may have been right about the cause of low wages, at least in their industries. If the textile machines (or perhaps "new and improved" machines that replaced older ones) raised both income and capital's share, and if the capital/labor ratio was low, then the introduction of the machines could have simultaneously raised total output and capital income on the one hand and reduced labor's income on the other hand. Indeed, the Luddite uprisings occurred fairly early in the Industrial Revolution, when the capital/labor ratio was low compared to the levels it subsequently reached, so this story is at least plausible. The subject merits further examination by economic historians.

III.C. *Total Factor Productivity as a Measure of Technical Progress.*

In general, one should expect that a real-world invention generally would alter all the parameters of the production function, not just one. We can call this mixed technical change. In our simple Cobb-Douglas case with two factors of production, there are only two parameters to consider: total factor productivity A and capital's share α. The qualitative results obtained in section III.A above are not altered by allowing A to change as well as α, so we need not dwell on the mathematical details here. However, mixed technical change does have an interesting implication for the usefulness of total factor productivity as a measure of technical progress.

The total differential for Y is

(25)
$$dY = \left(\frac{1}{1-\alpha}\right) Y\{(1/A)dA + [1 + \ln(K/L)]d\alpha\}$$
$$\gtrless 0 \quad \text{as} \quad dA + A[1 + \ln(K/L)]d\alpha \gtrless 0$$

It is immediately clear that total factor productivity A may be a misleading indicator of technical progress. It is possible for Y to increase in response to a technical change even though the change reduces total factor productivity. The condition necessary for this outcome is

$$dA + A[1 + \ln(K/L)]d\alpha > 0 \tag{26}$$

$$\Leftrightarrow \quad dA > -A[1 + \ln(K/L)]d\alpha$$

Recall that for this discussion we are assuming $d\alpha > 0$. If the coefficient on $d\alpha$ is positive, the economy can experience an increase in output even if dA is negative, as long as dA is less negative than the right side of the second line above. We thus can have situations in which technical progress raises total output but is nonetheless associated with a *decline* in total factor productivity. Symmetrically, we can have situations in which Y falls even though A rises[9]. In such cases, TFP does not just mismeasure the magnitude of technical change but even gets the direction wrong. We thus have another problem to add to the list of reasons to distrust TFP as a measure of technical progress.

IV. Three Factors of Production

We now return to the full model in which physical and human capital enter separately in the production function. Most of the characteristics of the steady state and the dynamic adjustment path are straightforward generalizations of the results obtained for the two-factor model, so the details can be skipped or relegated to the Appendix. The three-factor model merits special attention because it offers possible explanations for some observed behavior.

---

[9]Galor and Moav (2000) obtain a similar result for very different reasons. In their model, a change in the rate of technical progress (measured as (dA/dt)/A) causes the initial level of TFP to change in the opposite direction.

IV.A. *Steady State and Dynamics.*

The steady state values of K and H are easily obtained:[10]

$$K = \alpha^{\frac{1-\beta}{1-\alpha-\beta}} \beta^{\frac{\beta}{1-\alpha-\beta}} \left(\frac{A}{\delta+\rho}\right)^{\frac{1}{1-\alpha-\beta}} L$$

$$H = \alpha^{\frac{\alpha}{1-\alpha-\beta}} \beta^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\frac{A}{\delta+\rho}\right)^{\frac{1}{1-\alpha-\beta}} L$$

The steady state ratio of K to H therefore is

$$\frac{K}{H} = \frac{\alpha}{\beta}$$

We are interested in how the economy responds to a technical advance that alters factor shares. The number of possible combinations of parameter changes is now considerably larger than previously; to keep the discussion tractable and to concentrate on the most interesting case, we restrict attention to a technical change that alters the relative shares of human capital and unskilled labor while leaving physical capital's share unchanged: β rises, $1\text{-}\alpha\text{-}\beta$ falls, and α is unchanged. We also simplify the discussion by supposing that total factor productivity *A* is unchanged. We thus have a situation that parallels that of section III.A above, with human capital taking the place of physical capital in the discussion.

The derivatives of K and H with respect β, written in percentage form, are

---

[10]See the Appendix.

(27) $$\frac{dK}{d\beta}\frac{1}{K} = \left[\frac{1}{1-\alpha-\beta} + \frac{\alpha}{(1-\alpha-\beta)^2}\log\alpha \right.$$

$$\left. + \frac{1-\alpha}{(1-\alpha-\beta)^2}\log\beta + \frac{1}{(1-\alpha-\beta)^2}\log\left(\frac{A}{\delta+\rho}\right)\right]L$$

$$= \frac{1}{1-\alpha-\beta}[1 + \ln(H/L)]$$

(28) $$\frac{dH}{d\beta}\frac{1}{H} = \left[\left(\frac{1-\alpha}{\beta}\right)\left(\frac{1}{1-\alpha-\beta}\right) + \frac{\alpha}{(1-\alpha-\beta)^2}\log\alpha \right.$$

$$\left. + \frac{1-\alpha}{(1-\alpha-\beta)^2}\log\beta + \frac{1}{(1-\alpha-\beta)^2}\log\left(\frac{A}{\delta+\rho}\right)\right]L$$

$$= \frac{1}{1-\alpha-\beta}\left[\frac{1-\alpha}{\beta} + \ln(H/L)\right]$$

As in the two-factor case, the signs of both these expressions are ambiguous in general and depend on the magnitudes of the production function's parameters and of the rate of time preference. However, $(dK/d\beta)/(1/K) < (dH/d\beta)/(1/H)$ because the two expressions differ only in the first term and the share $1-\alpha$ of income going to H and L exceeds the share β going to H alone. Consequently, it is possible for $dH/d\beta$ to be positive and $dK/d\beta$ to be negative, but the converse is not possible.

The derivative of Y with respect to β is

$$(29) \qquad \frac{dY}{d\beta} = \alpha AK^{\alpha-1}H^{\beta}L^{1-\alpha-\beta}\frac{dK}{d\beta} + \beta AK^{\alpha}H^{\beta-1}L^{1-\alpha-\beta}\frac{dH}{d\beta} + aK^{\alpha}H^{\beta}L^{1-\alpha-\beta}\ln(H/L)$$

$$= Y\left[\alpha\frac{dK}{d\beta}\frac{1}{K} + \beta\frac{dH}{d\beta}\frac{1}{H} + \ln(H/L)\right]$$

$$= Y\left(\frac{1}{1-\alpha-\beta}[1+\ln(H/L)]\right)$$

$$= Y\frac{dK}{d\beta}\frac{1}{K}$$

which is positive if and only if $dK/d\beta$ is positive.

The dynamic adjustment path to the steady state is considerably more difficult to work out. A complete characterization is not possible, but the adjustment path can be either monotonic in both K and H or cyclic in both. The proof is given in the Appendix. Except for the possibility of cyclical adjustment, the results are not much different from what was obtained in section III.A above. As in the two-factor model, a technological change that lowers output initially may still be worth adopting if it raises output in the steady state.

IV.B. *An Application to Unskilled Labor's Share.*

The three-factor model offers an explanation for the widely noted fact that, in the United States over the last thirty years or so, the real income of unskilled labor has been stagnant while total income and the incomes of physical capital and especially human capital (skilled labor) all have risen. This event is regarded by at least some people as evidence of a breakdown in "the system" that has left unskilled workers behind and that must be corrected by some sort of government intervention. The three-factor model suggests the event in question is not compelling evidence of a defect in the workings of the economy. A technical change that raises total output and the income share of skilled labor can reduce the income of unskilled labor.

Labor income is total income multiplied by labor's share, $(1-\alpha-\beta)Y$. Its derivative with respect to β is

$$\frac{dY^L}{d\beta} = -Y + (-\alpha-\beta)\frac{dY}{d\beta}$$

$$= Y\ln(H/L)$$

Comparing this expression with (29) shows that $dY^L/d\beta$ can be negative at the same time that $dY/d\beta$ is positive. If H/L has been sufficiently small in the U.S., and if technical change recently has raised β, then the drop in unskilled labor's share is unsurprising and also does not signify any failure in the workings of the economy. In principle, we could check whether this explanation applies to the U.S. by seeing if $\ln(H/L) < 0 < 1 + \ln(H/L)$. In practice, this check is difficult to do because data on human capital H are sketchy at best.

Of course, real-world technical advances generally will be more complex than that discussed in this section; in particular, we would expect total factor productivity *A* and probably physical capital's share α to change in most cases. Indeed, the data show an upward trend in *A* and a downward trend in α. We have seen, however, that allowing *A* to change does not alter the general character of the results. Allowing α and β to change simultaneously complicates the results but again does not change the main conclusion that it is quite possible that a technical advance can reduce the income of unskilled labor even though it raises total output and the incomes of other factors of production.

This possibility also may explain Berman's (2000) recent finding from a three-dimensional panel (industry, country, time) that, in the decade of the 1980s covered by the panel, technical change was strongly biased against less-skilled workers and in favor of both physical and human capital. If research and development is conducted largely by industries in developed countries for the benefit of those same industries, it will tend to favor technical advances that increase the income shares of the relatively abundant factors - that is, of physical and human capital. Such behavior would be consistent with the literature on induced technical change. For example, Kiley (1999) has shown that, when factor shares are fixed, endogenous technical change that alters factor shares will be chosen to favor the abundant factor.

IV.C. *Failure of Underdeveloped Economies to Adopt New Technology*.

A serious issue in development economics is the failure of underdeveloped countries to adopt recent technological advances, leaving those countries ever farther behind the developed world. Increases in total factor productivity would not lead to this result; everybody gains from an increase in TFP. Share-altering technical progress, in contrast, explains this seemingly self-defeating behavior by underdeveloped countries.

Suppose Country A and Country B have the same technology (the same functional form of the production function with the same parameter values), but suppose Country A has a higher ratio of human capital to unskilled labor than Country B. This disparity could arise, for example, if one (or both) of the countries was not at its steady state or if the two countries had different steady states because they had different sets of distorting taxes. If an invention is discovered that raises human capital's share β, it could be the case that Country A would find it optimal to adopt this new technology but Country B would not. If Country A finds it optimal to adopt, it does so and thereby raises its output. As in the two-factor model, adoption will raise the steady state value of H/L and thus dispose Country A toward adoption in the future if another invention is discovered that raises β again. In the meantime, Country B does not adopt and stays disposed against adopting any future inventions that raise β. If actual inventions generally raise β, we would see Country A adopting them and increasing its income while Country B stagnates. The gap between the two countries' incomes would widen.

There is some recent evidence consistent with this scenario. Caselli and Coleman (2003) find that countries with a relative abundance of unskilled labor are relatively inefficient at using skilled labor and capital, that is, they have relatively low values of α and β, suggesting they have not adopted technical advances that have raised those parameters in other countries. In another paper, Caselli and Coleman's (2001) find that countries with relatively low human capital are relatively slow to adopt computer technology, which they argue increases the emphasis on human capital in production. In terms of our model, such an increase in emphasis is captured by an increase in β. These bits of evidence are by no means decisive, but they are consistent with the behavior one would expect from share-altering technical change.

Differing adoption behavior across countries has an important implication for cross-country studies of aggregate production or the determinants of economic growth. The usual procedure in such studies is to assume that all countries have production functions of the same form and with the same values for all parameters other than TFP. However, if technical change involves alteration of factor shares, and if some countries do not find adoption of such change in their interest, then the parameter values of the production functions in various countries will diverge. The assumption of common production functions then is invalid. In that case, the concept of beta-convergence (either absolute or conditional) is meaningless. Knowledge of how fast one country is converging to its steady state tells us nothing about other countries' behavior. Cross-country variation in the adoption of share-altering technical change may explain Lee, Pesaran, and Smith's (1998) finding that speeds of convergence differ across countries. The speed of convergence depends on the share parameters of the production function (Barro and Sala-i-Martin, 1995). Share-altering technical change would lead us to expect different adoption histories across countries and therefore also would lead us to expect different adjustment speeds, as Lee, Pesaran, and Smith report.

V. Economic Growth

Share-altering technical change has interesting implications for the analysis of economic growth. Here we briefly examine two - the possibility of asymptotic balanced growth and the usefulness of the popular quality ladder model.

V.A. *Asymptotic Balanced Growth.*

We start with the simpler two-factor model; a few remarks on the three-factor model follow. We also suppose for simplicity that total factor productivity A is constant.

Balanced growth is not possible in the presence of share-altering technical change because the shares must lie in the interval [0,1]. If one share is growing, the other must be falling. The growth rate of the share that is falling can be constant as that share goes asymptotically to zero, but the growth rate of the other share must die out as the share approaches its upper bound of one. Thus not all variables can be growing at constant rates. However, persistent growth that converges asymptotically to balanced growth is possible. To see how this behavior arises, divide both sides of (22) by Y to get the growth rate of output, which we can write in the following form by using (20) to substitute for K/L:

(30) $$\frac{dY}{dt}\frac{1}{Y} = \left(\frac{\alpha}{1-\alpha}\right)\left[1+\ln\left(\frac{\alpha A}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}\right]\left(\frac{d\alpha}{dt}\frac{1}{\alpha}\right)$$

Let us suppose that α grows over time and that $\alpha A > \delta+\rho$. Then Y also grows over time. Because α is bounded above by one, its growth rate must go asymptotically to zero. The long-run behavior of the growth rate of Y is not immediately obvious because the coefficient of the growth rate of α in (30) is a positive function of α, going to infinity as α goes to its upper limit of one (given our assumption that $\alpha A > \delta + \rho$):

$$\begin{aligned}\lim_{\alpha\to 1}\left(\frac{\alpha}{1-\alpha}\right)\left[1+\ln\left(\frac{\alpha A}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}\right] &= \lim_{\alpha\to 1}\left(\frac{\alpha}{1-\alpha}\right) + \lim_{\alpha\to 1}\frac{\alpha}{(1-\alpha)^2}\ln\alpha \\ &\quad + \lim_{\alpha\to 1}\frac{\alpha}{(1-\alpha)^2}\ln\left(\frac{A}{\delta+\rho}\right) \\ &= \infty + \infty + 0 + \infty \\ &= \infty\end{aligned}$$

However, the growth rate of Y can be any constant value γ if the growth rate of α satisfies

$$\frac{d\alpha}{dt}\frac{1}{\alpha} = \gamma\left\{\left(\frac{\alpha}{1-\alpha}\right)\left[1+\ln\left(\frac{\alpha A}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}\right]\right\}^{-1}$$

This expression goes to zero as α goes to one, so it satisfies the necessary bound on the growth rate of α.

If the growth rate of Y is constant, the growth rate of K is not. From (24) we have

$$\frac{dK}{dt}\frac{1}{K} = \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{1}{\alpha} + \ln\left(\frac{\alpha A}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}\right]\left(\frac{d\alpha}{dt}\frac{1}{\alpha}\right)$$

$$> \frac{dY}{dt}\frac{1}{Y} \quad \text{because} \quad \frac{1}{\alpha} > 1$$

However, $1/\alpha \to 1$ as $\alpha \to 1$, so the growth rate of K goes asymptotically to the growth rate of Y as α goes to its upper bound. In the limit, then, we have the growth rate of K approaching the constant growth rate of Y and the growth rate of α going to zero. Balanced growth thus is possible asymptotically. Also, the whole production function goes asymptotically to the AK model, where growth is self-sustaining even without R&D-induced changes in the production function's parameters. These results suggest that it would be very interesting to extend the analysis to a model of endogenous technical progress in which firms can expend R&D resources on changing α.

The three-factor model, though potentially richer and more realistic, is far more difficult to analyze. With the production function now given by (1), the growth rate of Y is (still assuming that total factor productivity is constant)

$$\frac{dY}{dt}\frac{1}{Y} = \alpha\frac{dK}{dt}\frac{1}{K} + \beta\frac{dH}{dt}\frac{1}{H}$$

$$= \alpha\left[1 + \frac{\alpha+\beta}{1-\alpha-\beta}\ln\left(\frac{K}{L}\right)\right]\left(\frac{d\alpha}{dt}\frac{1}{\alpha}\right) + \beta\left[1 + \frac{\alpha+\beta}{1-\alpha-\beta}\ln\left(\frac{H}{L}\right)\right]\left(\frac{d\beta}{dt}\frac{1}{\beta}\right)$$

Both α and β can change; they can do so in the same direction or opposite directions. Both are free to go to zero, but they cannot both go to one. Indeed, if either goes to one, the other must go to zero, at least if linear homogeneity of the production function is to be preserved. The behavior of the coefficients of the growth rates of α and β depends on whether α and β are growing or falling and on whether they are moving in the same or opposite directions. A theory in which R&D expenditure on α and β was endogenously determined might put useful restrictions on the relation between the growth rates of α and β. However, without some restrictions on the paths of α and β, not much can be said about the path of the growth rate of output Y.

V.B. *A Doubt About Some Endogenous Growth Models.*

To be useful, most endogenous growth models require existence of a balanced growth path, which often requires knife-edge assumptions that are functions of factor share parameters. For example, the usual quality ladder model of endogenous growth has an aggregate production function of the form

$$Y = AL^{\alpha}\sum_{i=1}^{N}(X_i^{q_i})^{1-\alpha}$$

where the X_i are intermediate goods produced in other sectors and the q_i are indices of the quality of the X_i. In this model, monopolistically competitive suppliers of intermediate goods compete to discover ways to advance up the quality ladder by increasing q_i. For balanced growth to be possible, we must impose the following knife-edge assumption on the form of the function describing the probability that R&D will succeed in raising the quality of intermediate good j:

(31) $$p_{jk_j} = Z_{jk_j}(1/\zeta)q^{-(k_j+1)\alpha/(1-\alpha)}$$

where $q > 1$ is constant, Z is the quantity of resources expended on R&D in industry j, k_j is the current quality step in that industry, ζ is the cost of a unit of R&D, and p_{jk} is the probability of advancing one step when the current step is at level k_j (see Chapter 7 of Barro and Sala-i-Martin, 1995, for details). Any departure from this form will lead either to a cessation of growth asymptotically or explosive growth. Assuming this functional form is defensible on the grounds that the balanced growth path that it permits can be considered an approximation, perhaps by appeal to a turnpike theorem, to the actual path of the economy. In the presence of share-altering technical change, however, the usefulness of this defense becomes questionable. Any time there is a technical change that alters α, the success function must change to reflect the change in α if balanced growth is to remain possible. In that case, though, the meaning of the balanced growth path as an approximation is lost because the balanced growth path itself is continuously shifting as α changes. Given the strong evidence for share-altering technical change discussed in the Introduction, one therefore has to question the usefulness of models such as the quality ladder model that rely on assumptions that require constant factor shares.

VI. Conclusion

The usual approach to modelling technical change assumes that technical change alters only total factor productivity, with no direct effect on factor shares. There is no particular reason to expect technical change to be of such a restricted nature, and indeed a substantial body of data and econometric analysis suggest that factor shares do vary as a result of technical change. The foregoing analysis has examined some of the properties and implications of technical change that is not share-neutral. Share-altering technical progress offers explanations for several economic events across both time and space: the Luddite uprisings of 19th century England, the growing gap between wages of skilled and unskilled labor in the U.S., and the failure of underdeveloped countries to adopt new technology. Share-altering technical change tends to perpetuate itself, can produce non-monotonic adjustment paths for output, and can lead to sustained growth in real output that is balanced asymptotically. We also have seen that share-altering change brings into question total factor productivity as measure of technical progress, possibly causing TFP even to have the wrong sign. Share-altering change also brings into question the usefulness of several classes of endogenous growth models.

These results suggest that share-altering technical is an important phenomenon. The nature and origin of such technical change therefore requires further study. In particular, it would be valuable to develop a model of endogenous technical change in which firms made deliberate decisions to allocate R&D resources to change that alters factor shares.

Appendix

A. Two-Factor Model: Optimality of Adoption when $0 < \partial Y/\partial\alpha$, $dY/d\alpha$

First suppose path P_L is the optimal adjustment path. Along that path, consumption and therefore utility is higher at all times than if the invention is not adopted and the economy remains at point E_0. Unambiguously, adoption is optimal. Now suppose that path P_U is the optimal path. Recall that optimality here is a conditional notion; *if* the invention is adopted, then the optimal path to follow is P_U. Along this path, initial consumption and therefore utility is lower with adoption than without it. It therefore is not immediately clear that adoption is optimal. However, notice that a path like P_L always is feasible even if not optimal. Along path P_L, consumption is always higher. Path P_L thus dominates the alternative of not adopting the invention and remaining at point E_0. Path P_L, however, is itself dominated by path P_U, so adoption is optimal *a fortiori*.

B. Three Factor Model: Steady State

The Hamiltonian is

$$V = U(C) + \lambda(I_K - \delta K) + \psi(AK^{\alpha}H^{\beta} - C - I_K - \delta H)$$

where 6 has been used to eliminate I_H. The dynamic equations are

$$dK/dt = I_k - \delta K$$
$$dH/dt = AK^{\alpha}H^{\beta} - C - I_K - \delta H$$
$$d\lambda/dt = \lambda(\delta + \rho) - \psi\alpha AK^{\alpha-1}H^{\beta}$$
$$d\psi/dt = -\psi[A\beta K^{\alpha}H^{\beta-1} - (\delta + \rho)]$$

In steady state, these must all equal zero. The first-order conditions are

$$\partial V/\partial C = U'(C) - \psi = 0$$
$$\partial V/\partial I_K = \lambda - \psi = 0$$

The initial and transversality conditions are

$$K_0 \quad \text{given}$$
$$H_0 \quad \text{given}$$
$$\lim_{t\to\infty} \lambda_t K_t e^{\rho t} = 0$$
$$\lim_{t\to\infty} \psi_t H_t e^{\rho t} = 0$$

We use the second of the first-order conditions to eliminate λ from the equation for $d\lambda/dt$. We then simultaneously solve the equations $d\lambda/dt = 0$ and $d\psi/dt = 0$ to obtain the steady state values of K and H given in the text.

C. Three Factor Model: Dynamics

Because L is constant, it plays no role in the dynamics. Therefore set L = 1 for simplicity. Then note the following relations:

(A1) $d\lambda/d\beta = -\psi\alpha AK^{\alpha-1}H^{\beta}\ln H < 0$

$$d\psi/d\beta = -\psi\beta AK^{\alpha}H^{\beta-1}\ln H - \psi AK^{\alpha}H^{\beta-1}$$

(A2)

$$= \frac{\beta}{\alpha}\frac{K}{H}\left(\frac{d\lambda}{d\beta}\right) - \psi AK^{\alpha}H^{\beta-1}$$

$$= \frac{d\lambda}{d\beta} - \psi AK^{\alpha}H^{\beta-1}$$

$$< \frac{d\lambda}{d\beta} < 0$$

(A3) $d\lambda/dK = -\psi(\alpha-1)\alpha K^{\alpha-2}H^{\beta} > 0$

(A4) $d\lambda/dH = -\psi\alpha\beta AK^{\alpha-1}H^{\beta-1} < 0$

(A5) $d\psi/dK = -\psi\alpha\beta K^{\alpha}H^{\beta-1} < 0$

(A6) $d\psi/dH = -\psi(\beta-1)\beta K^{\alpha}H^{\beta-2} > 0$

Suppose β rises with no change in α.

(1) At the original K, H, λ, ψ, we have dψ/dt < dλ/dt < 0.

(2) After re-optimizing, if the new λ, ψ are such that λ > ψ, then dψ/dt << dλ/dt. (To see this, notice that at K_0, H_0, the term $\alpha AK^{\alpha-1}H^{\beta}$ in the dλ/dt expression now is less in magnitude than the term $A\beta K^{\alpha}K^{\beta-1}$ in the dψ/dt expression. Both are multiplied by ψ. The δ+ρ term in the two expressions are the same; one is multiplied by ψ and the other by λ.) Given λ > ψ, all investment goes to K, none to H, by the bang-bang nature of the problem. Therefore K is rising and H falling, implying that dλ/dt is becoming less negative and dψ/dt is becoming more negative, which leads to explosive behavior and failure to satisfy the transversality condition. So we must have $\lambda_0 < \psi_0$.

(3) So take $\lambda_0 < \psi_0$ as given. This relation tends to make dλ/dt < dψ/dt, from the two dynamic equations for λ and ψ. However, we cannot have dλ/dt < dψ/dt because it leads to explosive behavior (λ < ψ ⇒ dK/dt = 0 < dH/dt by the bang-bang property, ⇒ dλ/dt becomes more negative, dψ/dt becomes less negative). So we must have both λ < ψ and dψ/dt < dλ/dt. We then have K falling and H rising (bang-bang) and also dλ/dt becoming more negative and dψ/dt becoming less negative. There are three possibilities to consider.

(3.1) K/H reaches the new steady state ratio K*/H* while ψ > λ. At that moment, dψ/dt = 0 but dλ/dt < 0 and dK/dt < 0 < dH/dt. Immediately, dψ/dt becomes positive, and dλ/dt continues to become more negative, implying impermissible explosive behavior.

(3.2) K/H reaches K*/H* at the same instant that λ = ψ. There are two possible ways for this to happen, one permissible and one not.

(3.2.a) Saddle path. The instant in question is the infinite horizon and the system goes monotonically to its steady state asymptotically. This possibility is permissible.

(3.2.b) The instant in question is finite. This possibility is not possible. Suppose it were possible. Then at that instant t#, we would have ψ = λ, dψ/dt = dλ/dt = 0, and K/H = K*/H*. If we also had K

= K* and H = H* separately, we would be in the steady state; however, steady state cannot be reached in finite time along an optimal path. Therefore, $K \neq K^*$, $H \neq H^*$, and both K and H are too high or both are too low (because their ratio is "just right" at K*/H*). Because $\psi = \lambda$, I_K is freely chosen. It is not possible to choose it so that K and H slide toward K* and H* while maintaining K*/H*. The reason is that doing that would leave $d\psi/dt = d\lambda/dt = 0$ and thus would also leave $\psi = \lambda$ = constant, implying that ψ is at its steady state value ψ^* and thus that C is at its steady state value C*. But if C = C* and $K \neq K^*$, $H \neq H^*$, then either (i) Y > Y* and Y - C* is too large, implying that K > K*, H > H*, in turn implying that we must have $I_K < \delta K$, so that C > C* (a contradiction), or (ii) Y < Y* and the opposite logic applies and C < C* (another contradiction). So $d\psi/dt$ and $d\lambda/dt$ must cease to be zero. Either ψ becomes larger than λ or vice versa. If $\psi > \lambda$, then H grows, K falls, making $d\lambda/dt$ smaller and $d\psi/dt$ larger, leading to impermissible explosive behavior. If $\psi < \lambda$, then once again we have an explosion, for opposite reasons.

(3.3) K/H reaches K*/H* while $\lambda < \psi$. Then K continues to fall, and H continues to rise, implying that K/H falls below K*/H*, which in turn implies that $d\psi/dt$ continues to become less negative and $d\lambda/dt$ continues to become more negative, but still $d\psi/dt < d\lambda/dt$. Eventually, ψ falls below λ, and we have a reversal of the investment pattern, with $I_H = 0 < I_K$. At that point, K starts growing, and H starts falling, leading to a cyclical adjustment path.

References

Acemoglu, D., "Labor- and Capital-Augmenting Technical Change," *Journal of European Economic Association* 1, March 2003, pp. 1-37.

Baldwin, R. E., and G. Cain, "Shifts in U.S. Relative Wages: The Role of Trade, Technology, and Factor Endowments," *Review of Economics and Statistics* 82, November 2000, pp. 580-95.

Barro, R. J., and X. Sala-i-Martin, *Economic Growth*, McGraw-Hill, Inc., New York, 1995.

Bartel, A. P., and N. Sicherman, "Technological Change and Wages: An Interindustry Analysis," *Journal of Political Economy* 107, April 1999, pp. 285-325.

Berman, E., "Does Factor-Biased Technical Change Stifle International Convergence? Evidence from Manufacturing," Working Paper #7964, National Bureau of Economic Research, October 2000.

Binswanger, H. P., "The Measurement of Technical Change Biases with Many Factors of Production," *American Economic Review* 64, December1974, pp. 964-76.

Blanchard, O., "The Medium Run," *Brookings Papers on Economic Activity* 2, 1997, pp. 89-158.

Blanchard, O., "Revisiting European Unemployment: Unemployment, Capital Accumulation, and Factor Prices," Geary Lecture, Economic and Social Research Institute, 1998.

Borjas, G. J., and V. A. Ramey, "Time-Series Evidence on the Sources of Trends in Wage Inequality," *American Economic Review* 84, May 1994, pp. 10-6.

Bound, J., and G. Johnson, "What Are the Causes of Rising Wage Inequality in the United States?" *Economic Policy Review* Federal Reserve Bank of New York, January 1995, pp. 9-17.

Caselli, F., "Technological Revolutions," *American Economic Review* 89, March 1999, pp. 78-102.

Caselli, F., and W. J. Coleman, II, "Cross-Country Technology Diffusion: The Case of Computers," *American Economic Review* 91, May 2001, pp. 328-35.

Caselli, F., and W. J. Coleman, II, "The World Technology Frontier," typescript, Harvard University and Duke University, July 2003.

David, P. A., and Th. van de Klundert, "Biased Efficiency Growth and Capital-Labor Substitution in the U.S., 1899-1960," *American Economic Review* 55, June 1965, pp. 357-94.

Duffy, J., and C. Papageorgiou, "A Cross-Country Emprical Investigation of the Aggregate Production Function Specification," *Journal of Economic Growth* 5, March 2000, pp. 87-120.

Dinopoulous, E., C. Syropoulos, and B. Xu, "Intra-Industry Trade and Wage Income Inequality," typescript, University of Florida, October 1999.

Galor, O., and O. Moav, "Ability-Biased Technological Transition, Wage Inequality, and Economic Growth," *Quarterly Journal of Economics* 115, May 2000, pp. 469-97.

Goldberg, S. A., "Long-Run Changes in the Distribution of Income by Factor Shares in Canada," in *The Behavior of Income Shares: Selected Theoretical and Empirical Issues*, Studies in Income and Wealth, vol. 27, National Bureau of Economic Research, Princeton University Press, 1964, pp. 189-272,

Harrigan, J., and R. A. Balaban, "U.S. Wages in General Equilibrium: The Effects of Prices, Technology, and Factor Supplies, 1963-1991," Working Paper #6981, National Bureau of Economic Research, February 1999.

Kahn, J. A., and J-S. Lim, "Skilled Labor-Augmenting Technical Progress in U. S. Manufacturing," *Quarterly Journal of Economics* 113, November 1998, pp. 1281-308.

Katz, L. F., and K. M. Murphy, "Changes in Relative Wages, 1963-1987: Supply and Demand Factors," *Quarterly Journal of Economics* 107, February 1992, pp. 35-78.

Kendrick, J. W., and R. Sato, "Factor Prices, Productivity, and Economic Growth," *American Economic Review* 53, December 1963, pp. 974-1003.

Kennedy, C., "Induced Bias in Innovation and the Theory of Distribution," *Economic Journal* 74, September 1964, pp. 541-7.

Kiley, M. T., "The Supply of Skilled Labour and Skill-Biased Technological Progress," *Economic Journal* 109, October 1999, pp. 708-724.

Krueger, A. B., "Measuring Labor's Share," *American Economic Review* 89, May 1999, pp. 45-51.

Kumbhakar, S. C., and A. Lozano-Vivas, "Deregulation, Markups and Productivity Change: The Case of Spanish Banks," *Research in Banking and Finance*, vol. 3, Iftekhar Hasan and William C. Hunter, eds. Elsevier Science, Ltd., Amsterdam, 2002, pp. 79-100.

Lee, K, M. H. Pesaran, and R. Smith, "Growth Empirics: A Panel Data Approach - A Comment," *Quarterly Journal of Economics* 113, February 1998, pp. 319-23.

Samuelson, P. A., "A Theory of Induced Innovation Along Kennedy-Weisacker Lines," *Review of Economics and Statistics* 47, November 1965, pp. 343-356.

Sato, R., "The Estimation of Biased Technical Progress and the Production Function," *International Economic Review* 11, June 1970, pp. 179-208

Sato, R., and M. J. Beckmann, "Neutral Inventions and Production Functions," *Review of Economic Studies* 35, January 1968, pp. 57-66.

Sato, R., and R. F. Hoffman, "Production Functions with Variable Elasticity of Factor Substitution: Some Analysis and Testing," *Review of Economics and Statistics* 50, November 1968, pp. 453-60.

Schultze, C. L., "Short-Run Movements of Income Shares," in *The Behavior of Income Shares: Selected Theoretical and Empirical Issues,* Studies in Income and Wealth, vol. 27, National Bureau of Economic Research, Princeton University Press, 1964, pp. 143-182.

Solow, R. M., "A Skeptical Note on the Constancy of Relative Shares," *American Economic Review* 48, September 1958, pp. 618-31.

Solow, R. M., "Perspectives on Growth Theory," *Journal of Economic Perspectives* 8, Winter 1994, pp. 45-54.

Stevenson, R., "Measuring Technological Bias," *American Economic Review* 70, March 1980, pp. 162-73.

Table 1
Capital's Share of Income, United States

Year	Capital's Share	Year	Capital's Share
1909	0.335	1935	0.351
1910	0.33	1936	0.357
1911	0.335	1937	0.34
1912	0.33	1938	0.331
1913	0.334	1939	0.347
1914	0.325	1940	0.357
1915	0.344	1941	0.377
1916	0.358	1942	0.356
1917	0.37	1943	0.342
1918	0.342	1944	0.332
1919	0.354	1945	0.314
1920	0.319	1946	0.312
1921	0.369	1947	0.327
1922	0.339	1948	0.332
1923	0.337	1949	0.326
1924	0.33	1950	0.363
1925	0.336	1951	0.345
1926	0.327	1952	0.317
1927	0.323	1953	0.311
1928	0.338	1954	0.305
1929	0.332	1955	0.329
1930	0.347	1956	0.319
1931	0.325	1957	0.311
1932	0.397	1958	0.301
1933	0.362	1959	0.316
1934	0.355	1960	0.309

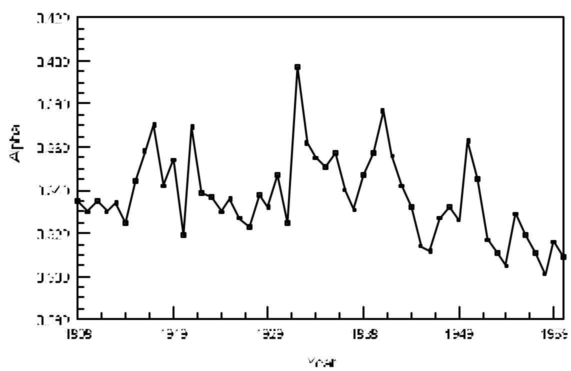

Figure 1: Plot of capital's share; Sato data.

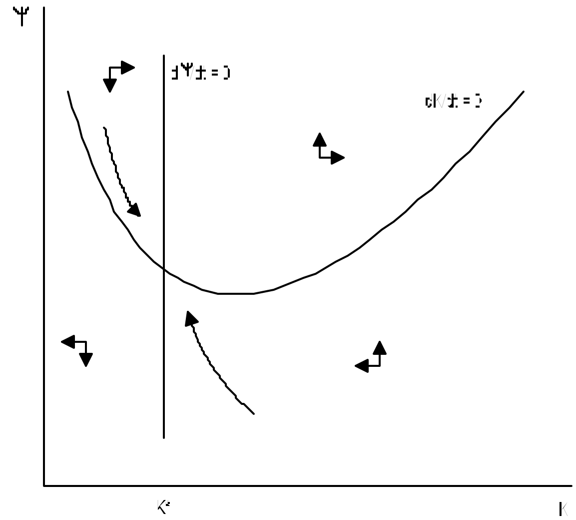

Figure 2: Generic phase diagram.

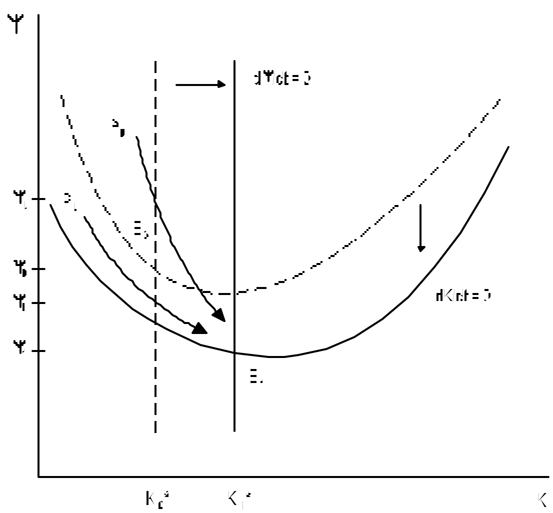

Figure 3: Technical advance with dY/dα>0.

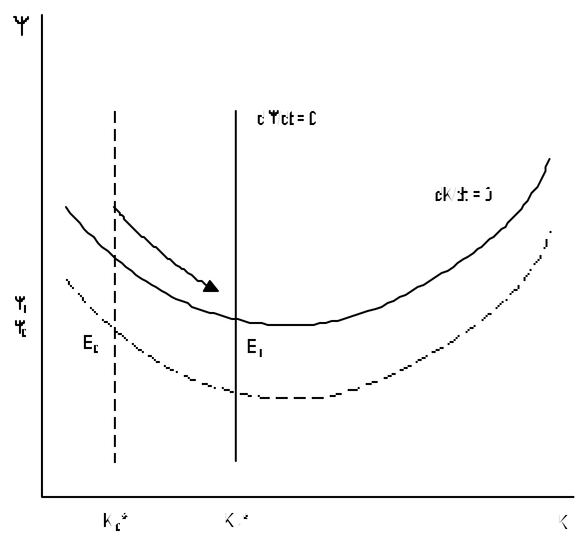

Figure 4: Increase in α with ∂Y/∂α < 0 < dY/dα; case where adoption is suboptimal.

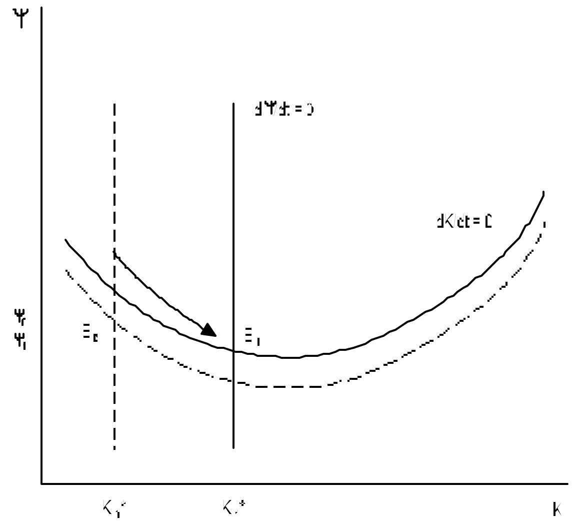

Figure 5: Increase in α with $\partial Y/\partial\alpha < 0 < dY/d\alpha$; case where adoption is optimal.