Inflation and Interest Rates

I. Inflation: Introduction

A. Definition

Inflation: the percentage rate of change in the general price level

$$\pi_t \equiv \frac{P_{t+1} - P_t}{P_t}$$

Notice that we can rewrite this as

$$\pi_t = \frac{P_{t+1}}{P_t} - 1$$

$$\Rightarrow \quad 1 + \pi_t = \frac{P_{t+1}}{P_t}$$

B. Causes

1. Anything that makes money supply different from money demand

2. Two examples

a. permanent decrease in Y^S - causes P to rise (positive inflation)
b. decrease in C^D - causes P to fall (negative inflation)

C. One-time inflation versus persistent inflation

1. One-time inflation (i.e., one-time change in the price level)

a. The economy receives a shock that changes the price level P
b. The increase in P brings to an end the process of price increases

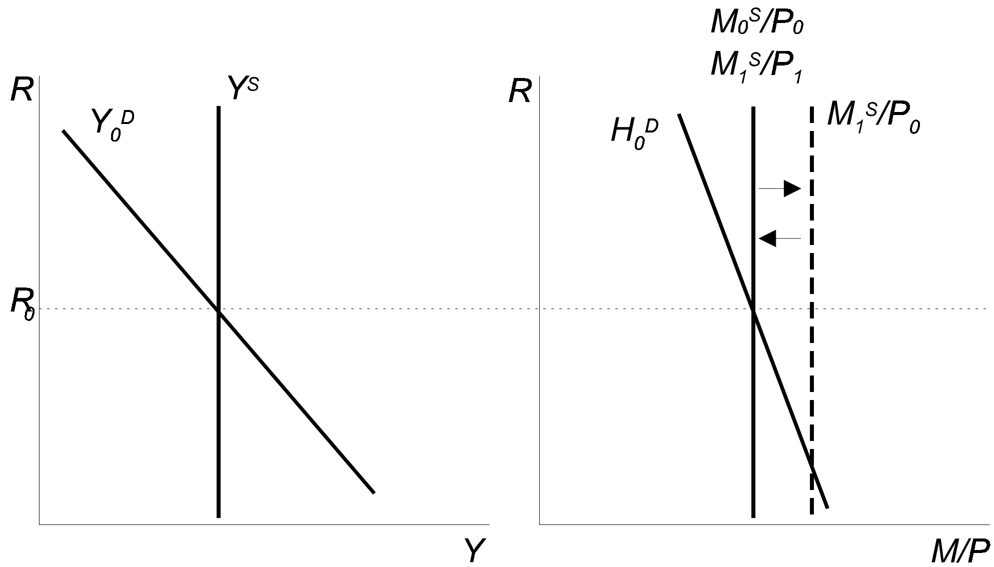
2. Persistent inflation (i.e., continuing change in the price level)

a. The economy is continuously shocked
b. The process of increasing prices continues indefinitely

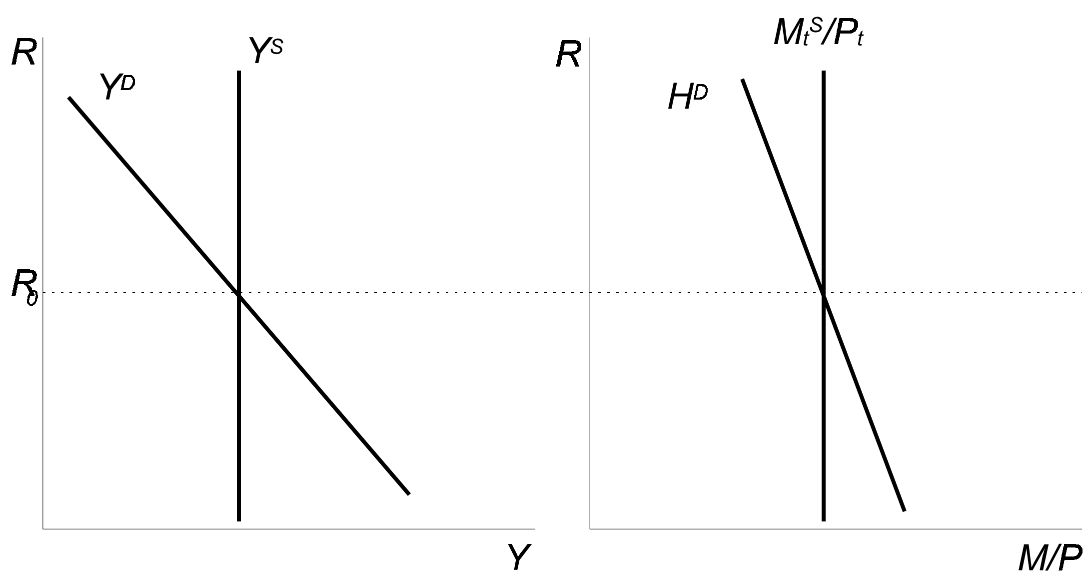
D. Persistent inflation

Caused by continuous increases in M^S

1. Succession of discrete increases in M^S



2. Continuous increases in M^S



II. Nominal Interest Rates, Real Interest Rates, and Inflation

A. Definitions

$r_t \equiv$ real interest rate

$R_t \equiv$ nominal interest rate
$\pi_t \equiv$ inflation rate

B. Relation

1. €1 buys $1/P_t$ goods today

2. €1 saved today yields $(1+R_t)$ Euros tomorrow and buys $(1+R_t)/P_{t+1}$ goods tomorrow

3. The *real return* from saving €1 is

$$\frac{1+R_t}{P_{t+1}} - \frac{1}{P_t}$$

4. Real *rate* of return is

$$r_t \equiv \frac{\frac{1+R_t}{P_{t+1}} - \frac{1}{P_t}}{\frac{1}{P_t}}$$

$$= (1+R_t)\frac{P_t}{P_{t+1}} - 1$$

$$= \frac{(1+R_t)}{(1+\pi_t)} - 1$$

$$= \frac{(1+R_t)}{(1+\pi_t)} - \frac{(1+\pi_t)}{(1+\pi_t)}$$

$$= \frac{R_t - \pi_t}{1+\pi_t}$$

$$\approx R_t - \pi_t \qquad \text{for } \pi_t \text{ small}$$

5. Expected (or *ex ante*) and actual (or *ex post*) real interest rates

a. Actual

$$r_t \equiv \frac{\dfrac{1+R_t}{P_{t+1}} - \dfrac{1}{P_t}}{\dfrac{1}{P_t}}$$

$$= \frac{R_t - \pi_t}{1+\pi_t}$$

$$\approx R_t - \pi_t \qquad \text{for } \pi_t \text{ small}$$

b. Expected

$$r^e{}_t \equiv \frac{\dfrac{1+R_t}{P^e_{t+1}} - \dfrac{1}{P_t}}{\dfrac{1}{P_t}}$$

$$= \frac{R_t - \pi_t{}^e}{1+\pi_t{}^e}$$

$$\approx R_t - \pi_t{}^e \qquad \text{for } \pi_t{}^e \text{ small}$$

6. Determination of the nominal interest rate

a. Rewrite the equation for r^e as

$$R_t = r_t{}^e + \pi_t{}^e + r_t{}^e \pi_t{}^e$$

b. The meaning of the terms on the right hand side:

r^e	the real rate of return in the absence of inflation
π^e	this term preserves the real value of the principal in the presence of inflation
$r^e\pi^e$	this term preserves the real value of the interest payment in the presence of inflation

C. Notation simplification

Hereafter, we ignore the distinction between expected and actual inflation and interest rates.

III. Real and Nominal Interest Rates in the Market-Clearing Model

A. Real and nominal rates compared

1. With no inflation, real and nominal rates are equal:

$r = R$
$r_M = R_M = 0$

2. With inflation, we must be careful

B. Product market

1. Previously, we said that output demand and supply depended on "the interest rate:"

$C^D(R_t, ...) = Y^S(R_t, ...)$

2. The argument was in terms of real interest rates - the relative price of consumption today in terms of consumption tomorrow.

3. Therefore, we now should change the notation to reflect the difference between real and nominal interest rates:

$C^D(r_t, ...) = Y^S(r_t, ...)$

C. Money demand: H^D_t

1. Previously, we argued that money demand depended on the spread between the "rate of return on bonds" and the "rate of return on money"

$R_t - R_{Mt}$

2. When there is inflation, real and nominal interest rates differ, so we write money demand in terms of the real spread:

$H^D_t = H^D(C_t , r_t-r_{Mt})$

3. However, the nominal interest rate on currency is always zero

$R_M \equiv 0$

4. Consequently, the real yield spread is

$$\begin{aligned} r - r_M &= (R - \pi) - (R_M - \pi) \\ &= R \\ &= r + \pi \end{aligned}$$

5. Therefore, we can write money demand as

$$H^D_t = H^D(C_t\,, r_t - r_{Mt})$$

$$= H^D(C_t\,, R_t)$$

so money demand depends on the *nominal* interest rate on bonds because that rate equals the spread between *real* rates when currency has a fixed nominal interest rate of zero.

6. Sometimes it is convenient to write money demand in terms of r and π:

$$H^D_t = H^D(C_t\,, R_t)$$

$$= H^D(C_t\,, r_t + \pi_t)$$

$$= H^D(C_t\,, r_t\,, \pi_t)$$

D. Complete system of equations

$$C^D(r_t, \ldots) = Y^S(r_t, \ldots)$$

$$H^D(C_t, R_t, \ldots) = M^S_t / P_t$$

IV. Money growth and inflation

Denote money growth rate by

$$\mu_t \equiv \frac{M_{t+1} - M_t}{M_t}$$

A. Money growing at fixed rate μ; otherwise, economy in steady state

1. Equilibrium condition for money market:

$H^D(C_t, R_t) = M^S_t / P_t$

2. LHS (Left Hand Side) is a given number

⇒ RHS (Right Hand Side) also a given number

⇒ P grows at same rate as M

⇒ $\pi = \mu$

3. Equilibrium with and without inflation

a. Without inflation

As just shown, to have no inflation we must have no money growth:

$0 = \mu = \pi = 0$

⇒ M and P both are constant, so M/P also is constant

⇒ $R = r + \pi$
$= r + 0$
$= r$ a constant

b. With inflation

We have $\mu = \pi > 0$

⇒ M and P grow at the same rate, so M/P still is constant

However, $\pi > 0$ means that

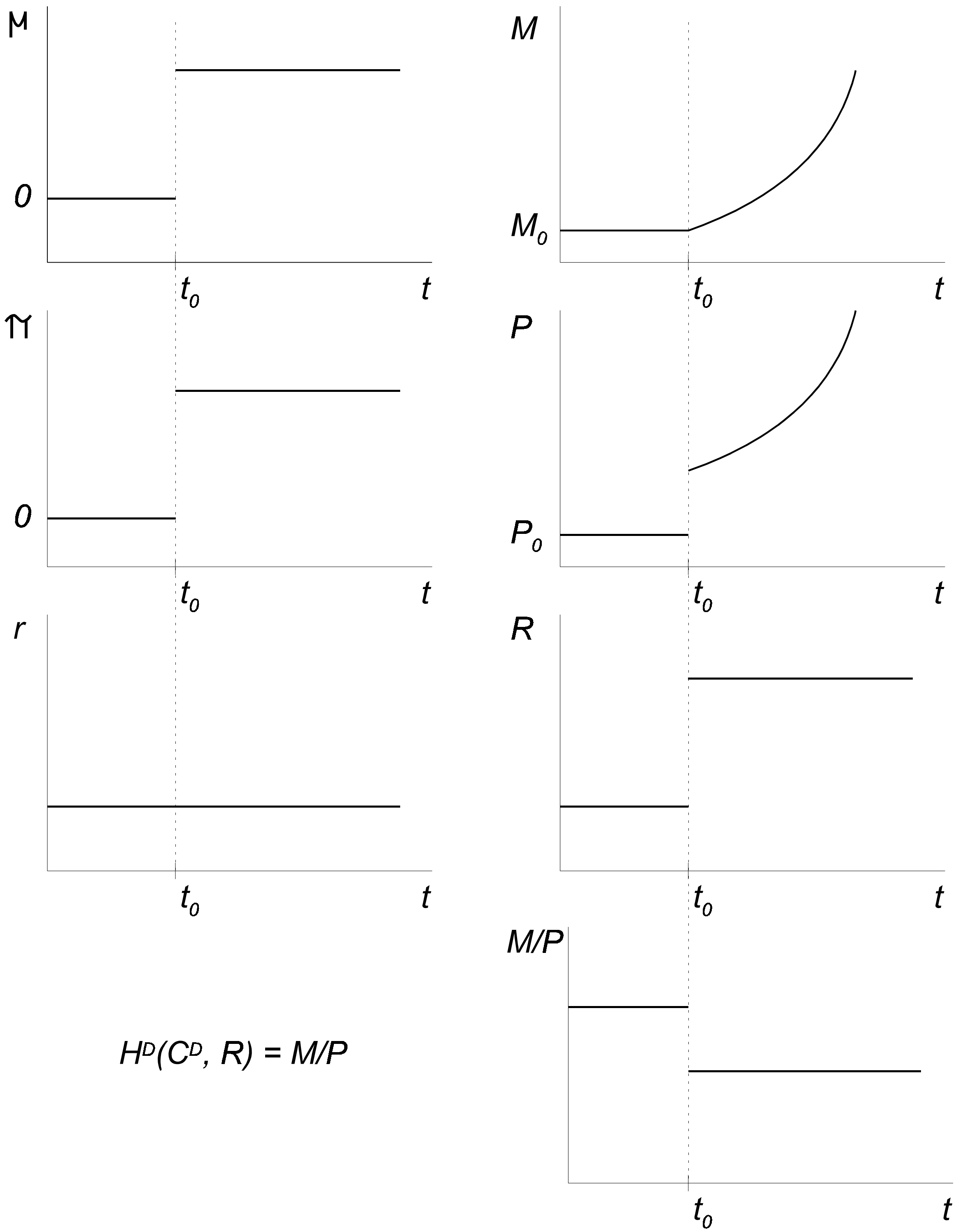
$R = r + \pi$ still a constant
$> r$

⇒ $H^D(\text{with } \mu > 0) < H^D(\text{with } \mu = 0)$

⇒ $M/P\ (\text{with } \mu > 0) < M/P\ (\text{with } \mu = 0)$

B. Change in money growth rate

1. If price adjustments are instantaneous (no short run effects)



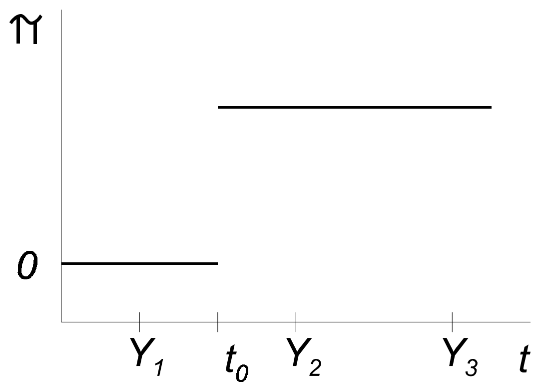
$$H^D(C^D, R) = M/P$$

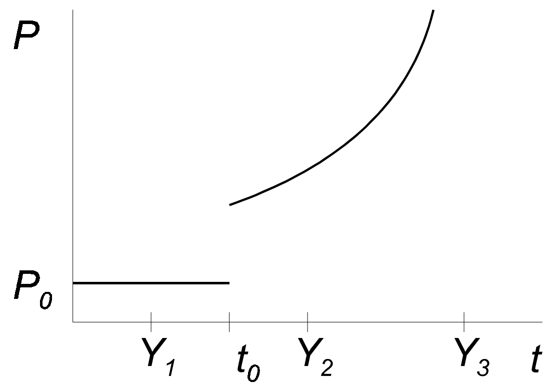
2. Some things to note:

a. None of the foregoing has any effect on the commodity market

At all times, $C^D = Y^S$ at the original interest rate.

b. *Measured* inflation differs between the first period and all succeeding periods





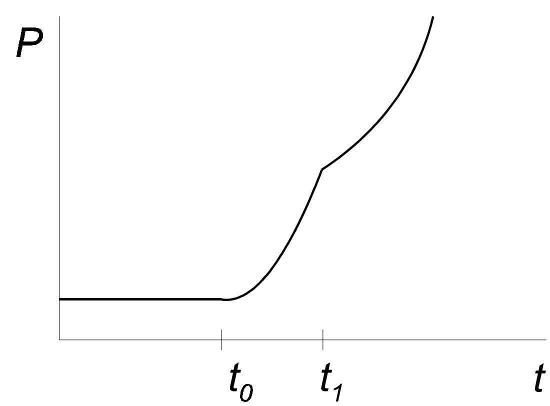
c. The nominal interest rate cannot reflect this high measured inflation

i. The measured inflation results from a discrete jump in the price level

ii. The jump is unexpected before it happens and therefore cannot be incorporated in the nominal interest rate before time t_0

iii. The jump is completed at the same time it is observed, so nothing remains to be incorporated into nominal rates after t_0.

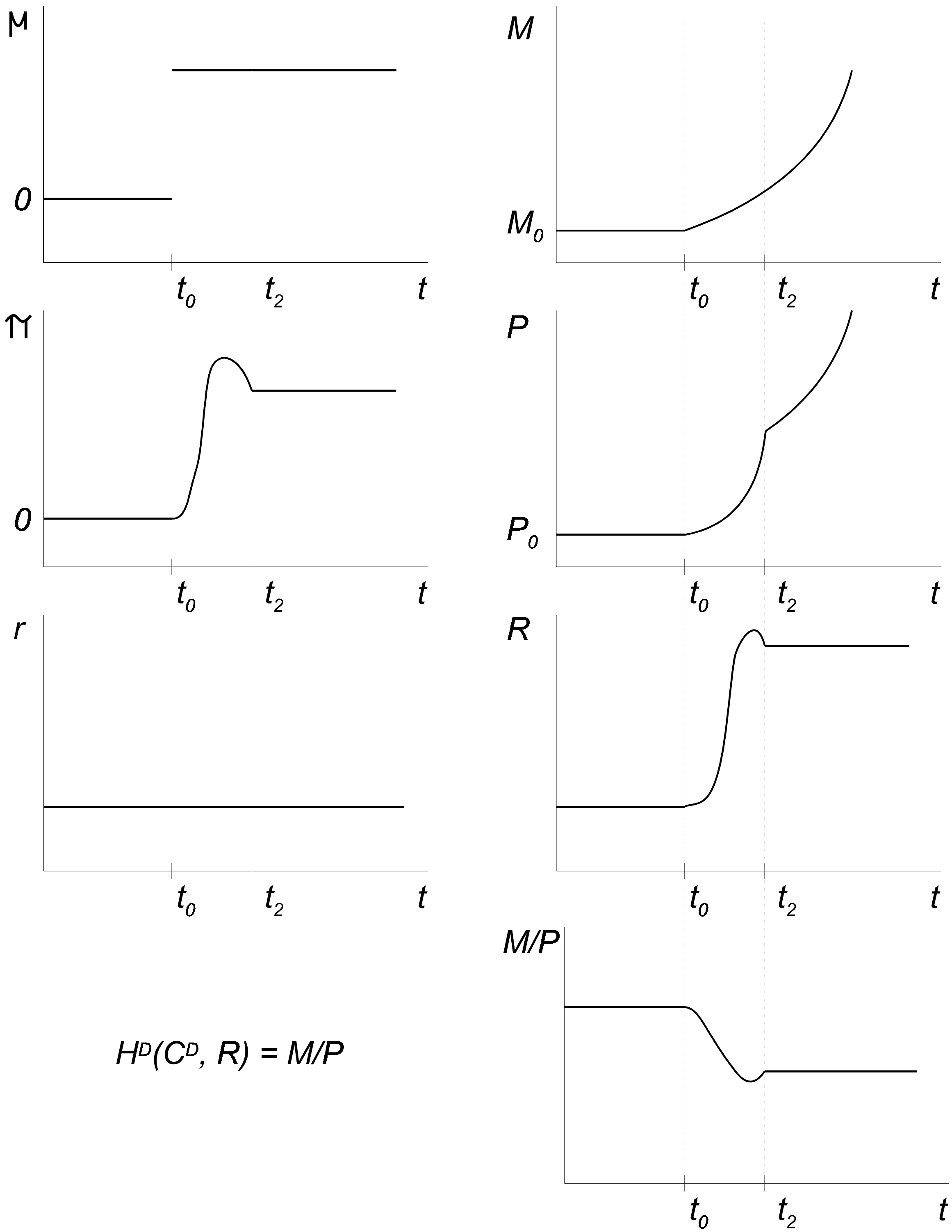
2. If prices adjust gradually



a. The "jump" in P is "stretched out"

⇒ measured inflation may exceed π for more than one period

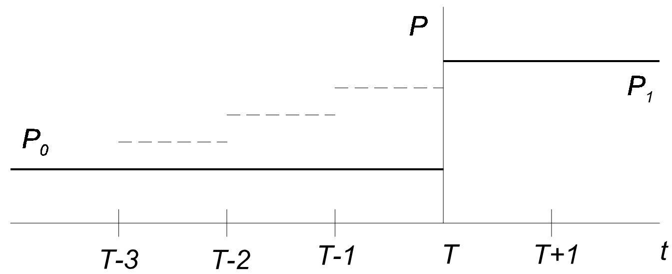
b. Also, nominal interest rate R now reflects the extra inflation during the period from t_0 to t_1



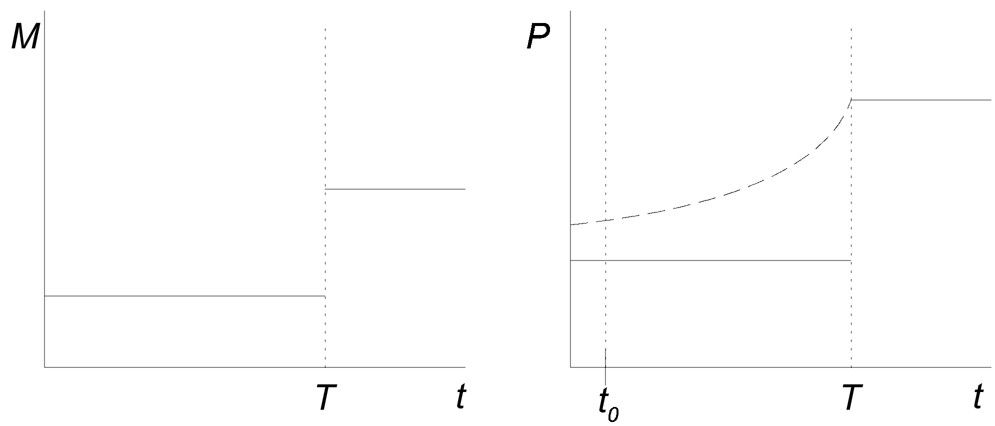
$H^D(C^D, R) = M/P$

V. Future Changes in Money Foreseen Now

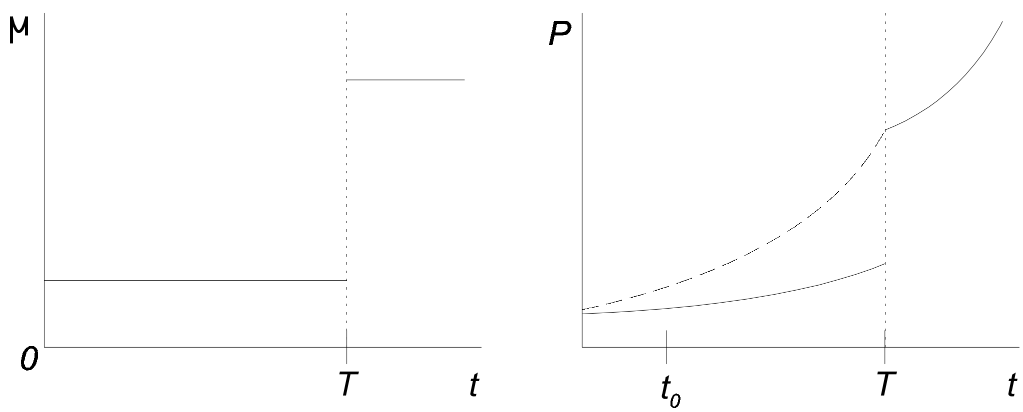
A. Preliminaries: Effect today of price change expected to occur in future



B. One-time increase in M



C. One-time increase in μ



VI. Inflation and the Term Structure of Interest Rates

A. A two-period problem

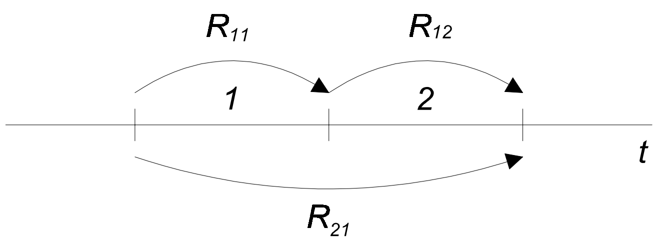
1. Interest rates

R_{ij} interest rate (in percent per year) carried by a bond of maturity i issued in period j

R_{1t} interest rate on a one-period bond issued in period t (i.e., a bond that will mature in period t+1)

R_{2t} interest rate on a two-period bond issued in period t (i.e., a bond that will mature in period t+2)

2. Two saving strategies



a. A sequence of one-period bonds, earning $(1+R_{1t})(1+R_{1t+1})$

b. A two-period bond, earning $(1+R_{2t})(1+R_{2t}) = (1+R_{2t})^2$

c. Competition guarantees that

$$(1+R_{1t})(1+R_{1t+1}) = (1+R_{2t})^2$$

$$\Rightarrow \quad (1+R_{2t}) = [(1+R_{1t})(1+R_{1t+1})]^{1/2}$$

= geometric average of $(1+R_{1t})$ and $(1+R_{1t+1})$

B. In general,

$$(1+R_{nt}) = [(1+R_{1t})(1+R_{1t+1})...(1+R_{1,t+n-1})]^{1/n}$$

or, if future interest rates are unknown,

$(1+R_{nt}) = [(1+R_{1t})(1+R^e_{1t+1})...(1+R^e_{1,t+n-1})]^{1/n}$

where R^e is the expected interest rate.

C. Term structure

Collection of rates quoted today:

R_{1t} , R_{2t} , R_{3t} , ... , R_{nt}

related by the foregoing formula.

D. Effects of changes in money growth on short and long interest rates

1. In our earlier discussion, R was R_{1t}

2. Recall that an increase in μ at first lowers R but ultimately raises it

$\Rightarrow$ R_{1t} falls but $R_{1,t+n}$ rises

$\Rightarrow$ R_{1t} falls but $R^e_{1,t+n}$ rises in period t

$\Rightarrow$ R_{1t} falls and R_{nt} rises in period t

3. An increase in μ thus

a. lowers short term interest rates initially but raises them ultimately

b. and therefore also raises long-term interest rates immediately

E. Implicit information about market expectations of the future

1. From observed R_{1t} and R_{2t} , can determine R^e_{1t+1} by using the above formula:

$R^e_{1t+1} = (1+R_{2t})^2/(1+R_{1t}) - 1$

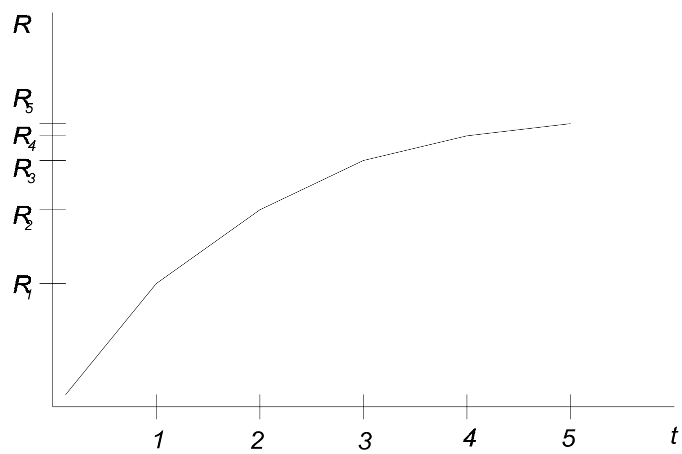
and similarly for any $R^e_{1,t+n}$.

2. Recall that $R^e_t \approx r^e_t + \pi^e_t$

$\Rightarrow$ Market yields contain information about the market's expectation of future inflation rates

F. Yield curves

1. Plot yield against time to maturity



2. So if the market decides that inflation will rise in the future, we should see

a. no change in current short rates

b. increases in current long rates (reflecting the increases in expected future short rates

VII. Some Interesting Inflations

A. Philip Cagan's study of six hyperinflations

	Austria	Germany	Greece	Hungary		Poland	Russia
	10/21-8/22	8/22-11/23	11/43-11/44	3/23-2/24	8/45-7/46	1/23-1/24	12/21-1/24
Average π/month	47.1	322.0	365.0	46.0	19,800	81.4	57.0
Average μ/month	30.9	314.0	220.0	32.7	12,200	72.2	49.3
Maximum π/month	134.0	32.4×10^{3}	85.5×10^{6}	98.0	41.9×10^{15}	275.0	213.0
μ in month of max π	72.0	1.3×10^{3}	73.9×10^{3}	46.0	1.03×10^{15}	106.0	87.0

B. Italy

Fratianni and Spinelli

	1914-20	1921-37	1938-49	1950-69	1970-80	1981-91
π	21.10	0.06	32.75	3.62	12.95	9.72
μ	22.70	3.3	30.4	12.40	17.40	10.30
γ	0.00	2.21	0.57	5.59	3.23	2.12