Money, Prices, and General Equilibrium

I. Money and Prices

A. Money

1. Nominal money supply M_t^S
2. Consider only currency (no checking accounts, credit cards, etc.)
3. Controlled by the government

B. Price level P_t

1. Commodities are priced in terms of money; the price level is P_t = money/commodity
2. For now, we suppose P has no trend.

C. Real money supply

1. The amount of output that the nominal money supply can buy:

$$M_t^S / P_t$$

II. Household Budget Constraints

A. One-period (discounted to the *beginning* of period t)

$$\frac{y_t}{(1+R)} + b_{t-1} + \frac{m_{t-1}}{P(1+R)} = \frac{c_t}{(1+R)} + \frac{b_t}{(1+R)} + \frac{m_t}{P(1+R)}$$

B. Infinite-horizon

1. As before, can derive the infinite horizon budget constraint:

$$b_{t-1} + \frac{m_{t-1}}{P(1+R)} + \frac{y_t}{1+R} + \frac{y_{t+1}}{(1+R)^2} + \frac{y_{t+2}}{(1+R)^3} + \ldots =$$

$$\frac{c_t}{1+R} + \frac{c_{t+1}}{(1+R)^2} + \frac{c_{t+2}}{(1+R)^3} + \ldots + \frac{R\frac{m_t}{P}}{1+R} + \frac{R\frac{m_{t+1}}{P}}{(1+R)^2} + \frac{R\frac{m_{t+2}}{P}}{(1+R)^3} + \ldots$$

2. The last terms on the right side

a. are the present values of the opportunity (foregone interest) costs of real money in each of the future time periods

b. equal m_{t-1} /P when the household wants to keep its money holding constant (i.e., when the household is satisfied with the amount of money it holds)

PROOF:
i. Constant money holding ⇔ m_{t-1} /P = m_t /P

ii. We already have seen that, when m_{t-1} /P = m_t /P, the infinite horizon budget constraint is

$$y_t + \frac{y_{t+1}}{1+R} + \frac{y_{t+2}}{(1+R)^2} + \ldots + \frac{(1+R)\,b_{t-1}}{P} = c_t + \frac{c_{t+1}}{1+R} + \frac{c_{t+2}}{(1+R)^2} + \ldots$$

iii. This constraint is a special case of the general constraint that lacks the term (m_{t-1} /P) on the left and all the terms of the form [R/(a+R)](m_{t+i}/P) on the right

$$\Rightarrow \qquad \frac{m_{t-1}}{P} = \frac{R\dfrac{m_t}{P}}{1+R} + \frac{R\dfrac{m_{t+1}}{P}}{(1+R)^2} + \frac{R\dfrac{m_{t+2}}{P}}{(1+R)^3} + \ldots$$

c. So, when the household is satisfied with the amount of money it holds, its budget constraint reduces to

$$y_t + \frac{y_{t+1}}{1+R} + \frac{y_{t+2}}{(1+R)^2} + \ldots + \frac{(1+R)\,b_{t-1}}{P} = c_t + \frac{c_{t+1}}{1+R} + \frac{c_{t+2}}{(1+R)^2} + \ldots$$

which is the same as we had when there was no money in the economy

⇒ in equilibrium, the household's consumption and labor decisions are independent of the amount of money it holds

d. This fact will be useful when we compare equilibria.

3. Lifetime budget constraint with time-varying interest rates

$$\sum_{j=0}^{\infty} c_{t+j}\left[\prod_{m=0}^{j}(1+R_{t+m})\right]^{-1} = b_{t-1} + \sum_{j=0}^{\infty} y_{t+j}\left[\prod_{m=0}^{j}(1+R_{t+m})\right]^{-1}$$

$$+ \left\{\frac{m_{t-1}}{P_t}\frac{1}{1+R_t} - \sum_{j=0}^{\infty}\left(\frac{m_{t+j}}{P_t}\left[\frac{R_{t+m+1}}{1+R_{t+m+1}}\right]\left[\prod_{m=0}^{j}(1+R_{t+m})\right]^{-1}\right)\right\}$$

III. Demand for Money

A. Motives

1. Transactions - need money to conduct daily transactions

2. Precautionary - uncertainty about timing of income or expenditures

3. Portfolio - money is one of many assets that competes for a place in the total portfolio

B. A Simple Transactions Model

1. Examine household demand for money
    a. firms' demand for money is very similar to household demand
    b. ignore it for simplicity

2. Some simplifying restrictions

    a. no uncertainty

    b. income and consumption expenditures are fixed at annual rates y and c

    c. income is paid at fixed intervals of length T (= fraction of year)
        → amount paid per payments period is y_T = Ty

    d. consumption expenditures occur continuously

        i. suppose for simplicity that cannot store consumption goods

        ii. obviously unrealistic but nothing important is lost

    e. two assets

        i. short-term saving asset b - pays interest at rate R

ii. money (m/P) - pays interest at rate $R_M < R$
→ if $R_M \geq R$, b would not be held, which is uninteresting

f. must use money to buy consumption goods
→ savings asset cannot be exchanged for goods, only for money

g. cost of converting bonds to money or money to bonds is γ per conversion
i. could make γ depend on size of transaction (like a brokerage fee)
ii. introduces complications without providing any extra insight

h. all interest income from holding (m/P) and b is saved in some long-term asset a
i. not spent on c
ii. can include interest earnings in the income that is spent on c without changing anything important (hard to do; see Jovanovic JPE, Romer QJE)

3. Sequence of events

a. Household is paid y_T at beginning of payments period

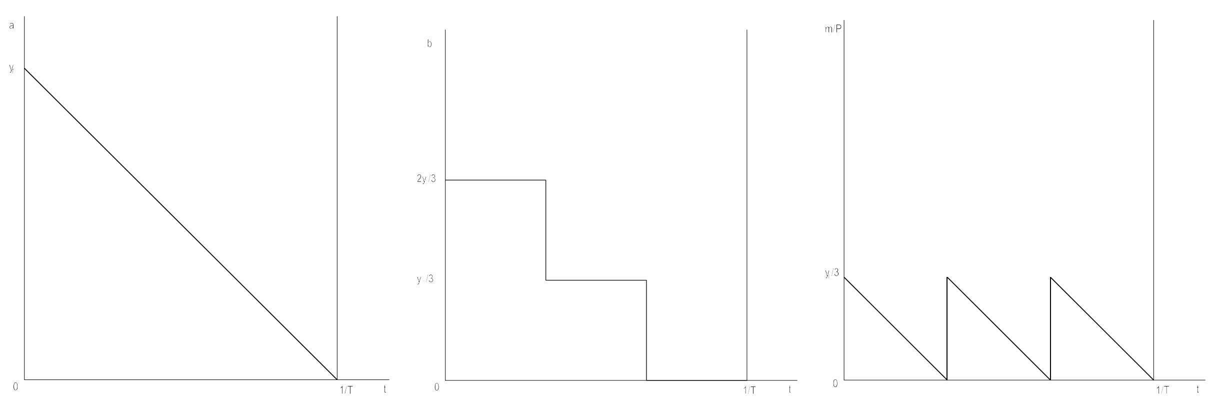
b. Household converts some part of y_T (yet to be determined) to b

c. Begins spending (m/P) at a constant rate to buy c

d. When (m/P) falls to zero, household converts some b to (m/P)

e. Process continues until the end of the payments period, when all assets are spent and the household is paid again

f. Process then repeats



g. A fact we will use but not explain (because doing so is tedious and unenlightening):

Conversions between b and (m/P) are equally spaced.

4. Objective

a. Household seeks to maximize profit from holding (m/P) and b:

$$\text{Profit} = \text{Interest income from b} + \text{Interest income from (m/P)} - \text{Total conversion costs}$$

$$= R\,T\,b_{Avg} + R_M\,T\,(m/P)_{Avg} - n\gamma$$

where

i. Interest rates are multiplied by the fraction of the year T to which they apply

ii. b_{Avg} is average holdings of b over the payments period

iii. $(m/P)_{Avg}$ is average holdings of (m/P) over the payments period

iv. *n* is the number of conversions between b and (m/P)

b. The nature of the optimization problem

i. On the one hand, holding all wealth as (m/P) minimizes conversion costs

ii. On the other hand, holding all wealth as (m/P) also minimizes interest earnings

c. Solution

i. To maximize profit, first find expressions for b_{Avg} and $(m/P)_{Avg}$

(a) $(m/P)_{Avg}$

(1) The amount of (m/P) withdrawn on each conversion is y_T/n

(2) Because c proceeds at a constant rate, (m/P) falls at a constant rate (= c)

(3) Therefore, $(m/P)_{Avg}$ is given by the expression

$$(m/P)_{Avg} = \frac{\text{Initial (m/P)} + \text{Terminal (m/P)}}{2}$$

$$= \frac{y_T/n + 0}{2}$$

$$= \frac{y_T}{2n}$$

(b) b_{Avg} is not so easy to compute because of the stair-step pattern it follows over time

(1) First calculate average total assets

$$a_{Avg} = \frac{\text{Initial a} + \text{Terminal a}}{2}$$

$$= \frac{y_T + 0}{2}$$

$$= \frac{y_T}{2}$$

(2) Then note that

$$a_{Avg} = b_{Avg} + (m/P)_{Avg}$$

$$\Rightarrow \quad b_{Avg} = a_{Avg} - (m/P)_{Avg}$$

$$= \frac{y_T}{2} - \frac{y_T}{2n}$$

ii. Substitute these expressions for $(m/P)_{Avg}$ and b_{Avg} into the expression for profit

$$\text{Profit} = R\,T\left(\frac{y_T}{2} - \frac{y_T}{2n}\right) + R_M\,T\,\frac{y_T}{2n} - n\gamma$$

This is an expression in one unknown, *n*.

iii. Maximize profit by finding the appropriate value of *n*

$$\frac{\partial \text{Profit}}{\partial n} = \frac{R\,T\,y_T}{2n^2} - \frac{R_M\,T\,y_T}{2n^2} - \gamma$$

$$= 0 \qquad \text{for max}$$

$$\Rightarrow \quad n^2 = \frac{(R - R_M)\,T\,y_T}{2\gamma}$$

$$\Rightarrow \quad n = \sqrt{\frac{(R - R_M)\,T\,y_T}{2\gamma}}$$

(only the positive root has economic meaning)

iv. Substitute this value into the expression for $(m/P)_{Avg}$ to obtain

$$(m/P)_{Avg} = \sqrt{\frac{y\gamma}{2(R - R_M)}}$$

$$= \sqrt{\frac{c\gamma}{2(R - R_M)}}$$

which is the famous square root rule.

→ Note that it is y, not y_T, that is in this expression.

v. We therefore have the following relations:

(a) $\dfrac{\partial[(m/P)_{Avg}]}{\partial c} > 0$

(b) $\frac{\partial[(m/P)_{Avg}]}{\partial\gamma} > 0$

(c) $\frac{\partial[(m/P)_{Avg}]}{\partial(R-R_M)} < 0$

d. We take $(m/P)_{Avg}$ to be "the demand for money"

i. represents average holding of money for any one person

ii. should be about the average holding of the representative person at any given time

e. Demand for the saving asset

i. The average stock of the savings asset depends positively on n:

$$b_{Avg} = \frac{y_T}{2} - \frac{y_T}{2n}$$

ii. From this, it is straightforward to show that

(a) $\frac{\partial b_{Avg}}{\partial c} > 0$

(b) $\frac{\partial b_{Avg}}{\partial\gamma} < 0$

(c) $\frac{\partial b_{Avg}}{\partial(R-R_M)} > 0$

C. Implications

1. Money demand depends on

a. y (or c) positively
→ volume of transactions

b. γ positively

i. higher γ makes conversion coslty
ii. make conversions less frequently
iii. take out more cash in each conversion
iv. higher average holding of (m/P)

c. the yield spread $R-R_M$ negatively
→ opportunity cost of holding money

2. Conversion costs γ

3. Note the square-root relation between (m/P) and y (or c)

a. ⇒ doubling y does not double (m/P) (raises it by square root of 2 ≈ 1.414)

b. Intuition (informal):

i. Suppose y doubles

ii. If we keep *n* unchanged, then (m/P) = $y_T/2$ also doubles

iii. costs paid

(a) opportunity cost paid doubles: $(R-R_M)(m/P)$

(b) conversion cost does not change: $n\gamma$

iv. Can lower total cost by letting *n* rise a little

(a) raises conversion cost

(b) lowers opportunity cost even more because of nonlinear relation between $(m/P)_{Avg}$ and *n*

D. The demand for money function

1. Household demand

a. $$h_{it}^D = h^D(c_{it}, R_t - R_M, ...)$$

b. Currency pays no interest, so $R_M = 0$ if the only type of money is currency.

c. Household money demand then simplifies to

$$h_{it}^{D} = h^{D}(c_{it}, R_t, ...)$$

2. Aggregate demand

a.
$$H_t^D = \sum_{i=0}^{I} h_{it}^D$$
$$= \sum_{i=0}^{I} h^D(c_{it}, R_t, ...)$$

b. When aggregate consumption C_t changes, household consumption c_{it} changes in the same direction, on average (because C_t is just the sum of the c_{it})

c. We therefore can write aggregate money demand as a positive function of C_t and a negative function of R_t:

$$H_t^D = H^D(C_t, R_t, ...) \qquad \partial H^D/\partial C > 0 \ ; \quad \partial H^D/\partial R < 0$$

E. Demand for money by firms

1. Suppose each firm

a. receives income at the rate q_T per payments period

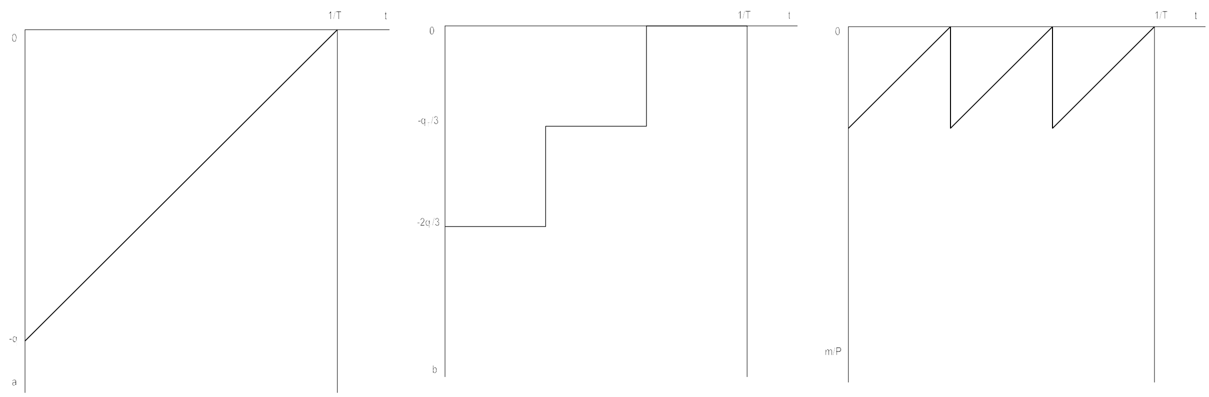
b. borrows q_T at the start of each payments period to pay their workers

c. continuously collects money from selling products

d. periodically transfers accumulate money to a savings asset b

e. money pays interest at the annual rate R_M and the savings asset b pays interest at the annual rate R

2. Firm portfolio management problem is symmetric to household problem:



3. It is straightforward to show that the firm's demand for money has the same form as household demand:

$$(m/P)_{Avg} = \sqrt{\frac{q\gamma}{2(R - R_M)}}$$

G. Aggregate demand for money

1. Aggregate demand for money is the sum of households' demand and firms' demand

2. The form for is the same as for households alone:

$$H_t^D = H^D(C_t, R_t, ...) \qquad \partial H^D/\partial C > 0 \ ; \quad \partial H^D/\partial R < 0$$

IV. General Equilibrium with Money

A. Total demand for bonds

1. Households now hold bonds for two reasons:

a. Consumption smoothing - the motive we studied earlier; long term

b. Cash management - the motive related to money demand; short term

2. Firms hold money only for cash management

3. Total demand for bonds is the sum of the three components:

B^D = (Household bond demand for consumption-smoothing) + (Household bond demand for cash management) + (Firm bond demand for cash management)

4. Net (or excess) bond demand is positively related to the interest rate and has the same form as before.

B. Walras' Law

Now requires that

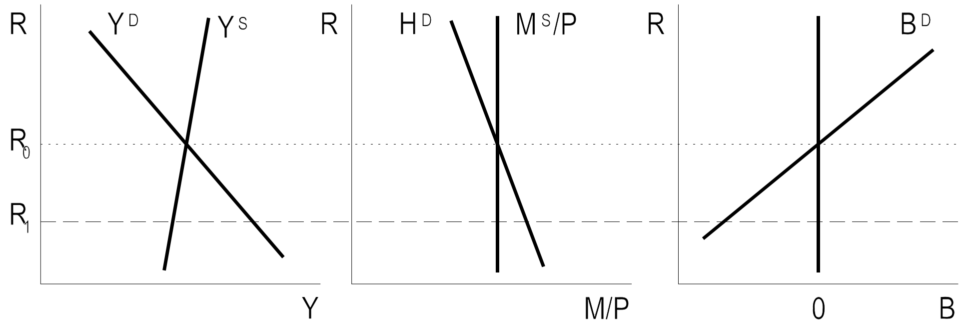
$$(C_t^D - Y_t^S) + (B_t^d - B_t^s) + (H_t^D - M_t / P_t) \equiv 0$$

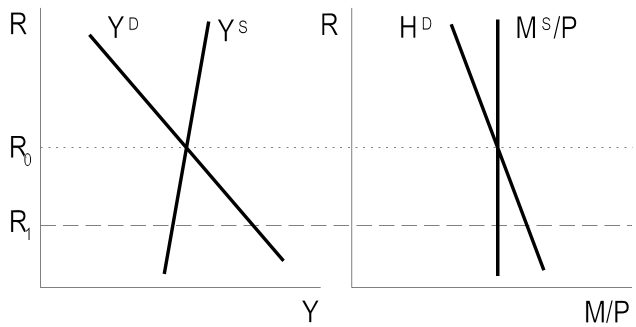
C. General Equilibrium

1. In general, Walras' Law implies that, with n markets, only n-1 excess demands are independent; equivalently, only n-1 of the markets are independent.

2. Here, we have three markets: output, money, and bonds.

3. Use Walras' Law to eliminate the bond market.





D. Example: Repeat earlier exercise current income y_t is unchanged but expected future income y_{t+i} rises

1. Demand for consumption rises

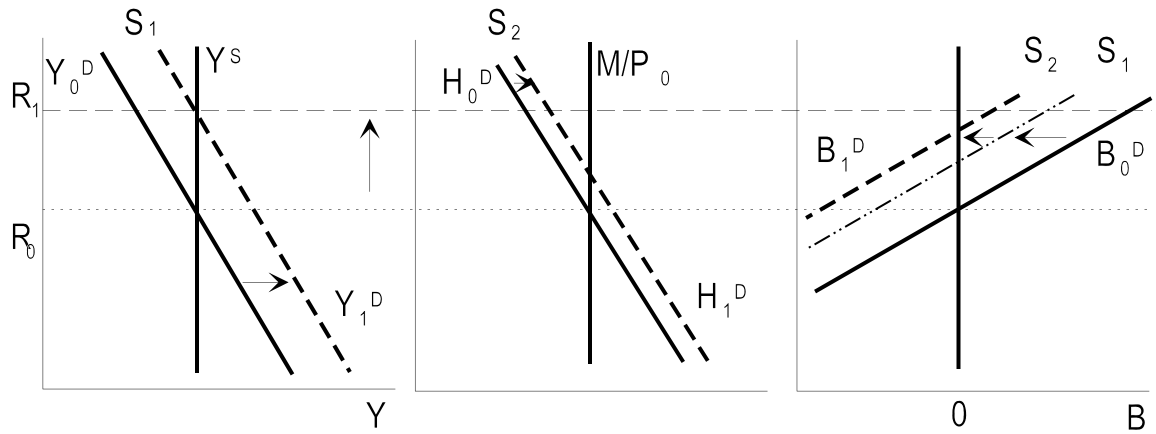
2. Because consumption demand rises, money demand also rises

3. Bond demand falls for two reasons:

a. to pay for the increase in consumption demand (shift S_1)

b. to pay for the increase in money demand (shift S_2)

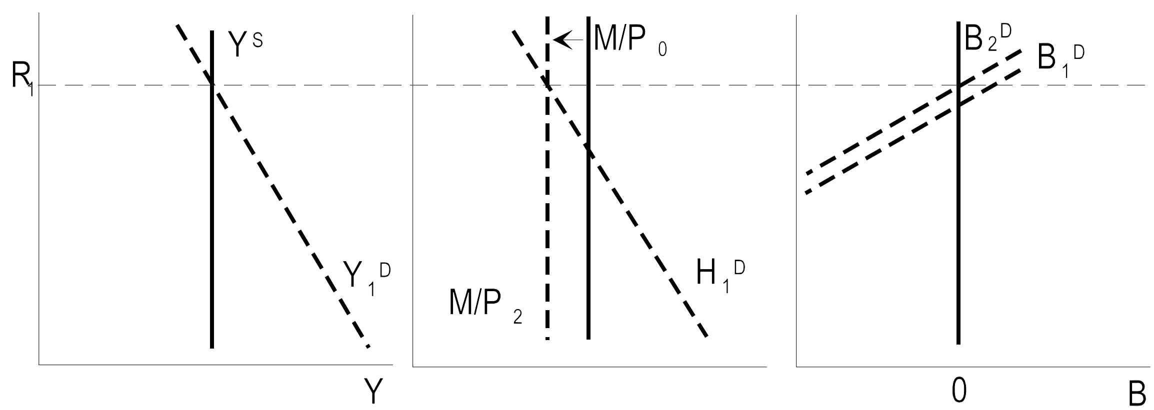
4. The economy is out of equilibrium with excess demand for consumption, excess demand for money, and excess supply of bonds. [Simplify for now by supposing that Y^S does not respond to the interest rate and is therefore vertical.]



5. We can see that equilibrium in the output market will be restored if the interest rate rises to R_1.

6. However, at R_1 *and* original price level P_0, we must have excess supply in the money market. The reason is that, at R_1, demand for output is back at its original level but interest rates are higher ⇒ lower quantity of money demanded (same Y^D, higher R). By Walras' Law, there must be excess demand in the bond market of the same magnitude as the excess supply in the money market.

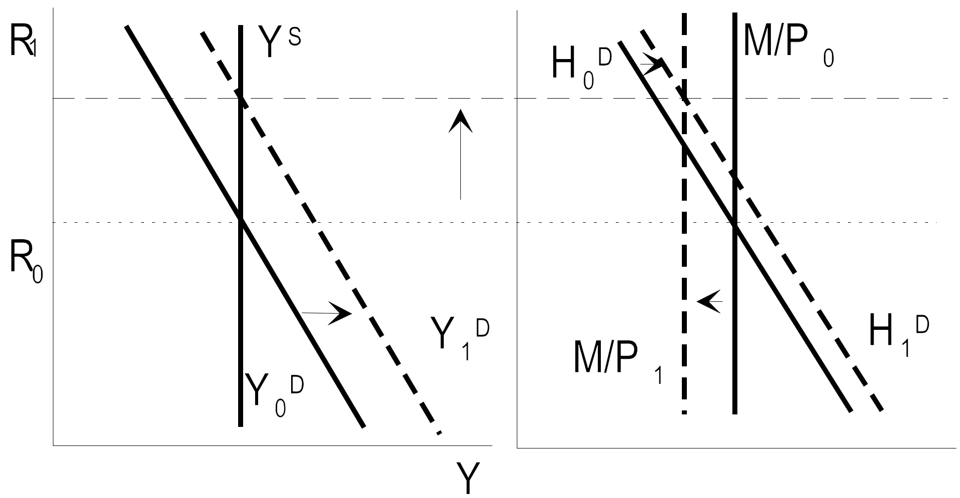
7. General equilibrium is restored by changing the other price, P, which rises. An increase in P reduces the real money supply M/P and simultaneously reduces the excess demand for bonds. P rises until general equilibrium is restored.



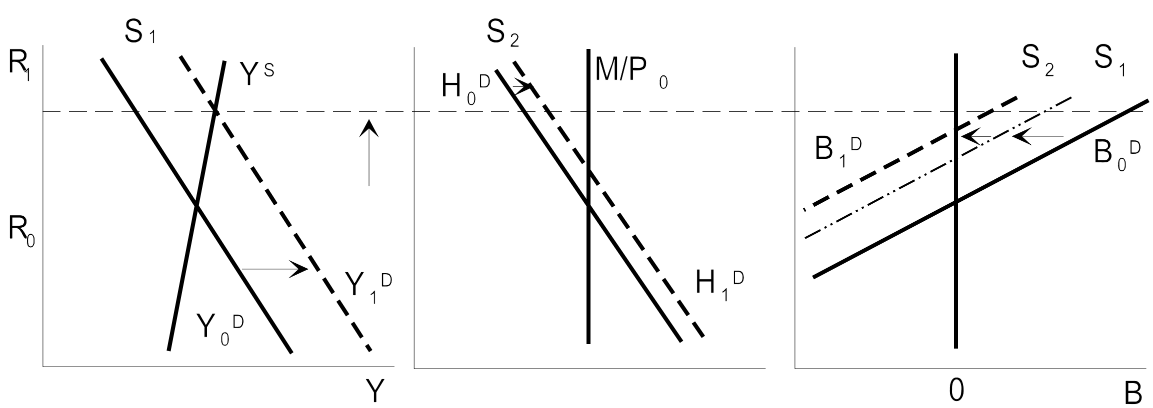
8. There are two prices in this economy: R and P. However, P does not affect the

output market, so R must take whatever value is necessary to establish equilibrium in the commodity market. P then changes to establish equilibrium in the money market.

9. Notice that we can do the entire analysis with just the output and money markets:



10. The results are essentially the same if Y^S responds to the interest rate:



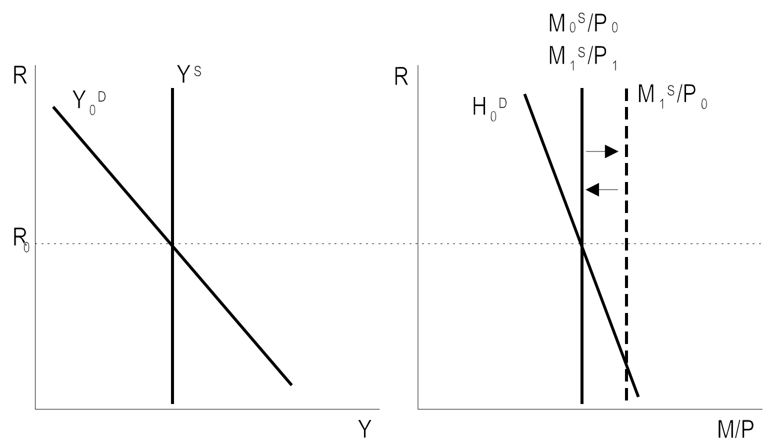
E. One-time increase in M

1. Central bank prints a *small* amount of money and gives it to people (“helicopter money”)

2. Because the gift is small (compared to normal income y)

   a. there is very little increase in C^D and therefore in H^D

   b. virtually all the M increase is absorbed by an increase in B^D



3. The disequilibrium in the money market is eliminated by an increase in P

4. Notice that no real variables are affected, a result called the *Neutrality of Money*. (The Neutrality of Money is the result that a one-time change in the nominal money supply ultimately has no real effects, only nominal effects.)

F. Increase in M: the short-run dynamics

We will discuss this later.