General Equilibrium

I. Definitions

A. Market Equilibrium

1. Demand = Supply

2. ⇒ the price clears the market

B. General Equilibrium

1. (Demand = Supply) in all markets simultaneously

2. We have two markets:

   a. Output (or "goods" or "commodities")

   b. Bonds (or Loans)

so general equilibrium requires that

demand for output = supply of output

demand for bonds = supply of bonds

at the same time.

3. We have one price - the interest rate R

4. The problem is to find a value for R that clears both markets simultaneously

II. Aggregate Markets

A. Output market (also called the "product market," "goods market," or "commodity market")

1. Aggregate demand for output

   a. Definition: the sum of all individual demands for output

   b. In our simple economy, the only demand for output is household demand for consumption, so aggregate demand is:

$$Y_t^D \equiv \sum_{i=1}^{I} c_{it}^D$$

$$\equiv C_t^D$$

2. Aggregate supply

a. Definition: the sum of all individual supplies of output

b. In our simple economy, each household supplies labor to its own production process to supply itself with output, so aggregate supply is:

$$Y^S \equiv \sum_{i=1}^{I} y_{it}^S$$

$$= \sum_{i=1}^{I} f(l_{it}^S)$$

B. Bond market

1. Bond demand

a. Some households have positive demand for bonds (i.e., they are lending). Denote this subset of households by I^+. Then

$b^D > 0$ for all households in I^+

b. Other households have negative bonds (i.e., they are borrowing). Denote this subset of households by I^-. Then

$b^D < 0$ for all households in I^-

2. Define aggregate bond demand and supply:

$$B_t^d \equiv \sum_{i \in I^+} b_{it}^D > 0 \qquad \text{aggregate bond demand}$$

$$= B^d(R_t, \ldots) \qquad \partial B^d / \partial R > 0$$

$$B_t^s \equiv -\sum_{i \in I^-} b_{it}^D > 0 \qquad \text{aggregate bond supply}$$

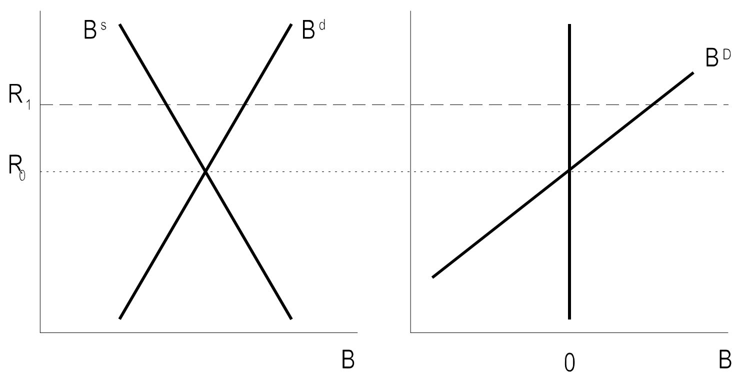
$$= \quad B^s(R_t, ...) \qquad \partial B^s/\partial R = -\sum \partial b_{it}^D/\partial R < 0$$

3. Also define net (or "excess") aggregate bond demand as

$$B_t^D \equiv B_t^d - B_t^s$$

a. In equilibrium, B^D must be zero, because in equilibrium $B^d = B^S$ (by definition of equilibrium)

b. Out of equilibrium, B^D can be positive or negative



C. Aggregate budget constraint and Walras' Law

1. Sum across all households to get the aggregate budget constraint

$$\sum_{i=1}^{I} b_{it}^D = \sum_{i=1}^{I} y_{it} + (1+R_t)\sum_{i=1}^{I} b_{it-1} - \sum_{i=1}^{I} c_{it}^D$$

$$\Rightarrow B_t^d - B_t^s = Y_t^S + (1+R_t)\sum_{i=1}^{I} b_{it-1} - C_t^D$$

$$= Y_t^S + (1+R_t)B_{t-1}^D - C_t^D$$

2. But in period t-1, the economy was in equilibrium (our entire analysis assumes prices moves quickly enough that the economy achieves equilibrium each period).

Therefore $B_{t-1}^{D} = 0$ and we can write

$$B_t^d - B_t^s = Y_t^S - C_t^D$$

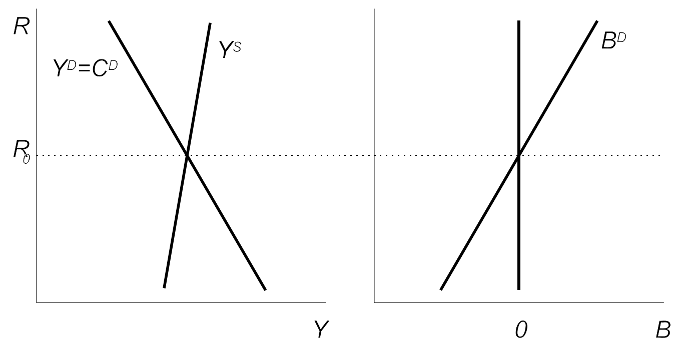
$$\Rightarrow \qquad B_t^D = Y_t^S - C_t^D$$

3. This last expression is a simple version of Walras' Law - "The sum of excess demands is identically zero."

$$(C_t^D - Y_t^S) + (B_t^d - B_t^s) \equiv 0$$

III. General Equilibrium

A. Graphical representation



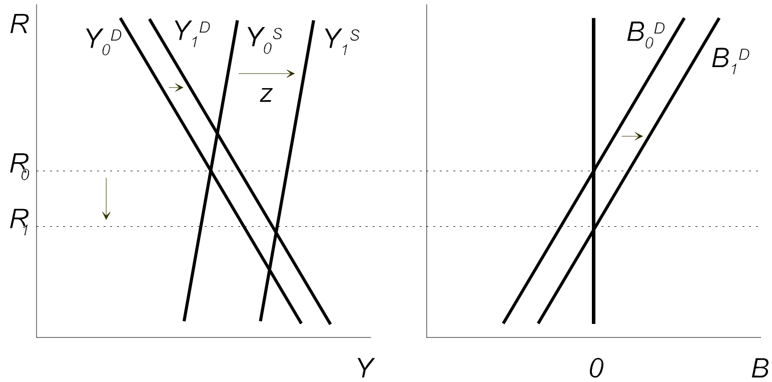
B. Effects of a disturbance to equilibrium

1. Suppose R = ρ so that C is constant over time. Income unexpectedly rises by z in the current period but is expected to return to its old value next period and then remain there (perhaps foreign aid from another country for one year).

   a. Current income rises by z

   b. Current consumption demand rises by [R/(1+R)]z < z, and bond demand rises by (z - [R/(1+R)]z) = z [1/ (1+R)] because everyone wants to save most of z, as we have seen previously.

   c. At the original interest rate R_0, the economy now is out of equilibrium:

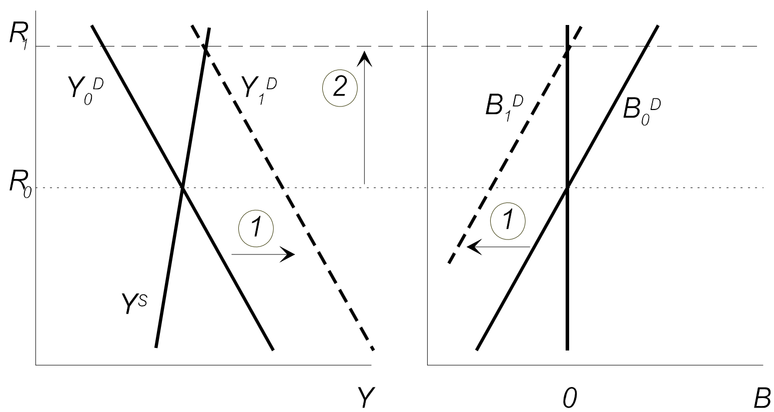


d. The interest rate falls to R_1 to restore equilibrium.

2. Suppose that current income y_t is unchanged but some expected future income y_{t+i} rises

a. People see an increase in lifetime wealth, so their demand for consumption rises today, even though current income has not changed. They reduce their bond demand (that is, they try to borrow) to pay for the higher consumption.

b. The economy is out of equilibrium with excess demand for consumption and excess supply of bonds:



c. The interest rate rises to reestablish general equilibrium.

IV. A Simplification

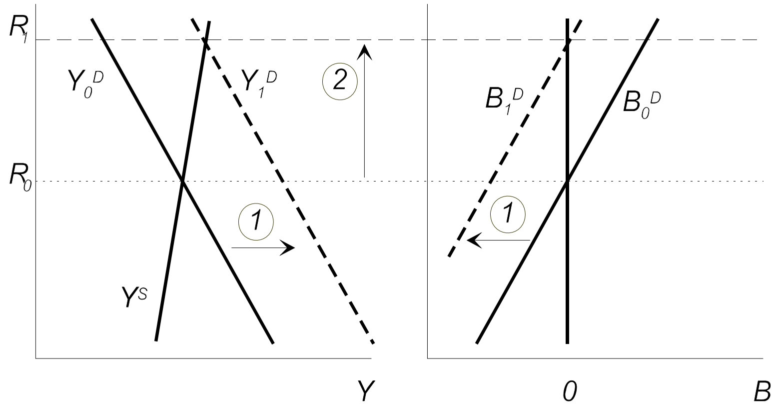
A. Implication of Walras' Law

$$(C_t^D - Y_t^S) + (B_t^d - B_t^s) \equiv 0$$

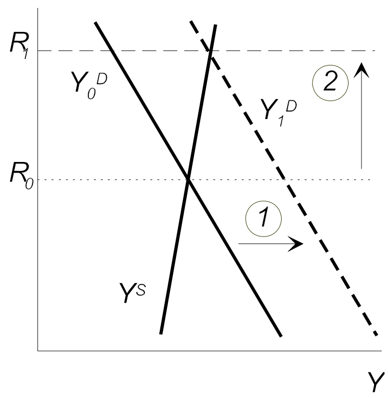
Note that *only one* excess demand is independently determined

B. Can do the whole analysis with only one market

1. Two graphs:



2. One graph:



V. Budget Constraint with Time-Varying Interest Rates

When interest rates change over time, the budget constraint is

$$\frac{c_t}{(1+R_t)} + \frac{c_{t+1}}{(1+R_t)(1+R_{t+1})} + \frac{c_{t+2}}{(1+R_t)(1+R_{t+1})(1+R_{t+2})} + \ldots$$

$$= b_{t-1} + \frac{y_t}{(1+R_t)} + \frac{y_{t+1}}{(1+R_t)(1+R_{t+1})} + \frac{y_{t+2}}{(1+R_t)(1+R_{t+1})(1+R_{t+2})} + \ldots$$

or

$$\sum_{i=0}^{\infty} c_{t+i}\left[\prod_{m=0}^{i}(1+R_{t+m})\right]^{-1} = b_{t-1} + \sum_{i=0}^{\infty} y_{t+i}\left[\prod_{m=0}^{i}(1+R_{t+m})\right]^{-1}$$

VI. Extensions: Investment and Government

A. Investment demand

$I^D_t = I^D(R_t, \ldots)$ $\qquad dI^D/dR_t < 0$

B. Government demand (government purchases)

$G^D_t = G_t$ $\qquad$ exogenous

C. Aggregate demand

$Y^D_t = C^D(R_t, \ldots) + I^D(R_t, \ldots) + G_t$

D. Graph and analysis are the same as before, except that now changes in investment demand or government demand can shift the aggregate demand curve Y^D.