

ECG590I - Asset Pricing

Problem Sets

1. Introduction to various derivatives

Hull 7th edition: Problems 1.4, 1.5, 1.6, 1.9, 1.22

2. Present value

1. Use continuous time to calculate the present value of the following income streams:

a. Three one-time payments of \$100, \$120, and \$140 received, respectively, 6 months, 1 year, and 2 years from now. The discrete-time interest rate is 10% (one payment per annum).

b. A continuous stream of income $x_t = 200 - 50t$ from $t = 0$ to $t = 3$ (three years). During the first 18 months, the discrete-time interest rate is constant at 10%. It then goes down to 5% until the end.

c. A continuous stream of income $x_t = 100t$ from $t = 0$ to $t = T$ with the continuous-time, time-varying interest rate equal to $r_t = 0.1t$.

2. In question 1.c, what is the current value of the income stream at time $t = T$, i.e., the moment the final payment is made?

3. Forward and futures prices

Hull 7th edition: Problems 2.3

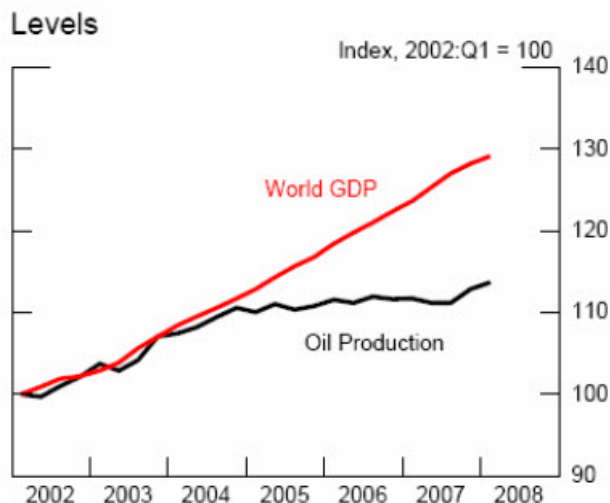
Hull 7th edition: Problems 5.9, 5.11, 5.14, 5.16, 5.20

4. Hedging using futures

Hull 7th edition: Problems 3.6, 3.13, 3.16

5. Demand, supply, and equilibrium

The graph below shows the behavior of world output (= world production) and petroleum output from the beginning of 2002 to the beginning of 2008. Explain how to use the information in it to draw a graph showing the changes in the market for petroleum between the beginning of 2005 and the beginning of 2008. Draw such a graph (that is, draw the demand and supply curves for petroleum in 2005 and 2008) and use it to explain the large increase in the price of petroleum over that period.

**6. Interest rates**

Hull 7th edition: Problems 4.4, 4.8, 4.9

7. Basics of options, including trading strategies

- (a) Hull 7th edition: Problems 8.1, 8.2, 8.12
- (b) Hull 7th edition: Problems 9.7, 9.16, 9.18
- (c) Hull 7th edition: Problems 10.4, 10.11, 10.12

8. Risk aversion

A lottery ticket pays \$4 with probability 1/2 and \$16 with probability 1/2. Consider two people with different utility functions. The first person has the utility function $U(C) = \sqrt{C}$, and the second person has the utility function $U(C) = \ln C$. Which person has the higher value for the lottery ticket? Explain the underlying reason for that outcome.

9. Arbitrage theorem

Suppose there are three underlying assets: a riskless bond B with an interest rate of 1% per year and two risky stocks, S_1 and S_2 . There are three states of the world: hot, warm, and cold. The matrix of current asset prices is

$$\begin{bmatrix} B_t \\ S_{1t} \\ S_{2t} \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.4 \\ 3.0 \end{bmatrix}$$

and the payoff matrix is

$$\begin{bmatrix} B_{t+1,H} & B_{t+1,W} & B_{t+1,C} \\ S_{1,t+1,H} & S_{1,t+1,W} & S_{1,t+1,C} \\ S_{2,t+1,H} & S_{2,t+1,W} & S_{2,t+1,C} \end{bmatrix} = \begin{bmatrix} 1.01 & 1.01 & 1.01 \\ 1.00 & 2.00 & 3.00 \\ 3.00 & 2.00 & 5.00 \end{bmatrix}$$

- (a) What is the vector of state prices?
- (b) What is the vector of risk-neutral probabilities?
- (c) What is the value in period t of a call option C_1 on S_1 with the payoff vector

$$\begin{bmatrix} C_{1,t+1,H} & C_{1,t+1,W} & C_{1,t+1,C} \end{bmatrix} = \begin{bmatrix} 0 & 0.50 & 1.50 \end{bmatrix}$$

10. Binomial trees

Hull 7th edition: Problems 11.4, 11.5, 11.14

11. Elements of asset pricing

Hull 7th edition: Problems 12.3, 12.4, 12.5, 12.10

12. Introduction to Black-Scholes

Hull 7th edition: Problems 13.4, 13.11, 13.18

13. Implied volatility

Hull 7th edition: Problems 18.1, 18.2, 18.6, 18.8, 18.15a

14. Greek letters and hedging

Hull 7th edition: Problems 17.14, 17.16, 17.23

15. Capital Asset Pricing Model and portfolio management

Gold pays no dividend and thus has a negative yield spread compared to the riskless asset: $\mu - r = -r < 0$. Gold also is a risky asset. Its price is a random variable, changing over time in ways that often are unpredictable. So gold is riskier than the riskless asset but pays a lower return. Nevertheless, gold is a popular asset. Perhaps investors are irrational or stupid. Perhaps instead there is some aspect of gold's behavior that we have overlooked. What characteristic does CAPM tell us gold would have to have for investors' behavior to be perfectly rational? Is there any evidence that gold has that characteristic?