

# CBMS Conference on Cluster Algebras and Applications

## Schedule of Talks

(all talks to be held in Room 201, Harrelson Hall, NCSU)

### June 13, 2006, Tuesday

- 8:30-9:00 Registration, Coffee and Refreshments, Rm 245
- 9:00-9:10 Welcome by Hien Tran, Associate Head
- 9:10-10:10 Andrei Zelevinsky  
Cluster Algebras 1
- 10:10-10:30 Coffee Break, Rm 245
- 10:30-11:30 Andrei Zelevinsky  
Cluster Algebras 2
- 11:30-2:00 Lunch
- 2:00-2:50 Monica Vazirani  
Affine tableaux & the double affine Hecke algebras
- 3:00-3:50 Sebastian Zwicknagl  
Quantum Symmetric algebras and Canonical Bases
- 3:50-4:20 Coffee Break

### June 14, 2006, Wednesday

- 8:30-9:00 Coffee and Refreshments, Rm 245
- 9:00-10:00 Andrei Zelevinsky  
Cluster Algebras 3
- 10:00-10:20 Break
- 10:20-11:20 Andrei Zelevinsky  
Cluster Algebras 4
- 11:30-2:00 Lunch
- 2:00-2:50 Christof Geiss  
From preprojective algebras to cluster algebras
- 3:00-3:50 Michael Shapiro  
Poisson properties of cluster algebras and semiconductor  
nets
- 4:10-4:20 Coffee Break
- 4:30-5:30 Discussion

## June 15, 2006, Thursday

8:30-9:00 Coffee and Refreshments  
9:00-10:00 Andrei Zelevinsky  
Cluster Algebras 5  
10:00-10:30 Break  
10:30-11:30 Andrei Zelevinsky  
Cluster Algebras 6  
11:30-2:00 Lunch  
2:00-2:50 Nathan Reading  
The combinatorics (and "dual" combinatorics) of clusters  
3:00-3:50 Ralf Schiffler  
Introduction to Cluster Categories  
3:50-4:20 Coffee Break  
4:30-5:20 Hugh Thomas  
Clusters and noncrossing partitions  
6:30- Banquet at 35 Chinese Restaurant  
1135 Kildare Farm Road, Cary  
Direction: I-440 (Beltline), exit US 1 S, right to Cary Park-  
way, right to Kildare Farm. Tel: 919-467-4262, 380-7378

## June 16, 2006, Friday

8:30-9:00 Coffee and Refreshments  
9:00-10:00 Andrei Zelevinsky  
Cluster Algebras 7  
10:00-10:30 Break  
10:30-11:30 Andrei Zelevinsky  
Cluster Algebras 8  
11:30-2:00 Lunch  
2:00-3:00 Dylan Thurston  
Cluster algebras from surfaces  
3:00-3:50 Gerard Helminck  
A flag variety relating matrix hierarchies and Toda-type  
hierarchies  
4:10-4:30 Coffee Break  
4:30-5:30 Discussion

**June 17, 2006, Saturday**

8:30-9:00	Coffee and Refreshments
9:00-10:00	Andrei Zelevinsky Cluster Algebras 9
10:00-10:30	Coffee Break
10:30-11:30	Andrei Zelevinsky Cluster Algebras 10

### **Transportation Information**

We have arranged airport pick-up service on June 12 and drop off on June 17. We will have a shuttle running periodically depending on need.

If you have not done so, please give your flight information with arrival time at Raleigh-Durham Airport. Your arrival will be either terminal A or C depending on the airlines. In either case, please come to the luggage claim area for pick-up.

If you take a taxi or are driving please note that conference housing is at the North Hall on Hillsborough Street opposite Bell Tower.

If you stay in Velvet Cloak Inn, they provide a free airport pick-up service. You will need to call 919-828-0333 in advance to save waiting time. Their van leaves for the airport at every hour from 7 am to 10 pm.

If we miss you, you can take a taxi (about \$25) or use the R & G Airport shuttle (\$ 18 one way, 1-800-840-2RDU) (1-800-840-2738 or 1-919-847-2226).

### **Registration**

Please go to the Math building (Harrelson Hall) Tuesday morning before 8:30 and pick up your name tags and conference materials.

### **Driving Direction**

The conference housing is in the North Hall in Hillsborough Street, approximately the opposite of Bell Tower.

The conference will be held in Harrelson Hall in Room 201. Harrelson Hall is about 10 min walking distance from the dorm. You can walk along Hillsborough westward for a few min. to Harrelson Hall. In order to park on campus you will need a parking permit (cost is about \$2.50 per day). You can purchase the parking permit when you check-in.

In the following we include the direction to the dorm. For direction to Velvet Cloak Inn or Brownestone Hotel please see the end.

#### **Coming From Interstate 85:**

1. Take I-85 (north or south, depending on where you started) until you get to the I-40 junction. From there, take I-40 East.

2. Continue until you get to Exit 289 - The Wade Avenue Exit. Taking this exit will lead you into the heart of Raleigh. After exiting off I-40, continue up Wade Avenue until you get to Faircloth street (it will be the second stoplight).

3. There will be a shopping center to your left, and signs indicating Meredith College to the right. Turn right at this stoplight onto Faircloth Street. Continue on Faircloth Street, and then turn left to Hillsborough Street. Drive until you see a Bell Tower on your right, the North Hall is on the left side of Hillsborough.

#### **Coming from I-40 Heading East:**

1. Take I-40 until you get to Exit 289 - Wade Avenue Exit. Then follow (2) and (3) above.

#### **Coming From Interstate 95:**

1. Take I-95 (north or south, depending on where you started from) until you get to the I-40 junction. Take the I-40 West exit. From there follow the I-40 directions below...

**Coming from I-40 Heading West:**

1. Take I-40 into Raleigh (you'll know you're in Raleigh when you see the sign "Raleigh Next 9 Interchanges").

2. You will then enter the beltline. Be sure to take I-40 West-Durham, be sure you do not take I-440 North and East Raleigh. In short, do not exit off I-40.

3. At Exit 295 - Gorman Street get off I-40. After exiting, take a right onto Gorman Street. Then drive on Gorman until you reach Hillsborough Street and then turn right to Hillsborough.

4. Drive on Hillsborough for a few min., the campus is now on your right. Before you see a Bell Tower, turn left to Enterprise Street. North Hall Dormitory is at the corner (opposite side from the Bell Tower)

**Coming from Raleigh-Durham International Airport:**

Leave the airport and get on Aviation Parkway (towards I-40). You can either then take Airport Boulevard to I-40 or follow Aviation Parkway to I-40. At the I-40 junction, take the I-40 East exit. This exit will get you on I-40 headed in the right direction.

1. Take I-40 until you get to Exit 289 - Wade Avenue Exit. Then follow (2) and (3) from the section of "**Coming From Interstate 85:**".

**Direction to Velvet Cloak Inn and Brownstone Hotel**

From the Beltline (I-440) exit at Hillsborough St. (this is the next exit to Wade Av. Exit if you are coming from north, or the next exit to Western Blvd from the south dir) towards NCSU or Meredith College direction or Downtown direction, proceed along Hillsborough St. Velvet Cloak Inn and Brownstone are next each other. Velvet Cloak Inn is at 1505 Hillsborough St, Raleigh, NC 27605 (Tel: 919-828-0333).

## Abstracts

**Monica Vazirani**

### **Affine Tableaux and the Double Affine Hecke Algebra**

The irreducible representations (irreps) of the symmetric group  $S_n$  are parameterized by Young diagrams  $\lambda$ . A given irrep has a basis indexed by Young tableaux of shape  $\lambda$ . In fact, this basis consists of weight vectors for a commutative subalgebra  $X$  of the group algebra  $CS_n$ .

The double affine Hecke algebra (DAHA) is a deformation of the group algebra of the *affine* symmetric group  $\widehat{S}_n$ , and also contains a commutative subalgebra  $X$ .

Not every irrep of the DAHA has a basis of weight vectors (and in fact it is quite difficult to parameterize all of its irreps), but if we restrict our attention to those that do, these are parameterized by “affine shapes”  $\widehat{\lambda/\mu}$  and have a basis of  $X$ -weight vectors indexed by the “affine tableaux” of that shape. In this talk, we will construct these irreps. This is joint work with Takeshi Suzuki.

**Sebastian Zwicknagl**

### **Quantum Symmetric Algebras and the Canonical Bases**

In this talk I will introduce quantum deformations of the symmetric algebras of a class of Lie algebra modules and discuss their relation to the dual canonical bases in quantum groups and quantum cluster algebras. I will start with quantum symmetric algebras introduced in my recent work with A. Berenstein, and then explain when quantum symmetric algebras are, indeed, flat deformations of the ordinary symmetric algebras. Surprisingly, my classification of such flat quantum symmetric algebras implies that each such algebra is equipped with a basis resembling the dual canonical one. I will conclude the talk with the expected relation between my flat algebras and quantum cluster algebras introduced by A. Berenstein and A. Zelevinsky in 2004.

**Cristof Geiss**

### **From preprojective algebras to cluster algebras.**

This is a report on a joint project with B. Leclerc (Caen) and J. Schroer (Bonn).

Let  $N$  be a maximal unipotent subgroup of a complex simply connected simple Lie group of Dynkin type  $\Delta$ . Berenstein, Fomin and Zelevinsky established a natural cluster algebra structure on the coordinate Ring  $C[N]$  with an initial see given by certain generalized minors.

On the other hand, let  $\Lambda$  be the preprojective algebra of type  $\Delta$ . This is a finite-dimensional selfinjective algebra, its stable module category is triangulated and 2-Calabi-Yau. By dualizing Lusztig’s construction of the semicanonical basis we can assign to each  $\Lambda$ -module  $x$  a function  $\delta_x \in C[N]$ . In particular,

we obtain in this way the above mentioned generalized minors for certain submodules of indecomposable injective  $\Lambda$  modules. The multiplication of the  $\delta_x$  is compatible with the cluster structure on  $C[N]$ . In this way we manage to "categorify" this cluster algebra, and it turns out that the cluster monomials correspond to rigid  $\Lambda$ -modules, and thus by construction belong to the dual of the semi-canonical basis.

**Hugh Thomas**  
**Clusters and Noncrossing Partitions**

Given an arbitrary Coxeter group, one can define a poset of noncrossing partitions. These were studied first by combinatorialists in types A and B, then by geometric group theorists. We study them from the point of view of representation theory of hereditary algebras. In finite or affine type, we show that there is a natural correspondence between noncrossing partitions of a simply laced Coxeter group and finitely generated exact abelian extension-closed subcategories of the representations of the associated quiver (with an orientation depending on a choice in the definition of the noncrossing partitions). One consequence of this description of noncrossing partitions is a type-free proof that noncrossing partitions form a lattice in finite type. (A type-free proof of this result was first given by Brady and Watt in 2005.) Our perspective also helps make clear why the lattice property fails once we leave finite type.

In finite type, the number of noncrossing partitions is the same as the number of clusters. In 2005, Reading gave a bijection between clusters and noncrossing partitions in finite type. Using our new description of noncrossing partitions, we reinterpret Reading's bijection, and show that it extends to affine type.

This is joint work with Colin Ingalls (UNB).

**Gerard Helminck**  
**A flag variety relating matrix hierarchies and Toda-type hierarchies**

Commutative subalgebras of the complex  $k \times k$ -matrices are known to generate both matrix and Toda-type hierarchies. In this paper a certain class of infinite chains of closed subspaces of a separable Hilbert space  $H$  will be introduced. To each such a flag one associates a sequence of solutions of the matrix hierarchy related to this subalgebra. They compose to a solution of the lower triangular Toda hierarchy corresponding to the transposed algebra. Both solutions can be expressed in determinants of suitable Fredholm operators, the so-called  $\tau$ -functions. These last functions also have a geometric interpretation in terms of line bundles over the flagvariety. They measure the failure of equivariance w.r.t. to the commuting flows of certain global sections.

**Nathan Reading**

### **The combinatorics (and "dual" combinatorics) of clusters**

A cluster complex of finite type can be realized concretely in terms of a crystallographic root system. Clusters of variables correspond to "clusters" of roots. The positive span of each cluster is a cone, and this collection of cones defines a complete fan with interesting geometric and combinatorial properties. Specifically, it is the normal fan of a simple polytope (the "generalized associahedron") and the number of its maximal faces is given by a simple formula (the "W- Catalan number") in terms of basic numerical invariants of the root system.

In this talk, I will briefly review the combinatorics of cluster complexes of finite type and some related "Catalan" combinatorics. I will then discuss a dual approach to the cluster complex. In the dual setting, the cluster complex arises not by specifying which sets of vectors span cones, but rather by coarsening the Coxeter fan associated to the root system. (The Coxeter fan is the fan defined by the hyperplanes orthogonal to the roots.) As an example of what makes the dual approach interesting: "g-vectors" arise naturally in the dual setting as expansions in the basis of fundamental weights, just as "denominator vectors" arise in the usual setting as expansions in the basis of simple roots. This is joint work with David Speyer.

### **Dylan Thurston**

#### **Cluster algebras from surfaces**

Abstract: Triangulations of surfaces with at least one puncture provide many interesting examples of cluster algebras, including many which are mutationally finite: there are only a finite number of different combinatorial types of clusters. We will survey some of these results and show how, by introducing variables associated to closed loops in addition to arcs, we can hope to prove positivity of these cluster algebras. We will also see how passing to the tropical limit corresponds to looking at laminations of the surface and see how we can start to deal with closed surfaces. A large part of this talk is joint work with Sergey Fomin and Michael Shapiro.

### **Ralf Schiffler**

#### **Introduction to cluster categories**

Abstract: Cluster categories are triangulated categories which, on one hand, provide models for the cluster dynamics of an acyclic cluster algebra and, on the other hand, give an interesting connection between cluster algebras and the representation theory of finite dimensional algebras. In this talk, we will define cluster categories and see why they are useful. As an example, we will give an ad hoc construction of the cluster category of type  $A_n$ .

### **Michael Shapiro**

#### **Poisson properties of cluster algebras and semiconductor nets**

Abstract: In the first part of the talk we will discuss a natural geometric Poisson structure on semiconductor nets that induces an r-matrix Poisson bracket on  $N \times N$  matrices compatible with a natural cluster algebra structure. In the second part of the talk we consider nets on annulus and discuss an inverse problem.