1. (25 pts) Let \( A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 4 & 4 \end{bmatrix} \).
   
   (a) Find some bases for \( N(A) \) and \( C(A^T) \).
   
   (b) Split the vector \( x = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} \) into the sum \( x_r + x_n \), where \( x_r \) is the row space component and \( x_n \) is the nullspace component.
   
   (c) Is it possible to find a non-zero vector in the intersection \( N(A) \cap C(A^T) \)?

2. (25 pts) Let \( S \) be the subspace of \( \mathbb{R}^4 \) spanned by \( (1, 0, -2, 1) \) and \( (0, 1, 3, -2) \). Find a basis for the orthogonal complement \( S^\perp \)?

3. (25 pts) (a) Find the projection of the vector \( (1, 4) \) onto \( (1, 3) \).
   
   (b) Let \( Q \) be the point on the line \( y = \frac{1}{3}x \) that is closest to the point \( (1, 4) \). Determine the coordinates of \( Q \).

4. (25%) (a) Find the projection of \( b = (1, 2, 3)^T \) onto the column space of
   
   \[ A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 0 & 2 \end{bmatrix}. \]

   (b) Find the projection of \( b \) onto the first column vector of \( A \), and the projection onto the second column of \( A \). What is their sum?