1. (20 pts) Judge if the following statements are true (T) or false (F).
   (a) For any matrix $A \in M_{mn}$ the map $T(x) = Ax$ defines a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.
   (b) Let a fixed $B \in M_{mn}$, then the map $T(A) = AB$ is a linear transformation from $M_{mn}$ to $M_{nn}$.
   (c) The map $f(x) = (x_1 - 2x_2 + 2, x_1 - x_3, 3x_1 + x_2)$ defines a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$.
   (d) The equation $Ax = b$ is always solvable when $A$ has full rank.
   (e) If nullity $(A) = 5$, then nullity $(A^T A) = 5$.
   (f) If rank $(A^T A) = 5$, then nullity $(A) = 6$.
   (g) The normal equation $A^T Ax = A^T b$ always has a unique solution.
   (h) If the vector $v_3$ is a linear combination of two linearly independent vectors $\{v_1, v_2\}$, then the Gram-Schmidt process on $\{v_1, v_2, v_3\}$ will give three non-zero orthonormal vectors.
      (i) If $T$ and $S$ are linear transformations from $V$ to $V$, then $2T + 3S$ is also a linear transformation. Here $2T + 3S$ is given by $(2T + 3S)(v) = 2T(v) + 3S(v)$.
      (j) The map $T(f) = 3f - 2f'$ is a linear transformation on the space $P_3$ of polynomials of degree less than or equal to 3. ($f'$ is the derivative of $f$)

2. (20 pts) Let $T$ be the linear transformation of $\mathbb{R}^2$ that reflects about the line $y = x$ and then rotates 30 degrees counterclockwise. Find the matrix $A$ for $T$ such that $T(x) = Ax, x \in \mathbb{R}^2$.

3. (20 pts) Solve the following least squares system

   $5x_1 + 4x_2 = 1$
   $x_1 - x_2 = 2$
   $2x_1 + 3x_2 = -2$
   $3x_1 - 5x_2 = 0$

4. (20 pts) Use the Gram-Schmidt process to find an orthonormal basis for $RS(A)$, where
   
   $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 0 \end{bmatrix}$

5. (20 pts) Let $f(x) = (x_1 + x_2, 3x_1 - 4x_2, 3x_1 - x_2)^T$ be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$.
   (a) Can you find all vectors $x \in \mathbb{R}^2$ such that $f(x) = (0, 0, 0)^T$.
   (b) Find all vectors $x$ such that $f(x) = (2, -1, 2)^T$. 