MA305, Test Two, Dr. Jing Name ________________________

10/19/2006, 3:00-4:15 Row Number ________________________

You must show all your work except the multiple choices problems.

1 (20 pts) Judge if the following statements are true (T) or false (F).

(a) $S = \{(x_1, x_2, x_3) | x_1 + x_2 + x_3 = 0, x_i's \quad \text{are real numbers}\}$ is a subspace of $\mathbb{R}^3$.

(b) $S = \{(x_1, x_2) \in \mathbb{R}^2 | x_1 + 3x_2 = 1\}$ is a subspace of $\mathbb{R}^2$.

(c) If S is a spanning set of a vector space V, and T is a subset of S, then T is also a spanning set of V.

(d) If A is a 3×3-matrix and $det(A) \neq 0$, then any vector in R^3 is in the row space of A.

(e) Any five vectors of $\mathbb{R}_4$ are linearly dependent.

(f) Let $L(x) = Ax$ and $A \in \mathbb{R}^{m \times n}$, then L is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.

(g) The set $S = \{(x, y) | x + y \leq 1\}$ is a subspace of $\mathbb{R}^2$.

2 (20 pts) (a) Find bases for row subspace RS(A), column subspace CS(A), and the nullspace NS(A) for the following matrix A

$$A = \begin{bmatrix} 1 & 3 & -2 & 4 \\ 0 & 2 & 1 & 3 \\ 4 & 0 & 0 & 3 \\ -7 & -5 & 8 & -9 \end{bmatrix}$$

(b) What are the rank and nullity of A?

3 (a) (12pts) Determine if the vector $(4, 11, 6)$ a linear combination of the following three vectors

$$v_1 = (1, 2, 3), v_2 = (2, 5, 4), v_3 = (1, -4, 1)$$

(b) (8 pts) What is the condition for the vector (a, b, c) being a linear combination of v_1, v_2, v_3?

4 (20 pts) (a) Find the transition matrix from the basis A

$$u_1 = (1, 2), u_2 = (3, 2)$$

to the basis B

$$v_1 = (1, 1), v_2 = (2, -3)$$

(b) If a vector v has the coordinate matrix $(1, -1)$ with respect to the basis A, what is the coordinate matrix with respect to basis B?

5 (20 pts) Solve the least squares solution to the system

$$-x + y = 10$$
$$2x + y = 5$$
$$x - 2y = 20$$