

Answer to MA305, Test 1

#1 $\begin{pmatrix} 3 & -1 & -1 & 7 \\ 1 & 2 & -3 & -7 \\ 1 & -2 & 1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & & & 2 \\ & 1 & & -3 \\ & & 1 & 5 \end{pmatrix}$

$$3 \cdot 2 + (-1)(-3) + (-1)5 = 6 + 3 - 5 = 4$$

$\begin{pmatrix} 3 & -2 & -1 & 7 \\ 2 & 1 & -3 & -4 \\ 1 & 2 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & & & 2 \\ & 1 & & -3 \\ & & 1 & 5 \end{pmatrix}$

$$3 \cdot 2 + (-2)(-3) + (-1)5 = 6 + 6 - 5 = 7$$

#2 $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

$$|A| = 2 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 6 - 2 = 4$$

(a) $\text{adj}(A) = \begin{pmatrix} 3 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 3 \end{pmatrix}$

(b) $A^{-1} = \begin{pmatrix} \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{pmatrix}$ so $x = \begin{pmatrix} \frac{3}{4} & -\frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{pmatrix} \begin{pmatrix} t \\ 1 \\ -t \end{pmatrix} = \begin{pmatrix} \frac{1}{2}t - \frac{1}{2} \\ 1 \\ -\frac{1}{2}t - \frac{1}{2} \end{pmatrix}$

#3 LU Decomp:

$$\begin{pmatrix} 1 & 0 & 0 \\ 4 & -3 & 0 \\ 6 & -4 & 1 \end{pmatrix} \begin{pmatrix} -2 & -3 & 0 \\ 0 & 3 & 6 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -3 & 0 \\ -8 & -21 & -18 \\ -12 & -30 & -23 \end{pmatrix}$$

$$\#4 \quad \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2^2 & 3^2 & 4^2 \end{vmatrix} = (3-2)(4-2)(4-3) = \boxed{2}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ x & 3 & 4 \\ x^2 & 3^2 & 4^2 \end{vmatrix} = (3-x)(4-x)(4-3) = (3-x)(4-x)$$

$$\begin{vmatrix} 1 & 1 & 1 \\ x & 3 & y \\ x^2 & 9 & y^2 \end{vmatrix} = (3-x)(y-x)(y-3)$$

$$\#5 \quad \begin{pmatrix} 1 & -2 & 1 & 1 \\ 3 & a & -1 & 2 \\ 2 & -4 & 5 & 3 \\ 2 & -1 & 1 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & & & \frac{20}{9} \\ & 1 & & \frac{7}{9} \\ & & 1 & \frac{1}{3} \\ 3 & a & -1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & & & \frac{20}{9} \\ & 1 & & \frac{7}{9} \\ & & 1 & \frac{1}{3} \\ 0 & \ominus & 0 & \uparrow \end{pmatrix}$$

$$2 + (-3)\frac{20}{9} - \frac{7}{9}a + \frac{1}{3} = 2 - \frac{60}{9} - \frac{7}{9}a + \frac{1}{3}$$

$$= 2 - \frac{19}{3} - \frac{7}{9}a = \frac{-13}{3} - \frac{7}{9}a$$

Hence, when ~~there is~~, ~~there is~~ $-\frac{13}{3} - \frac{7}{9}a = 0 \Rightarrow a = \left(\frac{7}{9}\right)^{-1} \left(-\frac{13}{3}\right)$

(i) when $a = -\frac{39}{7}$, there is a unique solution. $= \frac{9}{7} \left(-\frac{13}{3}\right) = \left(-\frac{39}{7}\right)$

(ii) when $a \neq -\frac{39}{7}$, there is no solution.