1. (15 pts) Solve the following linear system.

\[ \begin{align*}
3x_1 - 2x_2 - x_3 &= 7 \\
2x_1 + x_2 - 3x_3 &= -14 \\
x_1 + 2x_2 + x_3 &= 1
\end{align*} \]

2. (25 pts) (a) Compute \( A^{-1} \) for the matrix (if it exists)

\[ A = \begin{bmatrix}
2 & 1 & 0 \\
1 & 2 & 1 \\
0 & 1 & 2
\end{bmatrix} \]

(a) Compute \( \text{adj}(A) \).
(b) Solve the system \( Ax = B \), where \( B = [t, 1, -t]^T \) for each \( t \).

3. (20 pts) Find the LU decomposition for the matrix (if it exists)

\[ A = \begin{bmatrix}
-2 & -3 & 0 \\
-8 & -21 & -18 \\
-12 & -30 & -23
\end{bmatrix} \]

4. (20 pts) Compute the determinant for the following matrix

\[ \begin{bmatrix}
1 & 1 & 1 \\
x & 3 & y \\
x^2 & 9 & y^2
\end{bmatrix} \]

5. (20 pts) For what values of \( a \) does the system have a unique solution, infinitely many solution or no solutions?

\[ \begin{align*}
x - 2y + z &= 1 \\
3x + ay - z &= 2 \\
2x - 4y + 5z &= 3 \\
2x - y + z &= 4
\end{align*} \]