Solution to Test 2A, MA 114, Oct 19, 2006

1. (a) Rewrite the problem as: Maximize $P = 4x + 5y$ subject to

$$\begin{aligned} -x - y &\le -5 \\ 5x + y &\le 21 \\ -x + y &\le 3 \\ x \ge 0,\ y &\ge 0 \end{aligned}$$

The initial simplex tableau is
$$\begin{pmatrix} -1 & -1 & 1 & 0 & 0 & 0 & -5 \\ 5 & 1 & 0 & 1 & 0 & 0 & 21 \\ -1 & 1 & 0 & 0 & 1 & 0 & 3 \\ -4 & -5 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

(b) The possible first pivot elements are -1 & -1 at position $(1,1)$ and $(1,2)$.

(c)
$$\begin{array}{c} \begin{array}{ccccccc} x & y & u & v & w & P & \text{const} \end{array} \\ \begin{pmatrix} -1 & \boxed{-1} & 1 & 0 & 0 & 0 & -5 \\ 5 & 1 & 0 & 1 & 0 & 0 & 21 \\ -1 & 1 & 0 & 0 & 1 & 0 & 3 \\ -4 & -5 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array} \xrightarrow[\substack{R_4 - 5R_1 \\ -R_1}]{\substack{R_2 + R_1 \\ R_3 + R_1}} \begin{pmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 5 \\ 4 & 0 & 1 & 1 & 0 & 0 & 16 \\ \boxed{-2} & 0 & 1 & 0 & 1 & 0 & -2 \\ 1 & 0 & -5 & 0 & 0 & 1 & 25 \end{pmatrix}$$

$$\xrightarrow[\substack{R_4 + \frac{1}{2}R_3 \\ -\frac{1}{2}R_3}]{\substack{R_1 + \frac{1}{2}R_3 \\ R_2 + 2R_3}} \begin{pmatrix} 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 4 \\ 0 & 0 & \boxed{3} & 1 & 2 & 0 & 12 \\ 1 & 0 & -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & -\frac{9}{2} & 0 & \frac{1}{2} & 1 & 24 \end{pmatrix} \xrightarrow[\substack{R_4 + \frac{9}{6}R_2 \\ \frac{1}{3}R_2}]{\substack{R_1 + \frac{1}{6}R_2 \\ R_3 + \frac{1}{6}R_2}} \begin{array}{c} \begin{array}{ccccccc} x & y & u & v & w & P & \text{const.} \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & \frac{1}{6} & \frac{5}{6} & 0 & 6 \\ 0 & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 0 & 4 \\ 1 & 0 & 0 & \frac{1}{6} & -\frac{1}{6} & 0 & 3 \\ 0 & 0 & 0 & \frac{3}{2} & \frac{7}{2} & 1 & 42 \end{pmatrix} \end{array}$$

(d) Basic solutions: $u = -5,\ v = 21,\ w = 3,\ P = 0,\ x = 0 = y$

$x = 0,\ y = 5,\ u = 0,\ v = 16,\ w = -2,\ P = 25$

$x = 1,\ y = 4,\ u = 0,\ v = 12,\ w = 0,\ P = 24$

$x = 3,\ y = 1,\ u = 4,\ v = 0,\ w = 0,\ P = 42$

3. (a) There are 2 constraints and 3 variables

(b) Since there negative numbers in the last column (except the bottom corner), the tableau is not final

(c)
$$\begin{pmatrix} \boxed{-\frac{1}{2}} & 1 & 0 & \frac{1}{2} & 0 & -\frac{3}{2} \\ -\frac{3}{2} & 0 & 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ 3 & 0 & 0 & 3 & 1 & -9 \end{pmatrix} \rightarrow \begin{array}{c} \begin{array}{cccccc} x & y & z & & & \end{array} \\ \begin{pmatrix} 1 & -2 & 0 & -1 & 0 & 3 \\ 0 & -3 & 1 & -1 & 0 & 5 \\ 0 & 6 & 0 & 6 & 1 & -18 \end{pmatrix} \end{array}$$

(d) The final answer is when $x = 3, y = 0, z = 5$

the minimum value $C = 18$. (or max value $-C = -18$)

<u>Remark</u> In 2(b) the final simplex tableau is

$$\begin{pmatrix} -\frac{2}{3} & 0 & 1 & \frac{1}{6} & -\frac{5}{6} & 0 & \frac{7}{2} \\ -\frac{1}{3} & 1 & 0 & -\frac{1}{6} & -\frac{1}{6} & 0 & \frac{3}{2} \\ 4 & 0 & 0 & 3 & 6 & 1 & -42 \end{pmatrix}$$

$u = 0, v = \frac{3}{2}, w = \frac{7}{2}$

$-C = -42$ or $C = 42$

(e) The min. problem is: Maximize $-C=-4x-5y$ subj to

$$-x-y\le -5$$
$$5x+y\le 21$$
$$-x+y\le 3$$
$$x\ge 0,\ y\ge 0$$

The initial simplex tableau is $\begin{pmatrix} -1 & -1 & 1 & 0 & 0 & 0 & -5 \\ 5 & 1 & 0 & 1 & 0 & 0 & 21 \\ -1 & 1 & 0 & 0 & 1 & 0 & 3 \\ 4 & 5 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

2.(a) The matrix for the primal problem

$$\begin{array}{cc} x & y \\ \end{array}$$
$$\left(\begin{array}{cc|c} -1 & -1 & -5 \\ 5 & 1 & 21 \\ -1 & 1 & 3 \\ 4 & 5 & 0 \end{array}\right)$$

the transpose is

$$\begin{array}{ccc} u & v & w \end{array}$$
$$\begin{pmatrix} -1 & 5 & -1 & 4 \\ -1 & 1 & 1 & 5 \\ -5 & 21 & 3 & 0 \end{pmatrix}$$

So the dual problem is: Minimize $C=-5u+21v+3w$

subject to the constraints

$$-u+5v-w\ge 4$$
$$-u+v+w\ge 5$$
$$u\ge 0,\ v\ge 0,\ w\ge 0$$

(b) $\left(\begin{array}{cccccc|c} 1 & -5 & 1 & 1 & 0 & 0 & -4 \\ 1 & -1 & -1 & 0 & 1 & 0 & -5 \\ -5 & 21 & 3 & 0 & 0 & 1 & 0 \end{array}\right)$

(c) By Problem 1(c) the solution is

$$u=0,\ v=\tfrac{3}{2},\ w=\tfrac{7}{2}$$

and $C=42$