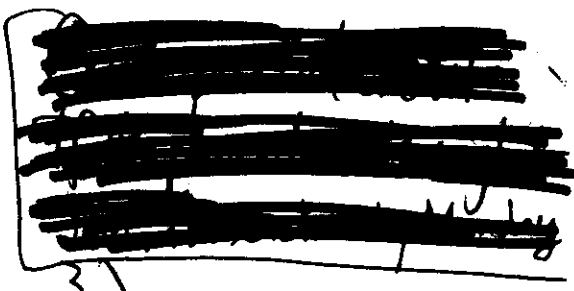
Test 1
Solution to MA114-003
Dr. Jing's class



1. $\begin{pmatrix} 3 & -1 & 2 & 14 \\ 1 & 2 & -3 & -7 \\ -1 & -1 & 1 & 2 \end{pmatrix} \to \cdots \to \begin{pmatrix} 1 & & & 3 \\ & 1 & & -5 \\ & & 1 & 0 \end{pmatrix}$

2. $\begin{pmatrix} 2 & 1 & 0 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 1 & 2 & 1 \end{pmatrix} \to \cdots \to \begin{pmatrix} 1 & & & \frac{2}{3} & -\frac{1}{3} & 0 \\ & 1 & & -\frac{1}{3} & \frac{2}{3} & 0 \\ & & 1 & \frac{1}{6} & -\frac{1}{3} & \frac{1}{2} \end{pmatrix}$

3. $\begin{pmatrix} 3 & +1 & -3 & -1 & 2 \\ 2 & 2 & -4 & 2 & 4 \end{pmatrix} \xrightarrow{(-1)R_2+R_1} \begin{pmatrix} 1 & -1 & +1 & -3 & -2 \\ 2 & 2 & -4 & 2 & 4 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 1 & -3 & -2 \\ 0 & 4 & -6 & 8 & 8 \end{pmatrix}$

$\to \begin{pmatrix} 1 & -1 & 1 & -3 & -2 \\ 0 & 1 & -\frac{3}{2} & 2 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -\frac{1}{2} & -1 & 0 \\ 0 & 1 & -\frac{3}{2} & 2 & 2 \end{pmatrix}$

$\therefore \begin{cases} x = \frac{1}{2}z + w \\ y = 2 + \frac{3}{2}z - 2w \end{cases}$

(a) Two solutions: $x=0, y=2, z=w=0$

$x=\frac{1}{2}, y=\frac{7}{2}, z=1, w=0$ etc

(b) $\begin{cases} x = \frac{1}{2}z + w \\ y = 2 + \frac{3}{2}z - 2w \\ z = z \\ w = w \end{cases}$

4. x = # boxes of soybeans

y = # boxes of rice

W = weight function

Minimize $W = 6x + 6y$

protein: $5x + 3y \geq 54$

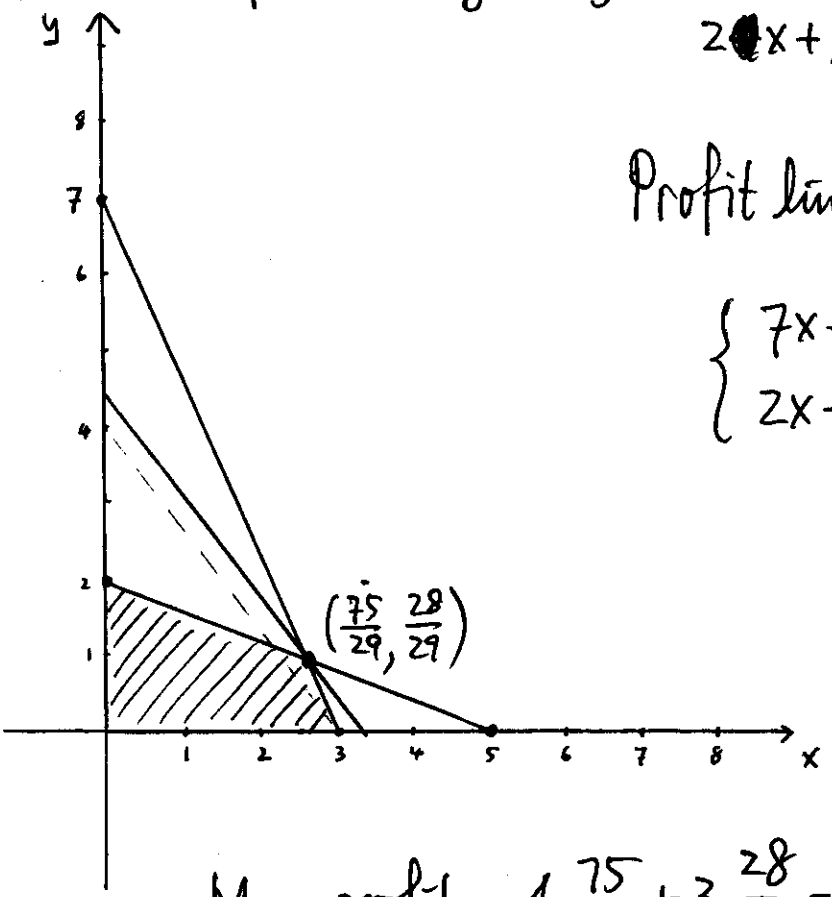
fat: $2x + y \geq 28$

carbohydrates: $2x + 3y \geq 35$

$x \geq 0,\ y \geq 0$

5. Maximize $P = 4x + 3y$ subj to $7x + 3y \leq 21$ $\quad x \geq 0, y \geq 0$

$2x + 5y \leq 10$



Profit line has slope $-\frac{4}{3} = -1.333$

$$\begin{cases} 7x + 3y = 21 \\ 2x + 5y = 10 \end{cases}$$

$$x = \frac{\begin{vmatrix} 21 & 3 \\ 10 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 3 \\ 2 & 5 \end{vmatrix}} = \frac{105 - 30}{35 - 6} = \frac{75}{29}$$

$$y = \frac{\begin{vmatrix} 7 & 21 \\ 2 & 10 \end{vmatrix}}{29} = \frac{70 - 42}{29} = \frac{28}{29}$$

Max. profit $= 4\frac{75}{29} + 3\frac{28}{29} = \frac{300 + 84}{29} = \boxed{\frac{384}{29}}$