Test A

Final Exam, MA 114 Finite Mathematics, Dr. Jing Name ____________

12/8/2005. 1:00-4:00 Id Number ____________________

You must show all your work.

1 (15 pts) Solve the following linear system

$$4x_1 - x_2 + 3x_3 = 21$$
$$2x_1 + 3x_2 - x_3 = 1$$
$$3x_1 + x_2 - 2x_3 = 8$$

2 (20 pts) In the following we consider the optimal problem: Maximize and minimize the function $x + y$ subject to the following constraints

$$2x + 3y \leq 14$$
$$x - y \leq 2$$
$$3x + 2y \geq 11$$
$$x \geq 0, y \geq 0$$

(a) Use the simplex method to find the maximum value of $P = x+y$ under the constraints.
(b) Use the simplex method to find the minimum value of $C = x+y$ under the constraints.
(c) State the dual problem of part (a) and solve it.
(d) State the dual problem of part (b) and solve it.

3 (20 pts) A class of 90 students are taking three courses: math, business and English. There are 55 students taking math, 45 taking business, 50 taking English, 26 taking math and business, 34 taking math and English, 20 taking business and English. and 10 students taking all three subjects.
(a) How many students don't take any of the courses?
(b) How many students take exactly one course?
(c) How many students take exactly two course?
(d) How many students take at least one course?

4 (20 pts) Two men (Jim and Bob) and three women (Ann, Beth, Carol) are on a committee. Two of the five are to be chosen to serve as officers. If the officers are selected randomly,
(a) what is the probability that both officers will be women?
(b) what is the probability that at least one officer will be a woman?
(c) what is the probability that both officers will be women if you know that Beth is an officer?

5 (15 pts) Michelle and Kim are playing tennis together, and Michelle is a better player that she always has a probability of 0.7 to win Kim.
(a) They decide that whoever wins two sets first will win the game. What is the probability that Kim wins the game?
(b) What is the probability that Michelle wins at exactly 4 sets when they play 7 sets?

(c) What is the probability that Michelle wins at at least 4 sets when they play 7 sets?

6 (10 pts) Eight people are eating in a restaurant.
(a) How many ways for they to sit along a straight line?
(b) How many ways for them to sit around a round table?

1. $\begin{pmatrix} 4 & -1 & 3 & 21 \\ 2 & 3 & -1 & 1 \\ 3 & 1 & -2 & 8 \end{pmatrix} \to \cdots \to \begin{pmatrix} 1 & & & 4 \\ & 1 & & -2 \\ & & 1 & 1 \end{pmatrix}$

2. a) ~~Minimize~~ ~~Maximize~~ Maximize $P = x + y$ subject to

$$2x + 3y \le 14$$
$$x - y \le 2$$
$$3x + 2y \ge 11$$

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 5 \\ 0 & 1 & \frac{1}{5} & -\frac{2}{5} & 0 & 0 & 2 \\ 1 & 0 & \frac{1}{5} & \frac{3}{5} & 0 & 0 & 4 \\ 0 & 0 & \frac{2}{5} & \frac{1}{5} & 0 & 1 & 6 \end{pmatrix}$$

The initial simplex is

$$\begin{pmatrix} 2 & 3 & 1 & 0 & 0 & 0 & 14 \\ 1 & -1 & 0 & 1 & 0 & 0 & 2 \\ -3 & -2 & 0 & 0 & 1 & 0 & -11 \\ -1 & -1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \to \cdots \to \cancel{\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 7 \\ 0 & 1 & 0 & \frac{1}{5} & -\frac{1}{5} & 0 & 1 \\ 1 & 0 & 0 & \frac{2}{5} & -\frac{1}{5} & 0 & 3 \\ 0 & 0 & 0 & -\frac{1}{5} & -\frac{2}{5} & 1 & 4 \end{pmatrix}}$$

the maximum is $P = 6$ at $x = 4, y = 2$.

b) Minimize $C = x + y$ subject to the same conditions

Initial simplex is

$$\begin{pmatrix} 2 & 3 & 1 & 0 & 0 & 0 & 14 \\ 1 & -1 & 0 & 1 & 0 & 0 & 2 \\ -3 & -2 & 0 & 0 & 1 & 0 & -11 \\ +1 & +1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \to \cdots \to \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 5 \\ 0 & 1 & 0 & -\frac{3}{5} & -\frac{1}{5} & 0 & 1 \\ 1 & 0 & 0 & \frac{2}{5} & -\frac{1}{5} & 0 & 3 \\ 0 & 0 & 0 & \frac{1}{5} & \frac{2}{5} & 1 & -4 \end{pmatrix}$$

Minimum is $C = 4$ at $x = 3, y = 1$.

c) Dual Problem:

Minimize $C = 14u + 2v - 11w$ subj to

$$2u + v - 3w \ge 1$$
$$3u - v - 2w \ge 1$$

$$\begin{array}{cc} x & y & \\ \end{array}$$
$$\begin{pmatrix} 2 & 3 & 14 \\ 1 & -1 & 2 \\ -3 & -2 & -11 \\ 1 & 1 & 0 \end{pmatrix} \to \begin{array}{c} \begin{array}{ccc} u & v & w \end{array} \\ \begin{pmatrix} 2 & 1 & -3 & 1 \\ 3 & -1 & -2 & 1 \\ 14 & 2 & -11 & 0 \end{pmatrix} \end{array}$$

Simplex $\begin{pmatrix} -2 & -1 & 3 & 1 & 0 & 0 & -1 \\ -3 & 1 & 2 & 0 & 1 & 0 & -1 \\ 14 & 2 & -11 & 0 & 0 & 1 & 0 \end{pmatrix} \to \cdots \to \begin{pmatrix} 0 & 1 & -1 & -\frac{3}{5} & \frac{2}{5} & -\frac{3}{5} & \frac{1}{5} \\ 1 & 0 & -1 & -\frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & \frac{2}{5} \\ 0 & 0 & 5 & 4 & 2 & 5 & -6 \end{pmatrix}$

Answer: $u = \frac{2}{5}, v = \frac{1}{5}, w = 0$ min $C = 6$ (see (a))

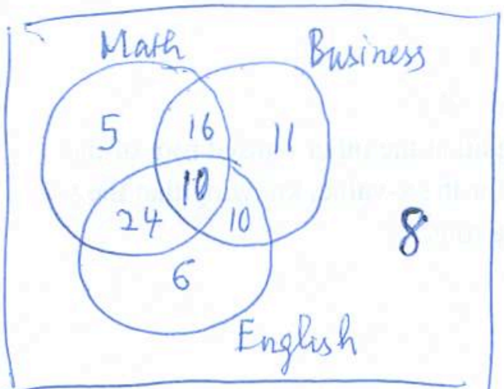
d) Minimize $C = x + y$ subj to
$-2x + 3y \geq -14$
$-x + y \geq -2$
$3x + 2y \geq 11$

$$\begin{pmatrix} -2 & -3 & -14 \\ -1 & 1 & -2 \\ 3 & 2 & 11 \\ 1 & 1 & 0 \end{pmatrix} \mapsto \begin{pmatrix} -2 & -1 & 3 & 1 \\ -3 & 1 & 2 & 1 \\ -14 & -2 & 11 & 0 \end{pmatrix}$$

i.e. Maximize $P = -14u - 2v + 11w$ subj. to
$$-2u - v + 3w \leq 1$$
$$-3u + v + 2w \leq 1$$

Answer $P = 4$ when $u = 0$, $v = \frac{1}{5}$, $v = \frac{2}{5}$.

3.



(a) 8

(b) $5 + 11 + 6 = 22$

(c) $16 + 10 + 24 = 50$

(d) $90 - 8 = 82$

4. (a) $p = \frac{C(3,2)}{C(5,2)} = \frac{3 \times 2}{5 \times 4} = \frac{3}{10}$

(b) $p = \frac{C(3,1)C(2,1) + C(3,2)}{C(5,2)} = \frac{3 \times 2 + \frac{3 \times 2}{2 \times 1}}{\frac{5 \times 4}{2 \times 1}} = \frac{12 + 6}{20} = \frac{\cancel{18}}{\cancel{20}} = \frac{9}{10}$

~~(c) A = event that both are women~~
~~B = event that one officer is a woman~~ ~~$P(A|B) = ?$~~

~~$P(A \cap B) = P(A) = \frac{3}{10}$~~

~~$P(B) = \frac{C(3,1)C(2,1)}{10} = \frac{6}{10} = \frac{3}{5}$~~ ~~$P(A|B) = \frac{\frac{3}{10}}{\frac{3}{5}} = \frac{1}{2}$~~

One officer is woman:
$p(A \cap B) = p(A) = \frac{C(3,2)}{C(5,2)} = \frac{3}{10}$

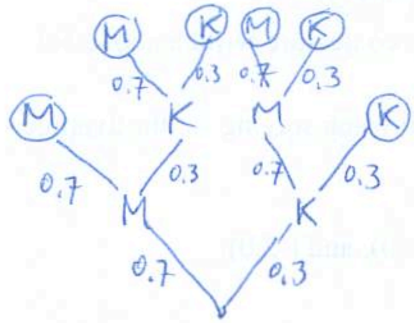
$P(B) = \frac{3 \cdot 2 + 3}{C(5,2)} = \frac{9}{10}$, $P(A|B) = \frac{1}{3}$

$\uparrow$ 1 is woman

(c) A = event that both are woman, B = event that Beth is an officer

$P(A \cap B) = \frac{2}{10} = \frac{1}{5}$, $P(B) = \frac{4}{10} = \frac{2}{5}$, $P(A \cap B) = \frac{1}{2}$

5.



(a) $p = 0.7\times 0.3^2 + 0.3^2\times 0.7 + 0.3^2$

$= 0.042 + 0.09 = 0.132$ Kim wins

(b) P(M wins exactly 4 sets in 7 sets)

$= C(7,4)\, 0.7^4\, 0.3^3 = \frac{7\times 6\times 5}{3\times 2\times 1} 0.7^4\, 0.3^3$

$= 35\; 0.7^4\, 0.3^3 \doteq 0.227$

(c) P(M wins at least 4 sets in 7 sets)

$= C(7,4)\, 0.7^4\, 0.3^3 + C(7,5)\, 0.7^5\, 0.3^2 + C(7,6)\, 0.7^6\, 0.3 + C(7,7)\, 0.7^7$.

6. (a) $8!$

(b) $7!$