

Review Outline, MA 241, Dr. Jing  
December 5, 2002

**Note: Final Exam, Thursday, Dec 12, 2002, 1-4 pm in HA 307**

The final exam will be a multiple choice exam with 25 questions. You should review all homework problems, test problems, quiz problems as well as examples covered in classes. *The following is only a partial list of topics to help you review and they do not represent the full range of questions in the final exam.*

1. Definite integrals.

Computations: basic formula; substitution method; Fundamental theorem; integration by parts; computation by partial fraction.

Applications: area; volume of revolution; work; hydrostatic pressure and force; arclength;

2. Improper integrals.

Convergence definition: type one and type two; convergence tests.

3. Differential Equations.

Separable equations; quadratic equations (characteristic roots); exponential growth and decay; the logistic equation.

4. Series.

Convergence tests: divergence test, integral test, comparison test, ratio test. Sum of series: telescopic series, geometric series. Absolute convergence.

Power series: radius of convergence, computation of radius of convergence. Integral of convergence. Representation of functions as power series (=Maclaurin series)

Taylor series: definition; basic Taylor series expansion; Methods of getting Taylor expansion: substitution, multiplication, differentiation, integration. Application of Taylor series: computation of  $f^{(n)}(0)$  without of direct computation.

5. Solving differential equations by power series.

Questions 1 to 4 are some of sample questions to show the format of the final exam. Questions 5 to 28 are other review questions. Note that they are not representative of the actual material covered in the final exam. The final exam will contain more questions from the topics mentioned above.

**Example** (0) Find the integral  $\int (x + 1)dx$ .

	A	B	C	D
(0)	$x + C$	$x^2 + C$	$\frac{1}{2}x^2 + x + C$	None of the above

The correct answer is C.

I Solve the following equations:

$$(1) \quad y'' - y = 0 \quad (2) \quad \frac{dy}{dx} = y$$

$$(3) \quad y'' - y = e^x \quad (4) \quad y' - xy = x$$

	A	B	C	D
(1)	$C_1e^t + C_2te^t$	$C_1e^t + C_2e^{-t}$	$Cte^t$	$Ce^t$
(2)	$Ce^y$	$Ce^x$	$\frac{y^2}{2} + C$	$\frac{x^2}{2} + c$
(3)	$C_1e^x + C_2e^{-x} + \frac{1}{2}xe^x$	$C_1e^x + C_2e^{-x} + e^x$	$C_1e^x + C_2e^{2x} + xe^x$	$C_1e^x + C_2xe^x + \frac{1}{2}xe^x$
(4)	$-1 + c \sum_{n=0}^{\infty} \frac{x^n}{n!}$	$-1 + c \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n)!}$	$-1 + c \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$	$-1 + c \sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$

Answer: 1. B, 2. B, 3. A, 4. D.

Other review questions are

5. Find  $\frac{d}{dx} \int_1^{x^4} \sec t dt$ .

6. Find the integral  $\int_0^{\pi/6} \cos^3 x dx$ .
7. Compute  $\int_0^1 \tan^{-1} x dx$ .
8. Is the integral  $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx$  convergent? If yes, what is the value?
9. Sketch the region enclosed by the curves  $y = x, y = x^2$ . Then find the area of the region.
10. Find the volume of the solid obtained by rotating the region bounded by  $y = x^2, y^2 = x$  about the  $y$ -axis.
11. Find the length of the arc of the parabola  $y^2 = x$  from  $(0, 0)$  to  $(1, 1)$ .
12. Find the average value of the function  $f(x) = \sin x$  on the interval  $[0, 2\pi]$ .
13. A heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft high. How much work is done in pulling the rope to the top of the building?
14. Find the center of mass for the lamina bounded by the curves  $y = e^x, y = 0, x = 0, x = 1$ .
15. Solve the differential equation  $\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}$
16. Find the orthogonal trajectories of the family of curves  $2x^2 + y^2 = C$ , where  $C$  is an arbitrary constant.
17. Polonium-214 has a half-life of 140 days. (a) If a sample has a mass of 200 mg, find a formula for the mass that remains after  $t$  days. (b) Find the mass after 100 days. (c) When will the mass be reduced to 10 mg?
18. Write the solution of the initial-value problem  $\frac{dP}{dt} = 0.08P(1 - \frac{P}{1000}), P(0) = 100$  and use it to find the population sizes  $P(40)$ . At what time does the population reach 900?
19. Determine whether the sequence  $a_n = \frac{n^2-1}{n^2+1}$  converges or diverges.
20. Write the number 2.3171717171717... as a ration of integers.
21. Find the sum of the series  $\sum_{n=0}^{\infty} \frac{3^n+2^n}{6^n}$ .
22. Determine if the series  $\sum_{n=1}^{\infty} \frac{3}{n(n+2)}$  is convergent.
23. Is the series  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$  convergent or divergent?
24. Test the convergence for  $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^2+1}$ .
25. Find the interval of convergence for the power series  $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$ .
26. Find the power series representation for the function  $f(x) = \frac{1}{(1-x)^2}$ .
27. Find the Maclaurin series for the function  $f(x) = \frac{1}{\sqrt{4-x}}$ .
28. Solve the equation  $y'' + y = 0$  using power series.

Answer:

5.  $\sec(x^4)4x^3$
6.  $\frac{11}{24}$
7.  $\frac{\pi}{4} - \frac{\ln 2}{2}$
8. Yes,  $\pi$
9.  $\frac{1}{6}$
10.  $\frac{3\pi}{10}$
11.  $\frac{\sqrt{5}}{2} + \frac{\ln(\sqrt{5}+2)}{4}$
12. 0
13.  $625 ft - lb$
14.  $(1/(e-1), (e+1)/4)$
15.  $y^2 + \sin y = 2x^3 + C$
16.  $x = Cy^2$
17. (a)  $50 \times 2^{-t/0.00014}$  (b)  $1.57 \times 10^{-20} \text{mg}$  (c)  $4.5 \times 10^{-5} s$
18.  $P(t) = \frac{1000}{1 + 9e^{-0.08t}}$ ;  $P(40) = 731.6$ ; 54.9
19. Converges to 1
20.  $\frac{1147}{495}$
21.  $\frac{3}{2}$
22. Convergent since  $\int_1^{\infty} \frac{1}{x(x+2)} dx$  is.
23. Convergent by alternating series test
24. Divergent since  $|a_n| \rightarrow 1$
25.  $[2, 4)$
26.  $\sum_{n=0}^{\infty} (n+1)x^n$
27.  $\frac{1}{2} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{(2n-1)!!}{n!8^n} x^n$
28.  $c_0 \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} + c_1 \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = c_0 \cos x + c_1 \sin x$