1. \[ f(x) = \frac{1}{4} x^{-1} \]
   \[ f'(x) = -\frac{1}{4} x^{-2} \]
   \[ f'(2) = -\frac{1}{4} \left(\frac{1}{4}\right) = \frac{-1}{16} \]

2. Slope: pay will increase by $0.50 for every additional unit sold.
   y-intercept: pay if the salesperson sells no goods.

3. \[ f(x) = 2x - 5 \]
   \[ f'(x) = \frac{5}{2} \]
   \[ y = \frac{1}{x} = x^{-1} \]
   \[ y' = -x^{-2} = \frac{-1}{x^2} \]

4. \[ y = \sqrt{x} \]
   Point (25, 5)
   \[ y' = \frac{1}{2} x^{-\frac{1}{2}} \]
   \[ y'(25) = \frac{1}{2} \cdot \frac{1}{\sqrt{25}} = \frac{1}{10} \]

5. \[ f(x) = 3x^2 + 1 \]
   \[ f'(x) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{3h^2 + 1 - 1}{h} = \lim_{h \to 0} 3h = 0 \]

6. \( \text{not differentiable: } x = -2, 1, 2 \)
   \( \text{not continuous: } x = -2, 1 \)
   \( \text{increasing } (-\infty, -2) \cup (1, 2) \)
   \( \text{decreasing } (-2, 1) \cup (2, \infty) \)

7. \[ S(t) = 50t - \frac{7}{t+1} = 50t - 7(t+1)^{-1} \]
   \[ W(t) = S'(t) = 50 + 7(t+1)^{-2} \]
   \[ v(1) = 50 + \frac{7}{4} \]
   \[ a(t) = v'(t) = -14(t+1)^{-3} \]
   \[ a(1) = -14 \cdot \frac{1}{8} = -\frac{7}{4} \]

8. \[ V = x^2 y = 250 \text{ ft}^3 \]
   \[ y = \frac{250}{x^2} \]
   \[ C'(x) = 4x - 500x^{-2} = 0 \]
   \[ 4x = \frac{500}{x^2} \]
   \[ x^3 = 125 \]
   \[ x = 5 \]

9. \[ f(x) = (5x+1)^4 \]
   \[ f'(x) = 4(5x+1)^3 \cdot 5 = 20(5x+1)^3 \]

10. \[ (2x^{-2})^2 = 2 \]
    \[ 2x^{-4} = 2 \]
    \[ x = \frac{5}{2} \]
12. $v(t) = 65 (1 - e^{-16t}) = 65 - 65e^{-16t}$

13. $3000 = 1000 e^{1.1t}$

14. $f(x) = 2e^{-3x}$
\[ f'(x) = 2e^{-3x} \cdot (-3) = -6e^{-3x} \]

15. $y = \ln(x^3 + 2x + 1)$
\[ y' = \frac{1}{x^3 + 2x + 1} \cdot (3x^2 + 2) = \frac{3x^2 + 2}{x^3 + 2x + 1} \]

16. (a) $P(t)$ represents the amount of a certain radioactive material present after $t$ years.
(b) $t$ is the time in years.

17. $V = \pi \int_0^4 (x^2)^2 \, dx = \pi \int_0^4 x^4 \, dx = \pi \left[ \frac{1}{5} x^5 \right]_0^4 = \pi \left( \frac{1}{5} 4^5 - 0 \right) = \frac{1024\pi}{5}$

18. $\int_0^2 \frac{2x}{(x^2 + 1)^2} \, dx$
\[ u = x^2 + 1 \quad x = 0 \quad u = 1 \]
\[ du = 2x \, dx \quad x = 1 \quad u = 2 \]
\[ \int_1^2 \frac{1}{u^2} \, du = \int_1^2 u^{-2} \, du = -\frac{1}{u} \bigg|_1^2 = -\frac{1}{2} \left( \frac{1}{4} - 1 \right) = \frac{3}{8} \]

19. $\int_0^3 2x \left(e^{3x^2}\right) \, dx$
\[ u = 3x^2 \quad x = 0 \quad u = 0 \]
\[ du = 6x \, dx \quad x = 1 \quad u = 3 \]
\[ \frac{1}{3} \int_0^3 e^u \, du = \frac{1}{3} \left( e^3 - 1 \right) \]

20. $C(t) = 1.1t + 2.4$
\[ t = 0 \quad \Rightarrow \quad C = 1.1 \cdot 0 + 2.4 = 2.4 \]

21. $V(t) = \int 2t \, dt = \int (2t + 1) \, dt = t^2 + t + C$
\[ V(0) = 0 + 0 + C = 0 \quad C = 0 \]
\[ V(t) = t^2 + t \]
\[ S(t) = \frac{1}{2} (t^2 + t) \, dt = \frac{1}{2} t^3 + \frac{1}{2} t^2 + C, \]
\[ S(0) = 0 + 0 + C = 0 \quad C = 0 \]

22. $S = \int_{-2}^{2.5} 2 \sqrt{x} \, dx = 2 \int_{-2}^{2.5} x^{1/2} \, dx = 2 \left[ \frac{2}{3} x^{3/2} \right]_{-2}^{2.5} = \frac{4}{3} \left( 2.5^{3/2} - 0 \right) = 5.27$
\[ y = x^2 + 1 \]

\[ y = 3 - x \]

25 \[ y = x^3 - 6x^2 + 9x + 3 \]
\[ y' = 3x^2 - 12x + 9 = 0 \]
\[ 3(x^2 - 4x + 3) = 0 \]
\[ 3(x - 3)(x - 1) = 0 \]
\[ x = 3, 1 \]
\[ y' \leftarrow + \quad - \quad + \rightarrow \]

\[ f(3) = 3 \quad \text{rel. min at } x = 3 \]
\[ f(1) = 7 \quad \text{rel. max at } x = 1 \]

26 The second derivative is the derivative of the derivative. It is also the rate of change of the rate of change. It tells us about concavity of a function.

27 \[ y = \frac{x + (x^5 + 1)^{10}}{3} = \frac{1}{3} x + \frac{10}{3} (x^5 + 1)^9 \]
\[ y' = \frac{1}{3} + \frac{10}{3} \left(5x^4 (x^5 + 1)^9 \right) \]

28 \[ y = \sqrt{1 - x^2} = (1 - x^2)^{1/2} \]
\[ y' = \frac{1}{2} (1 - x^2)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{1 - x^2}} \]

29 \[ y = x^4 - 5x^3 + 7 \]
\[ y' = 4x^3 - 15x^2 \]
\[ y'' = 12x^2 - 30x \]
\[ y'' (3) = 108 - 90 = 18 \]

30 \[ \frac{da}{dt} (a^2 t^2 + b^2 t + c^2) = 2a^2 t + b^2 \]

31 \[ y = \frac{3x^2 - 2x}{x + 3} \]
\[ y' = \frac{(x+3)(6x-2)-(3x^2-2x)(1)}{(x+3)^2} \]
\[ = \frac{3x^2 + 18x - 6}{(x+3)^2} \]
32 \[ y = (x^2 + 1)(x^2 - 1) \]
\[ y' = \left[(x^2 + 1)(2x) + (x^2 - 1)(2x)\right] = 2x(x^2 + 1 + x^2 - 1) = 4x^3 \]

33 \[ f(x) = 2e^{1-3x} \]
\[ f'(x) = 2e^{1-3x} \cdot (-3) = -6e^{1-3x} \]

34 \[ e^{2x} = 5 \]
\[ 2x = \ln 5 \]
\[ x = \frac{\ln 5}{2} \]

35 \[ \ln 3x = 2 \]
\[ 3x = e^2 \]
\[ x = e^2/3 \]

36 \[ y = \ln (x^2) = 2 \ln x \]
\[ y' = \frac{2}{x} \]

37 \[ P(t) = 300e^{0.01t} \]

38 The derivative represents the instantaneous slope or rate of change.

39 \[ g(t) = 5t - \sqrt{t} = 5t - t^{1/2} \]
\[ g'(t) = 5 - \frac{1}{2} t^{-3/2} \]
\[ g'(3) = \left[5 - \frac{1}{2(3)^{3/2}}\right] \]

40 \[ y = z^3 - 4z^2 + 2 - 3 \]
\[ y' = 3z^2 - 2 \quad y'(3) = 27 - 24 + 1 = 4 \]

41 \[ y = 4(5x+1)^2 \]
\[ y' = 4 \cdot 2(5x+1) \cdot 5 = 40(5x+1) \]
\[ y'(-1) = 40(-4) = -160 \]

42 \[ \int (x^3 + 6x^2 - x) \, dx \]
\[ = \frac{1}{4}x^4 + 2x^3 - \frac{1}{2}x^2 + C \]

43 \[ \int_0^1 e^{x/3} \, dx = 3e^{x/3} \bigg|_0^1 = 3\left(e^{1/3} - 1\right) \]
44 \[ \int \frac{1}{2x+1} \, dx = \ln \left| \frac{2x+1}{2} \right| + C \]

45 \[ \int_0^1 8x(x^2+1)^5 \, dx \]
\[ u = x^2 + 1 \quad x = 0 \quad u = 1 \]
\[ dx = 2x \, dx \quad x = 1 \quad u = 2 \]
\[ \frac{1}{2} \, dx = x \, dx \]
\[ \frac{1}{2} \int_1^2 3u^5 \, du = \frac{3}{2} \left( \frac{1}{6} \right) \left( \frac{1}{6} \right)^{1/2} = \frac{1}{4} (2^6 - 1) = \frac{63}{4} \]

46 \[ \text{Area} = \frac{2}{3} (f(1) + f(1^{1/3}) + f(1^{2/3})) \]
\[ = \frac{2}{3} \left( 1 + \frac{125}{27} + \frac{343}{27} \right) = \frac{110}{9} \]

47 \[ \int_0^\infty e^{-x} \, dx = \lim_{b \to \infty} \int_0^b e^{-x} \, dx = \lim_{b \to \infty} -e^{-x} \bigg|_0^b \]
\[ = \lim_{b \to \infty} -e^{-b} + 1 = 1 \]

48 \[ i = \frac{.09}{12} \]

a) new balance = previous balance + interest - payment
\[ y_{n+1} = y_n + \frac{.09}{12} y_n - 350 \]
\[ y_{n+1} = 1.0075 y_n - 350, \quad y_0 = 38,000 \]

b) \[ a = 1.0075 \quad b = -350 \]
\[ y_n = \frac{b}{1-a} + \left( y_0 - \frac{b}{1-a} \right) a^n \quad \text{(for } a \neq 1) \]
\[ y_n = \frac{-350}{1-1.0075} + \left( 38,000 - \frac{-350}{1-1.0075} \right) (1.0075)^n = 416,166.67 + (38,000 - 416,166.67)(1.0075^n) \]

49 \[ \text{initial } y_0 \quad i = .06 \quad \text{withdrawal} = 120 \]

a) \[ y_{n+1} = y_n + .06 y_n - 120 \]
\[ y_{n+1} = 1.06 y_n - 120, \quad \text{initial } y_0 \]

b) \[ \frac{b}{1-a} = \frac{-120}{1-1.06} = \frac{120}{.06} = 2000 \]

50 \[ \text{Avg. value} = \frac{1}{3-1} \int_1^3 (x^3+1) \, dx = \frac{1}{2} \left( \frac{1}{4} x^4 + x \bigg|_1^3 \right) = \frac{1}{2} \left( \frac{81}{4} + 3 - \left( \frac{1}{4} + 1 \right) \right) \]
\[ = \frac{1}{2} (20 + 2) = \boxed{11} \]