Final Exam - Review Worksheet

The problems for the final exam will be similar to the problems listed here. These problems should in no way be the extent of your studying, but more of starting point for practice problems.

1. Find the derivative of $f(x) = \frac{1}{x}$. Evaluate it at $x = 2$.

2. A salesperson’s weekly pay depends on the volume of sales. If she sells $x$ units of goods, then her pay is $y = 5x + 60$ dollars. Give an interpretation of the slope and the $y$-intercept of this line.

3. Find the derivatives of each of the following:
   (a) $f(x) = 2x - 5$
   (b) $f(x) = x^2$
   (c) $y = \frac{1}{x}$

4. Write the equation of the line tangent to the curve $y = \sqrt{x}$ at the point $(25, 5)$.

5. If $f(x) = 3x^2 + 1$, find $f'(0)$ by using the definition of the derivative. (Hint: think limits)

6. Given the graph of $f$ to the right, determine each of the following:
   (a) At which points is $f(x)$ not differentiable?
   (b) At which points is $f(x)$ not continuous?
   (c) Where is $f(x)$ increasing? decreasing? (use interval notation)

7. Find the velocity and acceleration of a car at $t = 1$ where the position of a car measured in kilometers, at time $t$ (in hours) is given by $s(t) = 50t - \frac{7}{t+1}$. Use proper notation.

8. Sketch the graph of a function with the following properties:
   (a) $f(-1) = 0$
   (b) $f'(x) < 0$ for $x < -1$
   (c) $f'(-1) = 0$
   (d) $f'(x) > 0$ for $x > -1$

9. A canvas wind shelter for the beach has a back, two square sides, and a top. Find the dimensions for which the volume will be 250 cubic feet and that requires the least possible amount of canvas.

10. Let $f(x) = (5x + 1)^4$. Find $f'(x)$.

11. Solve for $x$: $(2^{x+1} \cdot 2^{-3})^2 = 2^1$.

12. Suppose that the velocity of a parachutist is $v(t) = 65(1 - e^{-0.16t})$ meters/second.
   (a) Calculate the parachutist’s velocity when $t = 9$ seconds.
   (b) Calculate the parachutist’s acceleration when $t = 9$ seconds.

13. One thousand dollars is deposited in a savings account at 10% interest compounded continuously. How many years are required for the balance in the account to reach $3000$?
14. Differentiate \( f(x) = 2e^{1-3x} \).

15. Differentiate \( y = \ln(x^3 + 2x + 1) \).

16. The amount (in grams) of a certain radioactive material present after \( t \) years is given by the function \( P(t) \).
   
   (a) What does \( P(t) \) represent?
   
   (b) What does \( t \) represent?

17. Find the volume of the solid generated by revolving \( y = \sqrt{x} \) around the x-axis from 0 to 4.

18. Calculate \( \int_0^1 \frac{2x}{(x^3+1)^3} \, dx \).

19. Calculate \( \int_0^1 2x(e^{3x^2}) \, dx \).

20. Let \( C(t) = 0.1t + 2.4 \) be the rate of change of the costs (in millions per year) for a given company. Let \( t = 0 \) correspond to the year 1958. Find the net change in the costs from 1980 to 1998.

21. A particle’s acceleration is given by \( a(t) = 2t + 1 \). If the position and velocity are 0 at time 0, find the position at time 6.

22. Calculate \( \int_0^{2.5} 2\sqrt{x} \, dx \).

23. Calculate the area between \( y = 13 - 3x^2 \) and \( y = 1 \).

24. Calculate the area between \( y = x^2 + x \) and \( y = 3 - x \).

25. Sketch a graph of \( y = x^3 - 6x^2 + 9x + 3 \) by using properties of the first and second derivative.

26. Describe the second derivative in words.

27. Find the derivative of \( y = \frac{x + (x^3 + 1)^{10}}{3} \).

28. Calculate the derivative of \( y = \sqrt{1 - x^2} \).

29. Let \( y = x^4 - 5x^3 + 7 \). Evaluate the second derivative at \( x = 3 \).

30. Evaluate \( \frac{d}{dt}(a^2t^2 + b^2t + c^2) \).

31. Find the first derivative of \( y = \frac{3x^2 - 2x}{x+3} \).

32. Calculate the first derivative of \( y = (x^2 + 1)(x^2 - 1) \).

33. Differentiate \( f(x) = 2e^{1-3x} \).

34. Solve for \( x \): \( e^{2x} = 5 \).

35. Solve for \( x \): \( \ln 3x = 2 \).

36. Differentiate \( y = \ln x^2 \).
37. A certain insect population can be modeled by \( P(t) = 300e^{0.01t} \).

(a) What is the initial insect population?
(b) What is the growth constant?
(c) What is the differential equation satisfied by this population equation?
(d) At what rate will the population be growing when the population is 100 insects?
(e) At what rate will the population be growing when the population is 1000 insects?

38. Describe the derivative in words.

39. Let \( g(t) = 5t - \sqrt{t} \). Find \( g'(3) \).

40. Evaluate the first derivative of \( y = z^3 - 4z^2 + z - 3 \) at \( z = 3 \).

41. Evaluate the first derivative of \( y = 4(5x + 1)^2 \) at \( x = -1 \).

42. Integrate \( f(x) = x^3 + 6x^2 - x \).

43. Determine the following: \( \int_1^2 e^x \, dx \).

44. Find \( \int \frac{1}{2x+1} \, dx \).

45. Integrate \( \int_0^1 3x(x^2 + 1)^5 \, dx \).

46. Estimate the area under the curve \( y = x^3 \) from \( x = 1 \) to \( x = 3 \) using Riemann sums with left endpoints. Use three equal subdivisions.

47. Evaluate \( \int_0^\infty e^{-x} \, dx \).

48. A bank loan of $38,000 at 9% interest compounded monthly is made in order to buy a house and is paid off at the rate of $350 per month for 20 years. The balance at any time is the amount still owed on the loan.

(a) Find the difference equation for \( y_n \), the balance after \( n \) months.
(b) Solve the difference equation.

49. A savings account earns 6% interest compounded yearly. Each year a withdrawal of $120 is made.

(a) Find the difference equation for \( y_n \), the balance after \( n \) months.
(b) How much money should you deposit initially if you want the account to contain a constant amount of money.

50. Find the average value of \( y = x^3 + 1 \) from 1 to 3.