Section 2.3 : Curve Sketching

1. Find $f'(x)$ and $f''(x)$

2. Set $f'(x) = 0$. For each $x = a$ such that $f'(a) = 0$, plot $(a, f(a))$. These are the possible relative extreme points.

3. Check values in $f''(x)$:
   (a) If $f''(a) > 0$, the graph is concave up, and we have a relative minimum at $(a, f(a))$.
   (b) If $f''(a) < 0$, the graph is concave down, and we have a relative maximum at $(a, f(a))$.
   (c) If $f''(a) = 0$, we cannot tell. Draw your $f'$ number line. Use test points to fill in +’s and -’s. See if you have a sign change at $x = a$.

4. Set $f''(x) = 0$. For each $x = b$ such that $f''(b) = 0$, $(b, f(b))$ is a possible point of inflection.

5. Draw your $f''$ number line. Use test points to fill in +’s and -’s. See if you have a sign change at $x = b$. If you do, you have an inflection point at $(b, f(b))$.

6. Find the x-intercept(s) by finding the x values for which $f(x) = 0$. Plot the x-intercept(s).

7. Find the y-intercept by finding $y = f(0)$. Plot the y-intercept.

8. Plot any holes or asymptotes.

Sketch each of the following using the above procedure:

1. $f(x) = x^3 + 6x^2 + 9x$
2. $f(x) = -x^3 + 12x - 4$
3. $f(x) = -\frac{1}{3}x^3 + 9x - 2$
4. $f(x) = \frac{1}{3}x^3 + 2x^2 - 5x + \frac{8}{3}$
5. $f(x) = x^4 - \frac{4}{3}x^3$