Conversations With Mara

A set of discussions about the Philosophy of
Non-Relativistic Quantum Mechanics

Written to fulfill the requirements of Phil 2627
At the University of Pittsburgh
Fall 2003

Jonathan Gaffney

Forward:

Presenting philosophical viewpoints to me is something that can be best done in the context of a conversation. It helps me frame my understanding of the subject and probe more deeply into its meaning by playing the role of both the presenter and the devil's advocate of any given viewpoint. I hope that writing in this way will be successful for the purposes of becoming acquainted with the philosophy of quantum mechanics in a fairly broad manner. The narrative framework in the following papers may not in all cases be completely necessary; however, it is my desire to present this semester's works in a coherent manner. To this end, there may be periods of narration that are off-topic but connect one week's conversation with the next. My ultimate goal is to create a series that facilitates my understanding of the subject matter, is fun to read and write, and can be shared with a fairly wide audience without losing academic intensity.

A bibliography will be included, and sources will be credited. I apologize for the loose formatting.

Items that don't directly "fit" but are important to be pointed out are in square brackets throughout the text; imagine them as editors' notes.

Thanks to Dr. Reutche for this class and for reviewing and notating on each chapter.

Conversations With Mara

Episode One:

"It's Always One Thing Or The Other: Bohr's Complementary Interpretation"

Jonathan Gaffney
September 8, 2003

\

Mara sat on the steps to Richardson Hall, waiting for me to show up after my eleven o'clock quantum physics class. Her knee was anxiously bouncing up and down as she took a drag from her cigarette. "What took you so long?" She rubbed the butt in the ground as I stepped out of the building's side door and walked to her.

I sighed as I pulled her to her feet. "I'm sorry; I was hung up in a class discussion about Bohr."

She brushed herself off. "It's always one thing or another with you isn't it?"

I laughed, catching the irony of the statement.

"What's so funny?" Mara scowled as we headed down the hill towards the cafeteria.

"What do you remember learning about Bohr?"

Mara thought for a moment. "Wasn't he the father of the hydrogen atom?"

"Yes, but one of his most powerful contributions was to the world of quantum physics. His interpretation of quantum physical phenomena was very influential in the way physics is now taught and understood."

Mara smiled as she pulled open the door for me. "Interpretation of quantum mechanical phenomena? What's so special about quantum physics? Why does it need a specific interpretation? Isn't classical mechanics simply quantum physics taken to the limit of h approaching zero and c approaching infinity?"

Mara was just toying with me at this point. She is actually a year ahead of me, but as a good mentor she makes me begin my discussions at the beginning so there's a solid foundation to build from.

"Well, of course it is. But our everyday experiences are with macroscopic objects moving much slower than light and all actions involved in any physical process are considered to be much larger than the quantum that Planck discovered at the beginning of the 20th century. This means that when initial progress into physical phenomena was made, folks like Newton were simply trying to explain the world as they saw it. What they saw was a causal world: objects have instantaneous velocities and positions which can be well-determined; therefore, we can derive a force law and predict what will happen in a certain situation with full certainty.

"However, when the universal quantum of action was discovered (and by this I of course mean Planck's constant), physical science changed dramatically. A certain amount of 'individuality'[1] of atomic processes was introduced, which is irreconcilable with the idea of causality."

Mara interrupted me as we sat down. "Whoa, just what do you mean 'individuality'? You sound like you're giving the atoms the ability to choose what they want to do."

I apologized. "It's a bit tricky to discuss what I mean precisely. For an example, think about electrons. In certain experiments they behave as waves and in others they act as particles. Now, if you set up an experiment to detect wave properties, that's what you'll find. Just as if you set up one to find particle properties, you won't see any of the wavelike behavior but you will see the particle behavior. You can't ever see both at the same time, and furthermore, what you see depends on the *actual experimental setup!*

"This means that your detecting equipment plays a role in what you're observing. To classic physicists this is an absurd notion. You can always, in theory, know both the location and momentum of this meatball exactly. If I were to throw it at that wall, you can tell me its momentum as it passes over that chair, no problem."

Mara kicked her salad around with her fork. "Sure, of course. All you need is a stopwatch and a ruler."

"Right, and using them isn't a problem. They don't interfere with what you're trying to measure. We are used to this 'classical setup,' where there is a clear demarcation between observed object and measuring device, and in fact we cannot ascribe an unambiguous meaning to what we have seen unless we use precisely these classical ideas.[2] The problem is that because of the finite size of the quanta, that demarcation fades away, and we're left with the difficult issue of being unable to describe quantum phenomena in classical terms."

Mara shot me a look. "You can use that jargon all you want; I'm looking for a concrete example where classical physics fails."

It took me only a few seconds to remember an example the professor mentioned in class. "Ok. Let's think about Rutherford's Atom. When he discovered the nucleus in 1911, he revealed the inadequacy of classical physics to explain why atoms are stable.[3] The electrons should radiate their energy away and collapse into the nucleus, leaving a halflife of well under a second. But as it stands, not only did quantum theory account for the stability, but it also provided for laws governing the spectra of the elements by introducing 'stationary quantum states.'"

Mara took a drink of her water. "Ok, fine. Let's go back to your meatball example. I want to be able to say that 'this electron is precisely at the origin with this specific momentum.' You're saying that because of the quanta, I can't do this."

"Right. Dr. Robert Griffiths had an interesting analogy of this. We can provide a good description of a mountain by taking pictures of it from various directions – north, east, south, etc. – and placing them all together on a sort of topological map. This is how our scientific minds have been made to work, in a manner of synthesis. There is nothing, classically, to prevent you from doing this. In fact, this is encouraged. But in quantum physics, you simply cannot synthesize information like that.[4]" I took a bite of food. I was convinced that Mara was keeping me talking so that I wouldn't get a chance to eat anything.

She sat in silence for a moment and then spoke. "Give me an example."

"Consider the Stern-Gerlach experiment. Here, we have an electron which is in some spin state, described by some basis. We shoot it through a specific magnetic field so that it either ends up in some region 'A' or 'B' on a screen depending on whether it was originally in a spin-up or spin-down state."

Mara stopped me. "Hold on there, buster. If the electron is not in an eigenstate of the z-component of spin angular momentum (I am assuming that's what you're measuring), then we don't know a priori whether it'll end up going to region A or region B. We can only figure out a probability, right?"

"I'm sorry, my terminology got a bit lax there for a moment. Of course you're right. But after it goes through the apparatus, we can be 100% sure that it is in an eigenstate of either +Sz of –Sz. Now, if we were to turn the Stern-Gerlach apparatus so that it is oriented along the x-axis instead of the z-axis, then we know (after we run the experiment) whether it's in an eigenstate of +Sx or –Sx. However, it is easy to show[5] that the x and z components of spin angular momentum are not compatible observables. We can know one or the other, but not both. Bohr calls this sort of 'relationship' which has no analogue in classical physics 'complementarity.'[6] We can know Sz or Sx, but not both."

Mara furrowed her brow. "What if you were to send it through an apparatus aligned with the z-axis and then with one aligned with the x-axis? Then, an electron might, say, go "up" (for +Sz) and then to the right (for +Sx). Don't we know then that the electron was in some initial eigenstate of *both* Sz and Sx?"

"Ah, no! When we did the first measurement, we acted on the electron. Our instruments affected the electron in such a way that, while we became 100% sure that it was in the +Sz state, we also became completely clueless about whether it was in the + or – Sx state. In fact, even mentioning the '+ or – Sx state' is something that we can't really do, since we don't really know anything about it. Then, when we measured the x-component of the spin angular momentum, we lost all knowledge about the z-component. Bizarre, isn't it?"

"Yes." Mara thought for a moment. "Does anyone have any idea why?"

"Well," I replied, "Dr. Robert Griffiths suggests that perhaps there is actually nothing there to measure; that is, when we measure the z-component that the x-component actually isn't there to measure.[7] Or, to any extent, we shouldn't ask any questions about it."

"This sounds fairly vague and ambiguous."

I shrugged. "You're not the only one who thinks so. In fact, Redhead chooses not to talk about it very much on the basis that Bohr's own formulations of the framework are vague and ambiguous, and that he even prohibits asking certain questions about quantum mechanical systems.[8] However, it can't be denied that Bohr's viewpoint provides the foundations for the prevailing view in the physics community – the Copenhagen interpretation, at least as far as pragmatics in concerned. Essentially, it is important to realize that quantum mechanics is an entirely different beast from classical mechanics. We can't know all sides of the mountain. In fact, due to the uncertainty principle, we can't even know the exact momentum and position of any given sub-atomic particle. It's difficult to begin to think about this because it's so contrary to our everyday experience of causality and objectiveness."

There was a long period of silence as we finished our meals.

Mara eventually said to me, "We can't know both, huh? That sounds ridiculous. I bet you I can think of a situation where you DO know two incompatible observables at once. What say you? Up for a few thought problems?" She stood up and began walking towards the exit.

I downed what remained of my milk and jumped up. "You bet! No matter what you try, I will show you that classical physics is simply not sufficient to describe quantum mechanical phenomena." I caught up to her and we headed back to Richardson, where she promised to give me a challenge.

1. Rüdinger, Erik, and Finn Aaserud, eds. Neils Bohr Collected Works. Vol 7. Amsterdam: Elsevier, 1996. Page 331. From "On the Notions of Causality and Complementarity" published in Dialectica, Vol. 2 1948. Page 313.
2. Faye, Jan. Niels Bohr: His Heritage and Legacy. Dordrecht: Kluwer, 1991. Pages 127-146.
3. Same collection as #1. "Albert Einstein: Philosopher-Scientist" (ed. P.A.Schilpp), The library of Living Philosophers, Vol. 7, Evanston, Illionis 1949, pp. 201-204.
4. Dr. Robert Griffiths. Quantum Mechanics lecture at CMU. September 5, 2003.
5. Any introductory-level undergraduate text. My personal favorite is: Griffiths, David. Introduction to Quantum Mechanics. NJ: Prentice Hall, 1995.
6. Same article as #1, page 314.
7. Same lecture as #4.
8. Redhead, Michael. Incompleteness Nonlocality and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics. Oxford: Oxford University Press, 1987.

Also, though no specifics come directly from him, I would like to acknowledge Dr. Frank Tabakin, whose lectures from August 25 – September 5 at the University of Pittsburgh were very instrumental (pun intended).

Conversations With Mara

Episode Two:

“Bohr’s Thorn: Einstein’s Objections”

Jonathan Gaffney
September 15, 2003

Mara unlocked the door to the advanced physics lounge and flipped on the lights. Immediately she grabbed a piece of chalk and started drawing on the single board in the front of the room. I walked to the back and grabbed a can of Mountain Dew and sat in one of the high-quality office chairs the club bought with leftover budget money. By the time I was settled, Mara had drawn a diagram on the board and began speaking.

"Consider this simple setup. You have a double slit experiment as you can see here. If I fire a stream of electrons at the diaphragm (which has two slits separated by some small distance a), they will appear on the screen as an interference pattern. Now, imagine that I put the diaphragm (which weighs very little) on a spring so that I can measure the momentum transfer in the following way: if an electron goes through the top slit, it will cause the diaphragm to recoil up, and if it goes through the bottom slit, it will cause the diaphragm to recoil down. Now, I understand that we're dealing with really tiny momentum transfers, but let's assume that we actually have the ability to detect them accurately, something that's theoretically possible."

I smiled and said, "I'll let that go for the sake of your argument, because I'm curious as to where you're going with this."

Mara thanked me and continued, "Now, we can look at the screen immediately after the electron goes through the diaphragm and see a small dark spot where it hit the photographic plate. Then, when we repeat the experiment a large number of times, we will observe the expected interference pattern. Thus, for each individual electron we can measure the momentum and the position precisely."

I chuckled at Mara, feeling a bit proud of myself. "I don't agree with you at all."

"Oh," replied Mara, "and just why not?"

"Easy. From your source, the electrons "split" and go either to the upper slit or the lower one. If the angle that describes that spread is Ω, then the difference in momentum transfer is Ωhs (s is the number of waves per unit length). Invoking the uncertainty principle, this means an indeterminacy of 1/Ωs for the location of the electron. Now, using the known results for the fringes appearing due to interference patterns, we know that there should be Ωs fringes. Obviously, if there's an indeterminacy of the same order, there will be no interference pattern. Our measurement has affected the result; if we measure the wave nature of the electrons, we get an interference pattern. If we measure the particle nature, by finding the momentum transfer, we lose the interference pattern.[1] Period."

Mara laughed as she handed me a piece of chalk. "You expect me to fold that easily? You had better prove to me the uncertainty principle if you plan on slipping it into arguments like that."

"Fair enough." I slapped my can down on the counter and walked to the chalkboard. Mara sat in my seat and took a swig from the Mountain Dew.

I figured it was a matter of formality that I do this; Mara knew the uncertainty principle was very real – she just wanted to see if I could derive it. I quickly ran down through a derivation from the single slit experiment. "If the width of the beam d is on the order of the deBroglie wavelength,

$$\Delta x\, \Delta p = (h/\lambda)\sin\alpha = (h/\lambda)(\lambda/d) = (h/d) = h/\Delta x$$

and thus $\Delta x \Delta p \geq h$. The first equality comes from the momentum being equal to h over the wavelength and the second coming from the optics equation for sine of the angle between minima^2. Lambda is the deBroglie wavelength. So, there it is." I walked back to Mara and slapped the chalk into her hand.

Mara smirked. "You think you've won a big battle, don't you? Well, let's think about this situation." She walked to the board, erased everything, and drew a picture of a clock inside a box hanging by a spring. Beside the box was a ruler, and an arrow that was attached to the box pointed to it in such a way that you could measure the distance that the box moved.

I laughed. "That thing looks really complicated. Just what do you intend on doing with it?"

"Well, first let me explain the apparatus. Inside this box we have some radiation source which sends out photons at a fairly slow rate. Here we have a precise atomic clock also in side the box, attached to a shutter. At a certain exact time that we decide, it triggers the shudder to snap open and shut so quickly that only a single photon of the radiation inside the box can escape. Now, when a photon escapes, it has an energy which is related to mass by the relation $E = mc^2$; thus, the mass of the box decreases by E/c^2, or $hw/2\pi c^2$, where w is the frequency of the photon. With the decrease of the mass of the box, it will move upwards slightly, and we can read the scale to find out what mass left. Hence, we know Δt precisely (from setting the atomic clock) and ΔE precisely (from reading the scale). As you well know, ΔE and Δt are incompatible observables like Δp and Δx are; your uncertainty relation says that $\Delta E \Delta t$ should be greater than h, but I've shown that $\Delta E \Delta t$ approaches 0." She slapped the chalk down in the tray and leaned forward on the lab table, crossing her arms. "So there." She stuck out her tongue, and I thought that Bohr must have felt much like I did now, eager to disprove my colleague and friend.

I sighed, knowing that I personally didn’t have the skills needed to disprove her latest claim. I had heard that this was an argument that Einstein presented at a conference in Solvay in 1930, but I couldn’t remember how the proponents of quantum theory answered him.

Luckily, just when I was about to admit defeat, the physics professor walked in. “I couldn’t help but overhear you, Mara, and I think Einstein had a more interesting argument than this one, one in which he attempted to show that quantum theory was incomplete as it stood. However, I believe that showing you both why this argument was refuted is in order, so I shall begin by doing that.”

Mara immediately responded. “Not fair, Dr. C, I had him there!” She went over to the couch and pouted, upset that she was about to be showed up.

“Quite simply,” the professor began, “Bohr managed to use Einstein’s own theories against him. General relativity says that the ticking of the clock is dependent upon where it is in a gravitational field, and since the release of a photon causes a change in where the box is located vertically due to the change in mass, the clock changes positions in the gravitational field just enough to introduce an uncertainty on the order of Planck’s constant. Clever, no?”

Mara shook her head. “Yes, it’s very clever, but I was hoping to tear down quantum theory with that example, and I knew I’d at least instill a little doubt if my buddy here couldn’t refute it immediately.”

Dr. C. cleared his throat. “Well, as I mentioned before, I’ve got a much sharper argument that Einstein, Podolsky, and Rosen whipped up that is commonly known as the EPR paradox.[3] I hope you will forgive me if I explain it in simpler terms than they originally used, since doing so is just as instructive but quite a bit easier to understand.”

Mara and I agreed that it would be ok.

“Ok. Consider the decay of a stationary neutral pion.[4] You know that because of conservation of angular momentum, the positron and electron that are created are necessarily in the singlet configuration. Now, if we choose to measure the spin of the electron, we immediately know the spin of the positron. For example, if we measure the spin of the electron to be up, then we immediately know that the spin of the positron is down. You with me so far?”

Mara and I nodded.

“Ok, now let’s think about what that means. If I measure a spin-up electron 5 meters away from the original pion decay, then I know that 10 meters away from me you have a positron that is spin-down. I know this immediately and definitely. However, information cannot be transmitted faster than the speed of light, so we are left at an impasse. We have a so-called ‘locality principle’ which states that, in the words of Redhead[5], ‘Elements of reality pertaining to one system cannot be affected by measurements performed “at a distance” on another system.’ Thus, either the formalism of quantum mechanics is incomplete, or else this principle of locality must fail.

“Einstein and his colleagues thought that is was kind of spooky that a measurement that one makes on one particle could affect what we know about another, and it’s not surprising because locality is a very ingrained part of classical scientific observations and knowledge. Einstein was a proponent of the ‘realist’ view of quantum theory – that the electron and pion always had a certain spin, be it up or down, and we simply had to connive a clever enough way of measuring them, and we would find them out. Of course, this EPR paradox was in strong agreement with that viewpoint, and it threatened any other viewpoint pretty heavily.”

I looked up and asked, “What do you mean by measurements? What do you mean by ‘at a distance’? Your ‘paradox’ seems very vague.”

Dr. C smiled. “You’re learning quickly. There are a couple unanswered questions. Bohr took advantage of one when he claimed that classical considerations have no place in a quantum problem such as this. Again, he claimed that the act of measuring influences the outcome and therefore this argument is irrelevant.[6]

“However, the issue of locality is an interesting one. There’s a famous example which involves locality…“

Mara interrupted him, “…called Bell’s Inequality. I’ll handle this one.”

1. Rüdinger, Erik, and Finn Aaserud, eds. Neils Bohr Collected Works. Vol 7. Amsterdam: Elsevier, 1996. pp. 356ff. "Albert Einstein: Philosopher-Scientist" (ed. P.A.Schilpp), The library of Living Philosophers, Vol. 7, Evanston, Illionis 1949, pp. 216ff.
2. Dr. Frank Tabakin. Quantum Mechanics lecture at Pitt. September 8, 2003.
3. Wheeler, J.A. and W. Zurek. Quantum Theory and Measurement. Princeton: 1984. "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" by Einstein, Podolsky, and Rosen. Physical Review, 47, 777-80 (1935).
4. Griffiths, David. Introduction to Quantum Mechanics. NJ: Prentice Hall, 1995. Page 375. (There's a very simple version of EPR here also).
5. Redhead, Michael. Incompleteness Nonlocality and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics. Oxford: Oxford University Press, 1987.
6. Rüdinger, Erik, and Finn Aaserud, eds. Neils Bohr Collected Works. Vol 7. Amsterdam: Elsevier, 1996. pp. 291ff. "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" by Neils Bohr. Physical Review, 48, 696-702 (1935).

Conversations With Mara

Episode Three:

“Bell’s Inequality”

Jonathan Gaffney
September 22, 2003

Mara stood and walked to the front of the classroom, fixing the pink bandana on her head. She took the chalk that Dr. C offered as he stepped out of the way, cleared her throat and immediately began. “Ok, let’s start by thinking about the pion decay that we considered in the EPR explanation. There, we simply detected the particles with parallel detectors. Let’s tack on a slight twist, and instead of having the detectors fixed on the same axis, let’s allow them to be rotated independently of one another. Obviously, when they’re parallel, we simply recover what we had in EPR: if we detect, say, the positron passing through the detector ‘A’, then we know conclusively that the electron will NOT pass the second detector (‘B’). Also, if they are aligned anti-parallel, then a positron passing through A conclusively implies that the electron will pass through B.”

I jumped in. “So, I imagine that if we align them so that they are neither parallel or anti-parallel that we’re not as sure whether one or both will pass the detectors?”

“Exactly. Let’s start with A and B anti-parallel so that both particles will pass. Now, imagine that we rotate A some angle Ω. Now, if we send out a bunch of particles, we’re not going to always see that the electron and positron have opposite spin orientations, because sometimes the electron will pass through B but the positron will not pass through A. We can call these ‘errors’ if we wish; there is a disagreement in the measurement of the two detectors. Now, what happens when we rotate B through an angle Ω in the opposite direction?”

I thought for a minute before replying, “we’re going to get even more ‘errors’ – in fact, I would guess that we would get twice as many.”

Mara looked at me slyly, and I knew she had something up her sleeve. “Why twice?”

"Well," I responded, "it's symmetrical with the first case: rotating A causes some of the particles that should have been transmitted to be absorbed. Rotating B then should cause the same number to be absorbed again."

Mara grinned. "You're double counting, but it's not very important. In reality, sometimes particles from the same pion decay will be 'errors' and cancel each other out. Nonetheless, the errors cannot be MORE than doubled."

The professor broke Mara's explanation. "Actually, what's important here is that you're presuming that the electron and positron have some definite 'polarization' this whole time. That is, you presume that there is something – call it a 'hidden variable' if you want – that describes the Sz state of the positron and of the electron. This is a subtle, but incredibly important point to make here, if you want to use this argument to harm the local hidden-variable theories." There was a moment of silence while Mara waited to see if he was finished. "Um, go on Mara. Sorry to interrupt you."

Mara was visibly a bit put out by this, and she continued speaking with, "Yes, thank you, I was going to get to that." She regained her composure and went on. "Anyway, quantum mechanics predicts that the 'error rate' may in fact more than double."

"Huh?" I was surprised by this, unprepared.

"The reason is that the electron and positron aren't in a definite state: when the positron passes through A, we then know that the electron must be anti-parallel, so for some angles, we can get a much larger error rate.[1]"

"Really?"

"Yes, that's what quantum mechanics predicts. Right, Dr. C?" Mara looked at the professor.

Dr. C nodded.

"So, have there been any actual experiments to see who was right?" I was curious now. Of course, my money was on quantum theory, but this was something that was worth thinking carefully about.

"Yes," Mara replied, "there have been lots of experiments, and most of them show that the quantum theory predicts the appropriate results, not classical logic. In fact, the two experiments that didn't agree are now largely thought of as having some form of systematic error.[2]"

"So what does it mean?" I was working to try to put everything together. "Surely, the Copenhagen Interpretation concedes a violation of locality – that's what we just discussed as a result of EPR."

Mara responded quickly. "Yes, it concedes to violating one form of locality – 'A previously undefined value for an observable cannot be defined by measurements performed at a distance.'[3] But then again, proponents of the hidden-variable theory have a much more disturbing problem to face.

"If they continue to insist that the polarization is well-defined before the measurement and that we just need to find it out, then they have to reconcile that with the violation of Bell's Inequality. There must be some way of transmitting information from the positron to the electron instantaneously. Of course, we have been indoctrinated with relativity: no information can be transmitted faster than the speed of light. So here's what's left for the realists: they can choose to give up their hidden variables, or they can give up a much more critical form of locality. A tough choice."

I had somehow found myself arguing for the realists. "But locality is still violated, no matter what[4]. Even if I don't assign values to the particles until I measure, I immediately know the implications of my measurement – it 'affects' something that's far away. Clearly, something is happening superluminally. Isn't it?"

Mara shrugged and looked at the professor. "How do you explain this?"

The professor said, "Well, there's a great explanation that David Griffiths[5] gives: a bug moving in front of a projector can cast a shadow on a screen far enough away that the shadow moves faster than the speed of light. However, no actual information is transferred. It's not causal: someone standing at one point of the screen cannot transmit a message to someone else on the screen by manipulating the shadow. Thus, we can call it, say, an 'ethereal' influence: there's no reason, per se, that it cannot travel faster than the speed of light.

"If it WERE causal, however, then we can't allow it. Otherwise, you could figure out a way to send a signal backwards in time and kill your infant self. That's clearly absurd."

Mara looked back at me. "What he said."

I chuckled. "So what about the *non-local* hidden variable theories? It doesn't seem that the violation of Bell's Inequality would say anything about them."

Mara replied, "No, of course not. Bell's Inequality speaks only about locality. It seems that nature itself is fundamentally non-local, something that is difficult for us to deal with, perhaps.[6] Speaking of which, this has been a lot for me. I need to go have a smoke. If we're going to talk about this, I at least need to be outside."

1. Little, Tom. "Quantum Weirdness: III. Bell's Inequality." Online essay at http://www.telp.com/philosophy/qw3.htm.
2. Redhead, Michael. Incompleteness Nonlocality and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics. Oxford: Oxford University Press, 1987. Page 108.
3. Same as 2, page 77.
4. Same as 2, page 117.
5. Griffiths, David. Introduction to Quantum Mechanics. NJ: Prentice Hall, 1995. Page 380.
6. Van Fraassen, Bas C. "The Charybdis of Realism: Epistemological Implications of Bell's Inequality." Pages 108-109 in the original source, unknown (handout).

Conversations With Mara

Episode Four:

“The quest for an end to hidden variables: Von Neumann through Kochen-Specker”

Jonathan Gaffney
September 29, 2003

I followed Mara outside, mulling everything over in my head. Her ripped jeans whooshed back and forth rhythmically, and I thought about Bell's Inequality some more, and its motivation. We got outside, and she sat on the steps as she pulled the pack of cigarettes out of her pocket. I stood in front of her, looking down at her two shoes. On the red one was written "Right Shoe" and on the blue was written "Left Shoe." I had stared at them many times; this was a common pose for us – her smoking and me thinking. For some reason, this time I noticed something else that was written on her shoes, amidst the pen scribblings and marker swaths, in reflective places on the two feet: "imagination" and "impossibility."

"What does that mean?" I asked, pointing at her feet.

Mara looked down at them, trying to see what I was referring to. "Oh, that! It's a reminder of a quote from Bell: 'What is proved by impossibility proofs is lack of imagination[1].'" She took a drag.

"Do you agree with that?" I was genuinely intrigued by the statement.

"Well," she said, "imagination is a vital part of science. We need to be creative in the way we choose to view the world. I think it's valuable to look at these hidden variable theories, for example; we can learn something from them. And you never know – one might revolutionize physics."

I shook my head. "I don't know about all that."

She sat silent for a moment before continuing. "You know, for a long time, there was a so-called 'no-go theorem' that kept many promising physicists from investigating hidden-variable theories at all. It was a 'proof' by Von Neumann so devious that it essentially shut off all attempts to think outside the emerging quantum-mechanics paradigm. At least, insofar as hidden variables were concerned.

"But, it turned out there was a little problem in it. For thirty years or so, people were turning a blind eye to the mere possibility of a hidden variable theorem, all because a silly assumption slipped through the grates." Mara flicked the ashes from her cigarette in disgust.

"I've never heard of that. Explain."

"The mistake that Von Neumann made was essentially the following. We know that the average value of two summed observables (say, A and B) is the same as the sum of the average value of them when we deal with them separately, right?"

"Right. That's a trivial QM identity."

"Now, if A and B commute, then for each measurement the sum of the observables must be equal to the sum of the individual measurements of A and B. However, Von Neumann made the silly assumption that what I just said is true even if A and B *don't* commute. But, if A and B don't commute…"

I interrupted, "then you can't measure them at the same time, so there's no reason to impose such a requirement![2] Wow. So, who finally caught this mistake?"

"Bell. Apparently, Grete Hermann raised a complaint about it, but no one listened." She rolled her eyes and twisted her cigarette out on the concrete step.

"Bell seems to be involved in an awful lot of this stuff. He eliminates one no-go theory, he develops the famous inequality…"

"…and he proved another famous no-go theorem, also. Independently, Kochen and Specker derived a similar argument in a somewhat different way a year later, so they usually get their name attached to it – we call it the KS theorem. Mermin describes a very simple version of it with astounding clarity, given how difficult the original version was to explain.” Mara sounded more than glad to fill me in.

“Wait, wait – before you go into the details of this new theorem, could you do me a favor and tell me *why* it matters? I mean, why was this one important? Bell’s Inequality destroys locality, and it’s hard to imagine a nonlocal hidden-variable theorem.”

“First of all, remind me to talk to you about the nonlocal hidden-variable theorem later – there’s at least one interesting idea by Bohm on the table that we can discuss. And as for why the KS theorem is important: it again sets out to show that hidden-variable theorems are pipe dreams. However, in doing so, it too must make a number of assumptions that, while pretty tight, leave a few little loopholes that can be slithered through. We’ll see after a brief explanation how we can deal with the result if the argument applies to us, and we can also discuss how the argument might *not* apply to us, in a manner of speaking.” Mara pulled the piece of chalk out of her grey hoodie’s pocket and began to draw on the concrete. “This might look crude, and I’m going to skip over some steps, but it’s all in the interest of brevity. I just want you to get a feel for the argument. Go look at Mermin’s paper, for example, if you want to see this in the detail it deserves.” This is what she drew[3]:

$$\begin{array}{cccc}
\sigma_{1x} & \sigma_{2x} & \sigma_{1x}\sigma_{2x} & 1 \\
\sigma_{2y} & \sigma_{1y} & \sigma_{1y}\sigma_{2y} & 1 \\
\sigma_{1x}\sigma_{2y} & \sigma_{1y}\sigma_{2x} & \sigma_{1z}\sigma_{2z} & 1 \\
1 & 1 & -1 &
\end{array}$$

"Let's think about a system of two spin-half particles. We can think of nine operators and set them in this pattern. Now, the operators in each row and each column mutually commute, so you could measure each of the observables in each row or column simultaneously. The number at the end of each row or column is the product of the operators. Now pay attention, cause here's where the problem is. Since the operators commute in the fashion I've mentioned, the values of the observables must satisfy the same constraints as the operators. Since each observable appears twice – once in a row and once in a column, it is clear that the total product has to be positive one. However, we can immediately see that the product of the operators is *negative* one! This is a contradiction which implies that there is no consistent way of assigning values to the operators."

"Hmm." I scratched my head as I looked at the chart. "Interesting. So, what does it mean? If we can't assign values to each of the operators, then there can't be a hidden variable theorem, since that theorem would say that there are assigned values which simply need to be measured, right?"

Mara put the chalk down and looked up at me. "Well, that was the hope. However, a few assumptions went into this no-go theorem that we need to shed some light on. Redhead[4] provides three different ways of dealing with the KS paradox, and Bohm provides what seems to be another, albeit somewhat related method."

I shrugged. "Ok, well, don't keep me in suspense."

Mara furrowed her brow. "Let's see if I can remember them all." She crossed her legs and thought for a moment. "Well, ok, so the first one is trivial. We can abandon all searching for a realist hidden-value theorem. We can just assume that our current quantum mechanical interpretation is as good as we can do, and just let it go. This is what most people have done. It's the easiest, and from the point of view of most physicists, any attempt to work around the KS paradox is more complicated than it is worth; the results from the experiments are the same regardless, so there's no need to really concern ourselves with interpretation.

"Another way to deal with KS is to deny what Redhead calls a 'reality principle,' which states that for each self-adjoint operator that has an associated number, then there exists an observable in reality that is measured by that operator. If we deny this, then there's no reason to believe that measuring something using a certain procedure yields anything 'real,' so who cares if two different measurements are the same? One could refer to reality, while another is just an artifact."

I responded to this. "Yeah, it is certainly a very strange feature of the theory if we can just throw off some items as nonphysical, while retaining others, if we have no significant reason to prefer one over the other. I mean, it would be tough for someone to convince me that one measurement obtained the 'real' value, while some other measurement obtained some extraneous result. No, I don't like it."

Mara shrugged. "The third way to circumvent KS is to not assume that there's a one-to-one relationship between operators and observables. Van Fraassen proposed this particular scheme, which we might call *ontological contextuality*, saying essentially that since nonmaximal operators can have more than one observable. Hence, it's no longer reasonable to think that when we find a certain value for an operator from one measurement that it will be the same as when we find the value for that operator using a different measurement. Thus, KS is blocked."

She stopped talking. I stood for a moment and then asked, “So, what about Bohm? You said his way of dealing with KS was somewhat different than those three options.”

Mara smiled. “Yeah, Bohm kind of takes a sidestep of the whole KS paradox, saying that it doesn’t apply to him because of the importance of the measuring apparatus in his interpretation. But, let’s talk about that some other time. I’m exhausted. Say, meet me for dinner? My place, seven o’clock?”

“Hmm, are you cooking?”

“Sure.”

I grinned. “Well, then I’d better bring myself a sandwich.”

She punched me in the hip. “Just for that, I want you to look into the measurement problem. There are some elements of the Copenhagen Interpretation that are less than desirable, and Bohm attempts to avoid some of these with his interpretation. I expect you to be able to talk about that tonight.”

“Yes, ma’am!” I rolled my eyes as I walked to the library. I was thinking to myself that sometimes I couldn’t help wishing that Mara cared as much about cooking as she does about intellectual discussions.

1. Mermin, N. David. “Hidden Variables and the two theorems of John Bell.” Rev. Mod. Phys. Vol. 65, No. 3. July 1993. pg 803-814.
2. Same as 1, page 805-806.
3. While this follows Mermin, I used the diagram and some language from: Bohm, D. and B. J. Hiley. The Undivided Universe. London: Routledge, 1993. Pages 118-120.
4. Redhead, Michael. Incompleteness Nonlocality and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics. Oxford: Oxford University Press, 1987. Pages 119-138.

Conversations With Mara

Episode Five:

“The measurement problem and the quantum Zeno effect”

Jonathan Gaffney
October 13, 2003

I showed up at Mara's apartment a little before seven in the evening to find her standing above a large pot of water that was on the stove. I simply shook my head at her and said, "Silly girl, don't you know a watched pot never boils?"

She shrugged and looked at me. "Did you look into the measurement problem like I told you to?" It was clear that the less attention I paid to her cooking, the happier she was going to be.

"As a matter of fact, I did." I went over to the sofa and lay down. "So it seems that the Copenhagen interpretation, which is what most physicists cling to, has one very serious issue which must be dealt with; one which is unsettling, even with the many various attempts to explain it that have emerged. That problem is the question of how to explain 'measurement.' Since every 'measurement' of a microscopic (quantum) system must necessarily yield a result, the state of the physical system in question must undergo some sort of 'collapse,' wherein the dynamical equations of motion are violated. The system has stopped evolving and simply becomes an eigenstate of the measured value."

"Ok," Mara said, "so what actually constitutes a measurement?"

"That is," I responded, "exactly the problem. Between the time a given system is in an eigenstate of observable A and the measurement of an incompatible observable B is carried out – entirely carried out, all the way through to the point of a 'conscious impression' of the new appearance of the apparatus, some wave function must have 'violated the dynamical equations of motion and collapsed.'[1] Where exactly that collapse took place is subject of some debate. One possibility, which was proposed by Wigner, is that the intervening of a conscious observer is what causes the actual collapse. Of course, this seems to be patent nonsense – what is meant by the word 'consciousness?'"

Mara laughed. “Well, tell me a better idea, then.”

“Well, what if we simply say that a measurement is an interaction between some microscopic state and some macroscopic state? That seems like a reasonable explanation; it eliminates the need for consciousness.”

Mara gave me her all-knowing look. “But what do you mean by ‘macroscopic’? It seems that this has the exact same problem as Wigner’s proposal, whether you want to admit it or not. We’re clearly not getting anywhere.”

“Well, then let me talk about GRW[2], a theory that makes no mention of measurement. Instead, it simply says that systems go along obeying the laws of dynamics according to the Schroedinger equation *most* of the time. Every so often the system collapses on its own. The probability per unit time is fixed (collapse is expected about once every $10^{15}/N$ seconds, where N is the number of particles in the system), though the collapses are random. By collapse, GRW means that the wave function is multiplied by a Gaussian (not a spike; this would cause well-defined positions and hence cause the nasty side-effect of indeterminacy on the energy and momentum[3] [or explain spontaneous combustion!]) which localizes the wavefunction.”

Mara leaned back on the counter and replied, “Well, I can see some attractive features in that. It sounds like it almost fits the bill – it gives us an explanation for the collapse of a wavefunction without forcing us to deal with measurements, per se. However, I say ‘almost’ because I’m not comfortable with it. Gaussians have tails; they’re not completely localized like delta-functions are. This means that there’s still a little bit of ‘superposing’ of states, even in macroscopic objects. It’s a bit absurd to imagine that this spoon doesn’t have a definite position, or that my cat is a ‘little bit dead.' So we either have to accept that vagueness, or we strike a nail through GRW as well."

"What are you implying, then?" I furrowed my brow at her.

Mara responded, "It just sounds to me as if we're running around, going nowhere. Our 'orthodox' view seems to do a great job, up to explaining the collapse of the wave function. Then it falls apart – or at least gets to a point where you're forced to say that it's 'elephants all the way down'."

I shot her a look. "I thought it was turtles."

"Turtles, elephants, whatever. You get the point."

I thought silently for a little while before responding. "Well, then let me tell you about the Quantum Zeno Effect[4], because it should be an observable consequence of the collapse of the wavefunction, which is key in the orthodox interpretation."

"Ok, I suppose." Mara broke the pieces of pasta into the water.

"Let's think about an excited electron in an atom. We know that left to its own devices, it will eventually fall to the ground state and emit a photon. The probability that it will do so in a time t significantly less than the lifetime tau is proportional to time over tau (t/τ). Then the chance that it's still in the excited state after time t is simply $1 - t/\tau$. Simple, right?"

"Yes, so what happens when we make a measurement at time t, and then another one at time 2t?"

"Nothing surprising. We can do the math, and $(1 - t/\tau)^2$ is very near $(1 - 2t/\tau)$, which is what we would have expected if we had only taken one measurement at time 2t; since t is pretty small we can just drop the t squared term, since it's going to be much smaller than t. However, what is interesting is that we can show that at extremely short times, the probability of the spontaneous transition is proportional to t *squared*.[5] Then, if we perform the same thought experiment, we suddenly see that the chance that the electron is in the excited state when we take one measurement is essentially $(1 - 4t^2)$, but if we take two measurements, the probability is $(1 - t^2)^2 = 1 - 2t^2$ (where we again drop the higher order term, because it's so small). Thus, if we take measurements successively and very quickly, we actually *slow* the transition rate!"

"So you're telling me that if we continually observe an excited electron, it will never fall back into its ground state?"

I pondered for a moment. "Essentially, yes, if we could perform continual measurements. A watched pot never boils, so to speak."

"That's just silly," Mara said. "If you consider a particle in a bubble chamber, we know that it decays, although we're continually observing it."

"Well, practically speaking, it's going to be very hard to actually construct a situation where we take continual measurements. The bubble chamber's atoms are intermittent, and so I could argue that the measurements aren't sufficiently quick."

Mara put her hands on her hips and said, "Great. So you've come up with an interesting thought-experiment that can't be tested. Pointless!"

"No, no, it's not impossible to test. In fact, two years ago there was an experiment[6] that showed both the Zeno effect *and* an inverse Zeno effect, which can also be shown for certain scenarios."

"What do you mean, *inverse* Zeno effect? That if we measure a system frequently, we can speed up decay?"

"Yes. At certain frequencies, measurement makes the 'pot boil faster;' at others, it makes the 'pot boil slower.' This is a great success for the orthodox interpretation; we can explain the quantum Zeno effect in terms of collapse: when we measure something, the wavefunction collapses, and then it has to begin evolving all over again. Isn't it great how nicely that all works out?"

Mara shook her head. "It still says nothing about *how* the collapse occurs, or what a measurement *is*. And, in fact, I can explain the Zeno effect using Bohm theory[7]. So I really don't think you've managed to convince me of anything. Now come here; the spaghetti's done."

1. Albert, D. Quantum Mechanics and Experience. Harvard: 1991. "The Collapse of the Wave Function (chapter 5)." Pages 80ff.
2. Albert, David and Barry Loewer. "Tails of Schrödinger's Cat." From Perspectives on Quantum Reality, R. Clifton, ed. Netherlands: Kluwer Academic, 1996. Page 83.
3. Same as 1, page 95.
4. This explanation follows: Griffiths, David. Introduction to Quantum Mechanics. NJ: Prentice Hall, 1995. Pages 383-385.
5. See any undergraduate physics text. In Griffiths, it's page 309-311.
6. Fischer, M. C., et. Al. "Observation of the Quantum Zeno and Anti-Zeno Effects in an Unstable System." Phys Rev Let Vol 87 No 4. 23 July 2001.
7. This is mentioned briefly in Bohm, D. and B. J. Hiley. The Undivided Universe. London: Routledge, 1993. Pages 131-132.

Conversations With Mara

Episode Six:

“Bohm Theory: An Ontological Interpretation”

Jonathan Gaffney
October 27, 2003

We sat down at the table with our dinners, and I expected the worst as I took my first bite of the spaghetti. I don't know how it's possible, but Mara always manages to make even pasta taste like shoe leather. At least there was wine. "Interesting," I said, referring to the meal.

Mara thought I was referring to her suggestion about Bohm theory and began to explain. "Well, the first thing to note is that although Bohm really got the ball rolling when it comes to a complete ontological interpretation of quantum mechanics, there were others who had similar, but somewhat different viewpoints. I'll pretty much just be telling you about Bohm, but bear in mind that there are variants.

"Let's begin with a simple analogy[1] that might explain where Bohm was coming from. The invention of the lens contributed greatly to scientific process. Why?"

I thought for a moment. "Because it allowed us to look at various parts of any object and then see how they relate to each other?" I already had an idea where she was going, so I figured I'd play along.

"Yes, and they also made the very small and the very far away observable; hence, scientists could conceive that a reductionist approach could be relevant in all cases. However, if we think about a *hologram*, we notice that the entire illuminated structure can be seen, even if we only view a small part of the photographic plate. The one-to-one relationship between 'parts of the object' and 'parts of the image' is lost. It's this transition from a 'reductionist' approach to a 'holistic' approach that Bohm was hoping to achieve. More than another way to interpret quantum theory, it was a scientific revolution that to some extent he was shooting for."

"Ok," I replied, "but is it possible to do this? Bohr's conclusions about reality are pretty firm; is there room for a theory that attempts to take a whole different approach to this whole matter? Wouldn't he just arrive at the same conclusion?"

"I can understand where you're coming from; Bohr's conclusions are definitely the most compelling we've seen. However, they were based on some assumptions about the quantum of action, namely, that it's indivisible and unpredictable (and thus uncontrollable). Bohm challenges that perhaps there is a more complex description in which a process that involves a 'quantum jump' might in fact be continuous and analyzable, and this ontological approach cannot be automatically ruled out[2]. Sure, it's going to have its own assumptions, but Bohm's hope is that his assumptions might be somewhat more pleasing than Bohr's."

I scratched my head. "All right, I suppose. Why don't you tell me what his approach *is*, and then we'll see how I feel about it."

"Bohm starts by making dynamics central to his theory. He claims that all particles travel in trajectories, much like we perceive them to in the classical world. What determines how these particles move around is something that we can call a 'ψ-field[3]' (much like an electromagnetic field, for example): $\psi=R(x,t)e^{1S(x,t)}$ where S is the action. Well, we know that the Schrödinger Equation is simply a diffusion equation with dispersion, and that the continuity equation is built into it. It shouldn't be a surprise then that you can solve it using just such a ψ-field. Splitting the Schrödinger Equation into a real part and an imaginary part (exactly as you did when solving for the WKB approximation[4]), you find that the imaginary part holds within itself a 'quantum potential term.' Hence, Bohm concludes that particles are defined by a precise position and that their velocities and trajectories depend on both the classical potential energy as well as some new 'quantum potential energy.'"

I choked down more of the food. "So what's the form of this new potential?"

Mara laughed, "It's a bit mysterious." After I gave her a questioning look, she added, "Let me explain. The potential has the form of $(\partial^2 R)/R$. Instantly you notice that it is independent of the *strength* of the field! So there's clearly a difference between this ψ-field and, say, the electromagnetic field. In that latter case, a stronger field clearly has more effect on an electron, but in the former case, it would not! It's only the phase that matters. This also means that it doesn't fall off with distance."

"So, what, you're telling me that particles are guided in trajectories like ships on auto-pilot are guided by radio waves[5]? I mean, in that case, the amplitude of the radio waves doesn't matter, just the form."

Mara thought for a second. "Yeah, it's kind of like that."

I added. "But that implies some 'inner structure' in the particles that we haven't found. I don't know; that sounds pretty far-fetched. Possible, yes, but a bit strange."

Mara shrugged.

"So you're also saying that quantum mechanics is purely deterministic. That is, if I were to give you an initial configuration and a ψ-field, you'd be able to tell me the configuration at any later time, right?" Mara affirmed, so I continued, "So how does he explain probability?"

"Ah, yes. Well, probability largely results from our inability to precisely know the original configuration of particles in phase space. It can, and does, explain the projection postulate without involving any sort of mysterious 'collapse,' because it doesn't take the projection postulate to be fundamental; it instead comes from the combination of theory and the fact that macroscopic measurements typically involve many particles. Take my word for it.[6]" Mara finished her glass of wine and went into the kitchen for the bottle.

"Ok," I said, "let's assume for the moment that I do. How do the measurements occur in Bohm's theory, if they don't involve a collapse?"

"Since it's a deterministic theory, we're really just finding where the particles end up. The only inherent property they have is position (in the minimalist interpretation), not spin or mass, or anything else. All of those things are a property of the wavefunction, so a measurement of 'spin-up in z' means that the particle has arrived at some detector at some location that somehow indicates 'spin-up in z.' What this means is that if we have a detector for spin-up in z, we can only measure spin-up in z. We can't measure the non-commuting observable spin-up in x, because the particle can't be at a detector that isn't there! Thus we only assign a value to one of the pair of non-commuting observables at a time.[7]"

I nodded my head. "Very clever. So that's how he gets around Kochen-Specker. Because the apparatus we choose influences the measurements we do, we get away without having to have previously defined values for all observables. Doesn't that cost us locality, though?"

"Yes, of course it does. But we already knew that coming into this discussion, and Bohm was more than willing to give it up." Mara took a bite of bread.

"Ok, so then how does Bohm theory explain the Zeno effect?" I leaned back in my chair.

Mara was not phased. “We need a quantum potential for the particle to make a transition; this is introduced by the perturbed wave function which as we know is proportional to t for small times. The times are very short, and therefore the wave function will never become large enough to create a quantum potential for the particle to make the transition.

“As Bohm and Hiley point out[8], this emphasizes that quantum interactions are *participatory*; our rapid measurements yield a very intense amount of participation. We aren’t simply ‘watching’ anything happen – we’re influencing it.”

I pondered for a moment, and then said, “One more question. Why is ψ squared a probability? I mean, couldn’t something else describe the probability? Ψ mostly helps us in this theory when we’re concerned with the trajectories of the particles; I don’t see how that relates to probability.”

Mara sighed. “That, my friend, is a tough question. Rather than explaining the work that others have done on it, I’m simply going to tell you that much of it rests on a hypothesis, namely that the probability measure over the set of all possible initial distributions of the universe is given by the initial wavefunction of the universe squared[9]. I know this raises a whole host of additional questions, and what it really comes down to it that Bohm’s theory is based on a certain set of assumptions which are hard to justify. Overall, though, it appeals to many people because it clears up the measurement problem and provides a possible view of how the quantum world works.”

I looked at the painting of the café in Venice on the wall and thought about what Mara was saying. “It seems that he’s trying to justify having a classical explanation for quantum events, what with the trajectories and all… I just don’t know if that’s really worth all the trouble. It seems easier to just say that ‘quantum events are different’ and leave it at that.” I shrugged. “But I guess I can see what he was trying to get at.” I took another mouthful of the spaghetti. “Interesting.”

1. Bohm, David. Wholeness and the implicate order. London: Routeledge, 1980. The analogy is on pages 143 and following.
2. Bohm, D. and B. J. Hiley. The Undivided Universe. London: Routledge, 1993. Pages 24-26.
3. Dickson, M. “The Bohm Theory” chapter (sorry, no full reference info). Page 107.
4. This process struck me because we just did it in my QM class; Tabakin, 10/9/03.
5. Dickson, 111.
6. See Dickson, 115, or Bohm and Hiley, 40.
7. Dickson, 117.
8. Bohm and Hiley, 132.
9. Dickson, 121.

Conversations With Mara

Episode Seven:

“Decoherence and Modal Interpretations”

Jonathan Gaffney
November 3, 2003

Mara stood from her seat as she threw her napkin down onto the table. “Well, I cooked. You know what that means.”

I sighed as I stood. “Yeah, yeah. I’ll try to do as good a job cleaning the dishes as you did cooking.” I took our plates over to the sink and began running the water. “So, are there any other interpretations of quantum mechanics that claim to avoid the measurement problem? I’m still not exactly blown away by what you’ve told me thus far.”

“As a matter of fact, there’s another route to pursue: modal interpretations and decoherence. The one tends to lead naturally into the other, and as such I tend to lump them together. You wanna know about them?”

I shot her a look. “Does it look like I’m going anywhere? Lay it on me.”

Mara began, “I want to establish, first, what modal interpretations and decoherence set out to accomplish. Clearly, they want to be consistent with the quantum mechanical rule that all measurements must result in observed values, without resorting to any sort of collapse of the wave function or ontological interpretation of the wave function. Ideally, this should provide some middle ground that everyone could be happy with to explain the transition from the quantum world to the classical world. Also, if decoherence is to be taken seriously, it is a threat to quantum computing, turning quantum computers into nothing more than classical computers[1] by invalidating the quantum superposition principle… so, in theory, we might be able to learn whether the Copenhagen interpretation (or decoherence, for that matter) has any merit to it simply through the pursuit of quantum computing, but this is a topic for another time.

"Ok. The starting point for everything is something called the Biorthogonal Decomposition Theorem. Basically, this theorem says that any vector in a tensor-product Hilbert space can be written as a linear combination of terms in some respective bases. Here's the kicker, though: 'if the absolute value of the coefficients in this linear combination are all unequal then the bases are unique[2].' What this means is that the state of a two particle system will pick out a unique basis – and therefore an observable – for each of the component systems, assuming the coefficients aren't the same. The upshot to this is that if we wanted to measure the z-spin of some particle with a detector, we can decompose the state of the particle and detector into appropriate bases: the pointer reading 'up' will correspond to the particle being in the up-spin, without the wavefunction of the particle actually collapsing!

"Now, we need to invoke something called decoherence here in order to help us. Instead of thinking about systems in isolation, we invite them to interact with their environments. When it does so, there is a 'tracing-out' of the phase relations of the quantum system; quantum coherence is 'lost[3].' This means that, if the states of the environment are orthogonal, our pointer will in fact show 'up' for the z-spin being up, if that's what our pointer is designed to show."

I turned off the water and looked at Mara with a confused look on my face. "Wait, so there's not actually a *measurement*; the detector just indicates a property of the particle? That seems to imply some sort of intrinsic properties and hidden variables… doesn't that just lead us back to where we were before with the whole Kochen-Specker problem?"

"Well, there are different ideas on this. Kochen believed that systems have no intrinsic properties; all properties are *relational*. Whether a measurement is occurring or not, any properties that exist are only in relation to the system that 'witnesses' it[4]. Thus, an electron might have properties relative to, for example, a z-spin detector.

Others, such as Dieks, choose to allow the particles to have intrinsic properties, but they find their own ways of getting around the Kochen-Specker paradox. One way is claiming that there is some preferred division of the universe. This essentially amounts to a sort of atomic interpretation, where only 'atomic' subsystems follow biorthogonal decomposition, and complex systems inherit their properties from that.[5]"

"Hmm." I was scrubbing a plate as Mara leaned on the counter. "Ok, let's assume then that we can get around the K-S problem one way or another. I'm still not sure I like this. What happens when the coefficients are the same, like in the case we deal with all the time: the singlet state?"

"Yeah," Mara admitted, "there's no clean way of dealing with those states. The best thing we can do is claim that we really don't see such pure states very often. We can consider an entire spectrum of values for the coefficients; the chances of two of them being equal is so small that we can just throw them out. This seems very much a 'for all practical purposes' answer, so I don't like it. Dieks fought with this some, and he concluded that in such a situation, a measurement will in fact give either 'up' or 'down,' but the spins of the particles themselves remain indefinite[6]… all we read from two detectors is the correlation: one will definitely read 'up,' and one will definitely read 'down.'"

"What?" I nearly dropped a glass. "You're telling me that a measurement won't actually tell us what the spin of the particle *is*, only information about the system? I don't like that one bit; I'd rather lose some locality than have this sort of indeterminacy dangling out there."

Mara shrugged. "That's your choice."

"Besides," I added, "I've got a bigger problem. Measurements aren't perfect; when we introduce an error term, it becomes clear that there is a nonzero chance that the pointer will point 'up' when the particle is actually in a 'down' state. And, in fact, while this in and of itself is annoying, we can go to the next level as well: even if we had a perfect measuring device, some situations are prone to this Albert-Loewer problem anyway[7]: there's no error, but still the pointer isn't in a definite state [or, it's in a definite state, just not one that matters to us; it implies a superposition]. How can you deal with this?"

Mara smiled. "This is where decoherence comes in. We can claim that the environment resolves this issue in the way I described just a little bit ago."

"But all of that is based on the orthogonality of environmental states; is this always the case?"

Mara sighed. "Unfortunately, no. However, it's been shown that the states tend to orthogonality very quickly. *Very* quickly.[8]"

"But do they ever reach orthogonality? It sounds like they approach some limit without actually getting there, and I don't know how you can be comfortable with that. There's going to be a tail, albeit for a very short time, where the cat isn't definitely alive or dead (or, if you prefer, the pointer is in a superposition of up and down). It's literally in a superposition – a classical object is in a quantum superposition!"

"I agree; it's unsettling. But the time scale we're talking about here is so small that any observation you would attempt to make of this superposition would require a huge uncertainty in energy. To be crude, you might blow it up when you probe it with some object designed to take a "snapshot" of it within the time window. But yes, you're right. It's the matter of principle that's upsetting; maybe it just requires us to accept that some things in the classical world aren't quite as definite as we like to think they are… at least not *all* the time." Mara walked over to the fireplace in the living room and began to prepare it for a fire. "Many people like looking into the modal and decoherence theories because they, for all the shortcomings that you've pointed out, provide a pretty nice way of holding on to much of the orthodox view of quantum mechanics while still avoiding collapse. It also provides a support to Everett's Many Worlds Interpretation and gives a delineation between classical and quantum physics for Bohr's point of view[9], so it's easy to be attracted to it."

I dried off my hands and walked into the living room. "But it still has those problems, and until those get resolved, I just won't feel comfortable with it."

Mara agreed. "Well, quantum information technology hopes to accomplish quite a few things, and if it fails, that might just be the break that's needed for people to seriously consider theories other than collapse."

1. Zurek, Wojciech H. “Decoherence and the Transition from Quantum to Classical – *Revisited*.” Los Alamo Science, November 27, 2002. Page 6.
2. Dickson, Michael, "Modal Interpretations of Quantum Mechanics", *The Stanford Encyclopedia of Philosophy (Winter 2002 Edition)*, Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/win2002/entries/qm-modal/>.
3. Bacciagaluppi, Guido and Meir Hemmo. “Making Sense of Approximate Decoherence.” PSA 1994, Volume 1. Page 346.
4. See #2.
5. See #2.
6. Dieks, D. “Resolution of the Measurement Problem Through Decoherence of the Quantum State.” Physics Letters A. Dec. 25, 1989. Page 445.
7. See, eg, Ruetche, Laura. “Measurement Error and the Albert-Loewer Problem.” Foundations of Physics Letters, Vol 8. No. 4, 1995.
8. #1 and #3 both show convincing results.
9. Zurek, page 21.

Conversations With Mara

Episode Eight:

“Weak Measurements”

Note: this episode is based largely on the pair of seminars given at CMU Oct. 30 and Nov. 6, 2003 by Drs. Wu and Griffiths.

Jonathan Gaffney
November 10, 2003

Mara stepped back as the fire took on a life of its own. I turned off the main light, leaving only the fireplace and a dim light in the hallway behind us to prevent us from being surrounded in darkness. Mara pulled a pencil and a piece of paper to the center of the glass coffee table. “I have one more thing I want to talk about,” she said.

“Ok,” I replied. It had been a lot of talking and physics thus far, and I was tired. But we seemed to be accomplishing something, so I wasn’t objecting. “What have you got this time?”

Mara began, “In 1990, two physicists named Aharonov and Vaidman published a paper that began serious consideration of a new way of making measurements. They called their procedure ‘weak measurements,’ because the measuring device only interacts weakly with the particle that it’s measuring. Schematically, as the particle goes through the detector, it weakly interacts with a particle inside the detector. Then the detector makes a ‘strong’ measurement on the second particle, collapsing it (or whatever), leaving the original particle almost unchanged. Of course, there is some change in the particle’s momentum (or whatever was measured), but the magnitude of that change is well beneath the uncertainty principle, so it’s fair game.”

I was quick to reply, “If it’s such a subtle interaction, how can we obtain any information from it? Wouldn’t the blip merely appear as something very slight – if we even saw it at all?”

“Yes, of course. But if we repeat the experiment over and over again, we’ll find that we actually *do* get a result. And not only do we get a result, but often the result agrees with what we would have gotten if we had performed a typical ‘strong’ measurement.”

I tilted my head. “So you do a measurement, only disturbing the system very little? That means that it’ll keep evolving like normal when it comes out of the other side of the detector? Has this actually been *done*, or is it all speculation?”

“Yes, essentially, that is the case, and yes, it has been done[1]. To this extent, weak measurements seem to fly in the face of Bohr’s ideas: now, when we take a measurement, we don’t need to significantly perturb the state.”

Mara continued, “We could stop there and have an interesting situation on our hands, but that’s not the end of the story. Weak measurements beg us to look more closely at what a measurement really is. In traditional quantum mechanics, the wavefunction collapses into some eigenstate of the observable we measure. That tells us the value of the observable *at the precise moment of collapse*. Although it’s tempting to travel the road backwards in time to make claims about what the state was doing before, we can’t really do that as it stands. However, weak measurements offer an intriguing alternative: maybe we *can* in fact learn about the state prior to the strong measurement.”

I looked up. “Explain.”

“Well, first we ‘pre-select’ (prepare) a state and send it through a weak measuring device. Then we send it through a ‘strong’ measuring device, which ‘collapses’ the wavefunction, if you like. Now, here’s the key: we need to ‘post-select’ a state. What this means is we check to see if the projected state after the strong measurement is of the form that we want. If not, we throw it out. The result of all this is, according to the Aharonov, that after we perform this sequence a large number of times, we can actually learn something about an *intermediate* time. He defined the weak measurement of an observable C in the following way (where F is the final state and D is the initial state).”

Mara scribbled on the paper and showed this to me:

$$\langle C\rangle_{wk}=(\langle F|C\rangle\langle C|D\rangle)/\langle F|D\rangle$$

Before I could respond, Mara said, “Now, think about this a for a little bit. I won’t go through the derivation, but it follows smoothly from quantum mechanics[2]. Are you comfortable with this?”

“Yeah, sure, I’ll take your word for it.” I shrugged.

“Then let’s consider the so-called ‘three box paradox.’ Imagine you have three boxes and one particle. Let’s go through the procedure outlined above, pre-selecting the state $(1/\sqrt{3})(|A\rangle+|B\rangle+|C\rangle)$ and post-selecting the state $(1/\sqrt{3})(|A\rangle+|B\rangle-|C\rangle)$. Notice that one-ninth of the strong measurements we take at the end will ‘collapse’ the state into what we want (that is, state F). Now, let’s ask the question, ‘is the particle in box C between the pre- and post- selected states?’” Mara smiled and turned to me; the flickers of the fire made her eyes look mischievous. “What does the formula tell you?”

I plugged in the numbers. “$\langle C\rangle_{wk}= (-1/\sqrt{3})\ (1/\sqrt{3})\ /\ (1/3) = -1$,” I said. “Wait, negative one? That doesn’t make any sense at all! I asked a yes or no question and expected yes (1) or no (0)! What does –1 mean?”

Mara had an answer. “Well, there are a couple of different thoughts on the matter. One is that this should be interpreted literally; there is a sort of *negative* particle in box C. In other words, there’s a negative pressure acting on the box[3]. There’s a positive pressure on boxes A and B, and the total number of particles still comes out to be one. A slightly less mysterious way of viewing this is to consider three boxes with N particles in them. If we add one particle in that state and send it through our procedure, we simply have (N+1) particles in boxes A and B and (N-1) in box C.”

I couldn't keep myself from laughing. "You can't be serious. Negative particles? Negative probability? That's quite weird, don't you think?"

"Well, quantum mechanics *is* weird, so I don't know how surprised I'd be about a mysterious event such as this. But, in this exact situation, I have to admit, I'm a bit skeptical as well."

"So then how are we to interpret it?" I recalled that Mara said there was another way out of this example.

"We can't really be sure that the weak measurement is measuring what it says it's measuring. We'd like to believe it is, because then it would definitely have meaning. As it stands, we're definitely reading a –1 from the pointer, but maybe it doesn't actually correspond to the chance of finding the particle in box C. As Griffiths pointed out[4], for 'the probability of something measured between states D and F' to have any meaning, we need to require a sort of 'consistency condition.' [Briefly, this condition is that the trace of the product (FEDÉF) be zero, where each is an outer product (e.g. F = |F><F|).] This leads to $<E>_w$ (the weak measurement of E between the initial and final states) being precisely zero or one. Therefore, the probability that the weak measurement in the three-box paradox describes is certainly not what we would consider to be 'usual;' at the best, the authors are creating a new set of probability laws that need to be properly expanded upon in some forthcoming theory…"

"And at worst, the pointer simply means nothing that we can understand." I was more than happy to interrupt. "So I guess one solution is to simply use weak measurements only when this consistency condition holds and throw out all the other cases." I paused and sat back in my seat. I could definitely tell that the wine was beginning to have an effect.

Mara yawned and lay back on the sofa. "Yes, that's a conclusion. Weak measurements don't seem to be completely worthless; at the very least they bring important questions and results to the forefront of experimental quantum physics. I think it's a good example to show that there are yet some philosophical questions that physics may help to answer. Or physical questions that philosophy might be able to help answer. Whichever."

I sat back in my seat. From Bohr through Aharonov, I thought about these people who constantly challenged the way we think about physics. "It's a shame," I said to Mara, after I had watched the fire die down some, "that more people studying physics don't bother to take the time to think about what the work they're doing *could* mean. Sure, sometimes it's a bit weird, but it's only by questioning and rethinking our beliefs that we can ever hope to grow." I sat, hearing no response, for a moment. "I guess I'd just like to see a world where the line of distinction between philosophy and physics doesn't seem so acute."

I looked over and saw that Mara had fallen asleep. It had been a long day. I pulled the afghan off of the back of the couch and draped it on her. I put on my shoes and walked home.

1. See paper by Aharonov and Vaidman, cited below
2. The origin of this formula was an item of some debate, but Griffiths claimed essentially this on Nov. 6. He did not go through its derivation.
3. This was what Dr. Wu seemed to be arguing on Oct. 30.
4. This was the main point of Griffiths' criticism.

Griffiths cited the following references for his talk:

Aharonov and Vaidman. Phys Rev A 41 (1991) page 11. (Attached)
Kastner, R. qph/0207182
Aharonov, et. Al. Phys Letters 301 (2002) page 130. qph/0104062