

North Carolina State University
MA 141 Section 004 Final

Read all directions carefully. **A graphing calculator may NOT be used on this exam.** You must Show All Work for credit and clearly indicate your final answer; no work equals no credit. When you are finished fold your exam, write your name on the outside, turn it in, and then you may leave quietly. Good Luck!

Limits and Continuity

1) Evaluate the following limits

a) (5 pts) $\lim_{x \rightarrow 0} \frac{x^3 - 5x - 7}{2\pi x - 6}$

b) (5 pts) $\lim_{x \rightarrow \infty} \frac{x+1}{x^2+x+7}$

2) Consider the function $f(x) = \ln(x^2 - 4)$.

a) (3 pts) Given that $\ln(x)$ and $x^2 - 4$ are both continuous functions, why is $f(x)$ continuous?

b) (2 pt) Find the domain of $f(x)$.

c) (3 pts) Evaluate $\lim_{x \rightarrow 6} f(x)$.

Derivatives

3) Find the derivative, $f'(x)$, for the following functions.

a) (5 pts) $f(x) = \sin(x) + x^5 + e^x$

b) (5 pts) $f(x) = \cos(x)\ln(x)$

c) (5 pts) $f(x) = \tan(5x^2 + 7)$

d) (5 pts) $f(x) = \frac{\log_2(x)}{4^x}$

4) (5 pts) Implicitly differentiate the following to find $\frac{dy}{dx}$

$$1 + x = e^y.$$

Integrals

5) Evaluate the following integrals.

a) (5 pts) $\int_1^5 \frac{3}{x} dx$

b) (5 pts) $\int \ln(x) dx$

c) (5 pts) $\int x e^{x^2} dx$

d) (5 pts) $\int_0^1 x e^x dx$

6) (4 pts) Evaluate the following integral using the method of partial fractions

$$\int \frac{x+1}{x^2(x-1)} dx$$

7) (4 pts) To evaluate the following integral use a trig identity

$$\int \cos^3(x)\sin^8(x) dx$$

8) (4 pts) Use the table of integrals given to evaluate

$$\int (1 + \cos(2x))\sin(7x)dx$$

9) Use the Fundamental Theorem of Calculus and the properties of a definite integral to evaluate the following.

a) (3 pt) $\frac{d}{dx} \int_{-\pi}^x t^3 e^t dt$

b) (2 pt) $\frac{d}{dx} \int_{-\pi}^{\pi} x^3 e^x dx$

Applications of derivatives

10) Let $g(x) = \frac{x^3}{3} + 3x^2 + 9x + \frac{e}{2}$.

a) (4 pts) For what x-values does the tangent line to $g(x)$ have slope 0?

b) (3 pts) Find all intervals where $g(x)$ is increasing.

11) For the function $f(x) = x^{1/3}(x - 4)$:

a) (5 pts) find all critical points.

b) (3 pts) find the x-value that will *minimize* $f(x)$.

12) (5 pts) A hot air balloon rising straight up from a level field is tracked by a range finder 500ft from the lift-off point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of $0.14 \frac{rad}{min}$. How fast is the balloon rising at that moment?