North Carolina State University
MA 242 Section 007 Exam 4 Form 2

Read all directions carefully. **A graphing calculator may NOT be used on this exam.** You must Show All Work for credit and clearly indicate your final answer; no work equals no credit. When you are finished fold your exam, write your name on the outside, turn it in, and then you may leave quietly. Good Luck!

True/False
1) (3 pts) All vector fields are conservative.
2) (3 pts) $\vec{r}(u,v) =< 2\cos(u), 2\sin(u), v >$ is a parametrization of $x^2 + y^2 = 4$ in R^3.
3) (3 pts) $\vec{r}_u \times \vec{r}_v$ is a normal vector to the tangent plane of the parametrized surface $\vec{r}(u,v)$.

Computation
1) Given two vector fields $\vec{F} =< -ze^{-x}, 3y^2, e^{-x} >$ and $\vec{G} = ye^{x^2}i + e^x j + z^4 k$
a) (10 pts) Which vector field is conservative? Why?
b) (10 pts) For the conservative vector field find the potential function f.

2) For the function $f(x,y,z) = \ln(xyz)$
a) (5 pts) Compute $\nabla(f)$
b) (10 pts) Integrate the line integal $\int_C \nabla f \cdot d\vec{r}$ over C where C is the twisted cubic
$\vec{r}(t) = ti + t^2 j + t^3 k \quad 1 \leq t \leq 2$.

3) Let C be the part of the circle $x^2 + y^2 = 16$ where $x \geq 0$ and $y \leq 0$.
a) (5 pts) Parametrize C in the counter clockwise direction.
b) (12 pts) Compute the line integral with respect to arclength of $f(x,y) = y + xy$ over C.
c) (12 pts) Compute the line integral of $\vec{F}(x,y) = yi + xyj$ along C.

4) Let S be the part of the surface $z = 4 - y^2$ inside the cylinder $x^2 + y^2 = 4$.
a) (5 pts) Parametrize S.
b) (10 pts) Set up (**but Do Not evaluate**) the integral that will give the surface area of S.
c) (12 pts) For the vector field $\vec{F} = xj - zk$ set up (**but Do Not evaluate**) the integral that will give the flux of $\vec{F}$ across S (with upward orientation).

BONUS
(+5 pts) Show $\int_C (1 - ye^{-x})dx + e^{-x}dy$ is independent of path from $(0,1)$ to $(1,2)$.