

North Carolina State University  
MA 242 Section 008 Exam 3 Practice

This test is entirely integration. My recommended study procedure is study how to set up the integrals **first**. You will have to set up more integrals than you evaluate on the exam. Once you can set up all the integrals then practice the actual integration. You will need to know the following integration techniques; u-sub, by parts, and trig integrals for example  $\int \sin^3(x)dx$ ,  $\int \cos^2(x)dx$ .

1) Integrate

a)  $\int_1^4 \int_0^{\sqrt{x}} e^{y/\sqrt{x}} dy dx$

b)  $\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2} dx dy$  **remember** if an integral is hard or impossible try writing as an equivalent integral (you will have to change the bounds) with the order of integration switched!

2) Integrate  $\iint_D \ln(x^2 + y^2 + 1) dA$  where D is the region bounded by the unit circle  $x^2 + y^2 = 1$ .

3) Find the volume of the wedge cut from the cylinder  $x^2 + y^2 = 2$ , by the planes  $y = -z$  and  $z = 0$ .

4) A solid in the first octant is bounded by the planes  $y = 0$ ,  $z = 0$ , and the surfaces  $z = 4 - x^2$  and  $x = y^2$ . Find the mass of the solid if the density function is  $\delta(x, y, z) = kxy$  where k is some constant.

5) Find the volume of the upper region cut from the unit sphere  $x^2 + y^2 + z^2 \leq 1$  and the cone (in spherical coordinates)  $\phi = \pi/3$ .

6) Find the volume trapped between the surfaces  $z = 4 - 4(x^2 + y^2)$  and  $z = (x^2 + y^2)^2 - 1$ .

7) Find the volume of the solid cut from the thick walled cylinder  $1 \leq \sqrt{x^2 + y^2} \leq 2$  and the cones  $z = \sqrt{x^2 + y^2}$  and  $z = -\sqrt{x^2 + y^2}$ .

8) The bonus will involve a coordinate transformation that is not one of the standard three. Evaluate  $\int_0^3 \int_0^4 \int_{y/2}^{(y/2)+1} (\frac{2x-y}{2} + \frac{z}{3}) dx dy dz$  by using the coordinate transformation  $u = \frac{2x-y}{2}$ ,  $v = \frac{y}{2}$ , and  $w = \frac{z}{3}$ .