Exam 3 9:27 ①

a) False, $V = \iiint_E dV$

b) True

c) False

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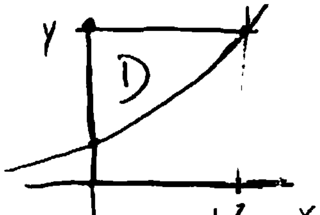
1) 

$0 \le x \le 2$

$e^x \le y \le e^2$

$$\boxed{\int_0^2 \int_{e^x}^{e^2} \arctan x \, dy\,dx}$$

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2) $D = \begin{cases} 0 \le x \le 1 \\ 0 \le y \le \sqrt{1-x^2} \end{cases}$

$0 \le r \le 1$

$0 \le \theta \le \pi/2$

$$\int_0^{\pi/2} \int_0^1 e^{-r^2} r\,dr\,d\theta = \int_0^{\pi/2} \frac{e^{-r^2}}{-2}\Big|_0^1 d\theta$$

$u = r^2$

$du = 2r\,dr$

$$= \int_0^{\pi/2} \left[\frac{e^{-1}}{-2} - \frac{e^0}{-2}\right] d\theta = \int_0^{\pi/2} \left[\frac{1}{2} - \frac{1}{2e}\right] d\theta$$

$$= \boxed{\left[\frac{1}{2} - \frac{1}{2e}\right] \pi/2}$$

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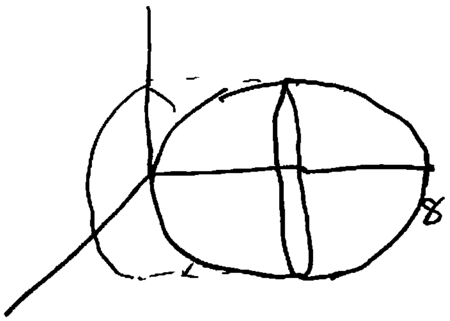
3) 

$D = \quad x^2 + z^2 = -x^2 - z^2 + 8$

$2(x^2+z^2) = 8$

$x^2+z^2 = 4$

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$r^2 \le y \le -r^2 + 8$

$$\boxed{\int_0^2\int_0^{2\pi}\int_{r^2}^{8-r^2} \underset{\uparrow \atop x}{r\cos\theta}\, y\, \underbrace{r\,dy\,d\theta\,dr}_{dV}}$$

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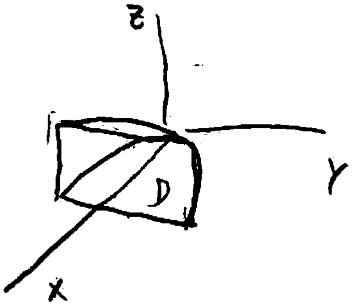
4) E



$D: \begin{cases} y^2 \le x \le 1 \\ -1 \le y \le 1 \end{cases} \leftarrow$ $\quad x = 1$, $x = y^2 \Rightarrow y = \pm 1$

$0 \le z \le x$

$$\text{Mass} = \int_{-1}^{1}\int_{y^2}^{1}\int_0^x x\,dz\,dx\,dy = \int_{-1}^{1}\int_{y^2}^{1} x^2\,dx\,dy$$

$$= \int_{-1}^{1} \frac{x^3}{3}\Big|_{y^2}^{1} dy = \int_{-1}^{1}\left[\frac{1}{3} - \frac{y^6}{3}\right]dy = \left(\frac{1}{3}y - \frac{y^7}{3\cdot 7}\right)\Big|_{-1}^{1}$$

$$= \left[\frac{1}{3} - \frac{1}{21}\right] - \left[-\frac{1}{3} + \frac{1}{21}\right] = 2\left[\frac{1}{3} - \frac{1}{21}\right]$$

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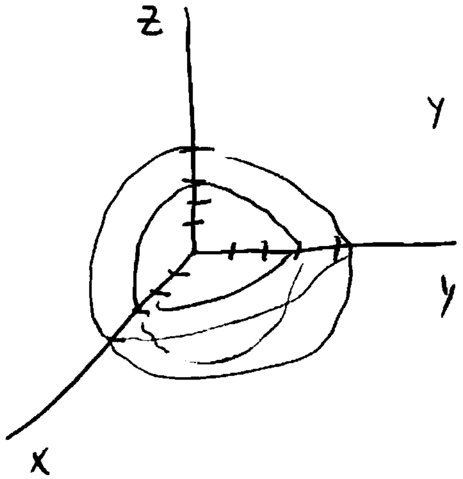
5)



$y \ge 0$

$3 \le \rho \le 4$

$0 \le \theta \le \pi$

$0 \le \varphi \le \pi$

$x^2 + y^2 = \rho^2\cos^2\theta\sin^2\varphi + \rho^2\sin^2\theta\sin^2\varphi$

$= \rho^2\sin^2\varphi$

$$\iiint_E x^2 + y^2\,dV = \boxed{\int_0^{\pi}\int_0^{\pi}\int_3^4 \rho^2\sin^2\varphi\;\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta}$$

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