Test 2 Form 2

T/F

1) T
2) T
3) F

---

1) normalize $\vec{A}$ $\quad \vec{u} = \frac{\vec{A}}{|\vec{A}|} = \frac{1}{\sqrt{5}}\langle -1, 2 \rangle$

$D_u f = \nabla f \cdot u$

$\nabla f = \left\langle \frac{1}{x^2+y^2}(2x), \frac{1}{x^2+y^2}(2y) \right\rangle$

$\nabla f(2,1) = \left\langle \frac{1}{5}(4), \frac{1}{5}(2) \right\rangle = \left\langle \frac{4}{5}, \frac{2}{5} \right\rangle$

$D_u f = \nabla f(2,1) \cdot u = \left\langle \frac{4}{5}, \frac{2}{5} \right\rangle \cdot \left\langle \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle = \frac{-4}{5\sqrt{5}} + \frac{4}{5\sqrt{5}} = 0$

---

2) $\frac{\partial z}{\partial y} = \frac{\partial z}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial y}$

$= e^{\theta}\left[(x+y)^2 + 2y(x+y)\right] + re^{\theta}\left[x \frac{1}{xy} x\right]$

---

3) $\frac{\partial f}{\partial x} = \frac{1}{2}\frac{y}{\sqrt{xy}}$, $\quad \frac{\partial f}{\partial y} = \frac{1}{2}\frac{x}{\sqrt{xy}}$

both $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are not continuous at $(0,0)$

so f is not differentiable at $(0,0)$

---

4) a) $\frac{\partial f}{\partial x} = 3x^2y + 24x$, $\quad \frac{\partial f}{\partial y} = x^3 - 8$

both continuous so critical pts occur when $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$

Solving

$$x^3 - 8 = 0 \quad \xrightarrow{\text{Plug into}} \quad 3x^2y + 24x = 0$$

$$x = 2 \qquad\qquad 3(4)y + 24(2) = 0$$

$$12y + 48 = 0$$

$$y = -4$$

Critical pt. at $(2,-4)$

b) $f_{xx} = 6xy + 24 \longrightarrow f_{xx}(2,-4) = 6(2)(-4) + 24 = -24$

$f_{yy} = 0$

$f_{xy} = 3x^2 \longrightarrow f_{xy}(2,-4) = 12$

$$D = f_{xx}f_{yy} - (f_{xy})^2 = 0 - 12^2 < 0$$

this shows $(2,-4)$ is a saddle pt.

---

5)
a) $$\left.\frac{xy}{x^2+y^2}\right|_{y=kx^2} = \frac{kx^3}{x^2+k^2x^4} = \frac{kx}{1+k^2x^2} \text{ for } x \neq 0$$

$$\lim_{x\to 0} \frac{kx}{1+k^2x^2} = \frac{0}{1} = 0$$

---

b) we cat say any thing else about it without additional work