

North Carolina State University
MA 242 Section 008 Exam 2 Form 1

Read all directions carefully. **A graphing calculator may NOT be used on this exam.** You must Show All Work for credit and clearly indicate your final answer; no work equals no credit. When you are finished fold your exam, write your name on the outside, turn it in, and then you may leave quietly. Good Luck!

- 1) For the function $f(x, y, z) = \frac{x}{y} - yz$
 - a) (12 pts) find the gradient of f at (4,1,1).
 - b) (12 pts) find the tangent plane to the level surface $4 = f(x, y, z)$ at (4,1,1).
 - c) (12 pts) find the directional derivative of f in the direction of $\vec{A} = \langle 2, -1, 2 \rangle$ at the point (4,1,1).
 - d) (12 pts) What direction(I want the vector \vec{u} =?) gives the greatest rate of change at (4,1,1)?
- 2) (15 pts) Given $z = e^r \cos(\theta)$ where $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(y/x)$ calculate $\frac{\partial z}{\partial x}$.
- 3) (12 pts) Is the function $f(x, y, z) = \sqrt{x + e^y} + z$ differentiable at (3,0,1)? You must justify your answer.
- 4) (10 pts) Suppose $f_x = \frac{1}{x} - g(y)$ and $f_y = \frac{-1}{y} + h(x)$. What can we say about any non-zero critical point of $z=f(x,y)$ (is it a local max, local min, saddle)?
- 5) (15 pts) Integrate the following integral over the region $R = \{(x, y) | 0 \leq x \leq 2, 0 \leq y \leq 1\}$.

$$\int \int_R \frac{xy}{x^2 + 1} dA$$

BONUS

(+5 pts) Show that every normal line to the sphere $x^2 + y^2 + z^2 = r^2$ passes through the center of the sphere. Hint: the normal line to a surface at a point is perpendicular to the tangent plane at that same point.