Asymptotic Capacity of Multi-User MIMO Communications

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Abstract

This poster introduces two theorems from Random Matrix Theory to derive capacity expressions of a class of multi-user communication schemes, among which single-user MIMO with multi-cell interference presented here. A Kronecker channel model is assumed between the base stations and the terminals. New asymptotic capacity formulas, independent of the instantaneous channel realization, are provided for single-user decoding and MMSE decoding at the terminal.

Introduction

- We consider a downlink scenario with multi-cell interference with
  - K base-stations, among which K − 1 called interferers
  - n_k, n_k > K antennas at the transmitter k and the receiver
  - Kronecker channel model H_k = R_k H_k, due to
  - limited distance between antennas
  - privileged directions of energy propagation

- Assuming large numbers of antennas, we study the asymptotic
  - single user (SU) decoding capacity, with uniform and optimal power allocation
  - MMSE decoder capacity

- Our objective is to
  provide deterministic equivalents to SU and MMSE capacities independently of the instantaneous channel realization

System Model

Figure: Single-user MIMO in multi-cell interference

\[ \mathbf{y} = \mathbf{H}_k \mathbf{s}_k + \sum_{j=2}^{K} \mathbf{H}_j \mathbf{s}_j + \mathbf{n} \]  

(1)

with \( \mathbf{H}_k = R_k^H \mathbf{H}_k \) ∈ \( \mathbb{C}^{n_k \times n_k} \), \( \mathbf{y} \) ∈ \( \mathbb{C}^{n_k} \) received signal by user 1, \( n_k \) ∈ \( \mathbb{C}^{n_k} \) AWGN, \( \mathbf{s}_k \) ∈ \( \mathbb{C}^{n_k} \), symbol of base station k.

Single-user and MMSE decoding

- (Per-antenna) single-user decoding capacity
  \[ C_{\text{SU}}(\sigma^2) = \frac{1}{n_k} \log_2 \det \mathbf{I}_{n_k} + \frac{1}{\sigma^2} \sum_{j=1}^{K} \mathbf{H}_j \mathbf{H}_j^H - \frac{1}{n_k} \log_2 \det \mathbf{R}_k + \frac{1}{\sigma^2} \sum_{j=1}^{K} \mathbf{H}_j \mathbf{H}_j^H \]  

(2)

- MMSE decoder capacity
  \[ C_{\text{MMSE}}(\sigma^2) = \frac{1}{n_k} \sum_{l=1}^{n_k} \log_2 (1 + \gamma_l) \]  

(3)

with

\[ \gamma_l = h_l^H \left( \sum_{j=1}^{K} \mathbf{H}_j \mathbf{H}_j^H - \mathbf{h}_l \mathbf{h}_l^H + \sigma^2 \mathbf{I}_{n_k} \right)^{-1} h_l \]  

(4)

\[ = \frac{\tau_l}{n_k} \text{tr} \left( \sum_{j=1}^{K} \mathbf{H}_j \mathbf{H}_j^H + \sigma^2 \mathbf{I}_{n_k} \right)^{-1} \]  

(5)

where \( h_l \in \mathbb{C}^{n_k} \) is the \( l \)-th column of \( \mathbf{H}_j \)

Theorem 1. Deterministic equivalent of the Stieltjes Transform

Let \( K, \ N \), \( \{ n_k \}_{k=1}^{K} \) ∈ \( \mathbb{N} \), \( c_k = n_k/N \), and let

\[ \mathbf{B}_N = \sum_{k=1}^{K} \mathbf{R}_k^H \mathbf{X}_k \mathbf{X}_k^H \mathbf{R}_k^H \in \mathbb{C}^{N \times N} \]  

(6)

\( \mathbf{T}_k \in \mathbb{C}^{n_k \times n_k} \), \( \mathbf{R}_k \in \mathbb{C}^{n_k \times \infty} \) nonnegative definite, \( \mathbf{X}_k \in \mathbb{C}^{n_k \times \infty} \) standard Gaussian. Denote, for \( z \in \mathbb{C} \setminus \mathbb{R}^+ \), \( m_k(z) = z^{-1} \text{tr}(\mathbf{B}_N - z \mathbf{I}_N)^{-1} \). Then, as \( n_k \to \infty, N \to \infty \) (\( c_k \) fixed),

\[ m_k(z) \to m^0_k(z) \]  

(7)

where

\[ m^0_k(z) = \frac{1}{N} \sum_{j=1}^{K} \frac{1}{z + \sigma^2 \mathbf{I}_{n_k}^{-1}} \left( \mathbf{R}_k - z \mathbf{I}_N \right)^{-1} \]  

(8)

and the \( \{ \mathbf{e}(z) \} \) form the unique solution to the equations

\[ \mathbf{e}(z) = \frac{1}{N} \text{tr} \left( \sum_{j=1}^{K} \frac{1}{z + \sigma^2 \mathbf{I}_{n_k}^{-1}} \left( \mathbf{R}_k - z \mathbf{I}_N \right)^{-1} \right) \]  

(9)

Theorem 2. Deterministic equivalent of the Shannon Transform

If \( \frac{1}{N} \text{tr}(\mathbf{T}_k) = \frac{1}{N} \text{tr}(\mathbf{R}_k) = 1 \), then \( \mathbb{V}(x) - \frac{1}{2} \lambda_0^2 = 0 \), where

\[ \mathbb{V}(x) = \frac{1}{N} \log \det \left( \mathbf{T}_k + z \mathbf{I}_N \right) \]  

(10)

and

\[ \mathbb{V}(x) = \frac{1}{N} \log \det \left( \mathbf{T}_k + z \mathbf{I}_N \right) \]  

(11)

Capacity with uniform/optimal power allocation

- Uniform power allocation
  - \( C_{\text{SU}}(\sigma^2) \) obtained directly from Theorem 2.
  - \( C_{\text{MMSE}}(\sigma^2) = \frac{1}{n_k} \log_2 \left( 1 + \sigma^2 \text{tr} \sum_{l=1}^{n_k} \mathbf{h}_l \mathbf{h}_l^H \right) \), with \( c_k \) defined in Theorem 1.

- Optimal power allocation for SU decoding
  \[ C_{\text{SU}}(\sigma^2) = \frac{1}{n_k} \log_2 \left( 1 + \sigma^2 \mathbb{E} \left[ \sum_{l=1}^{n_k} \mathbf{h}_l \mathbf{h}_l^H \right] \right) \]  

(12)

Optimal Strategy: align eigenvectors of \( \mathbf{P}_1 \) to \( \mathbf{T}_1 \) and \( (\rho_1) = \text{diag}(\mathbf{P}_1) \) as

\[ (\rho_1) = 0 \]  

(13)

\[ (\rho_1) = (1 - \alpha_1) \left( \frac{1}{n_k} \sum_{l=1}^{n_k} (1 - \alpha_l) \right)^{-1} \]  

otherwise

for \( \alpha_k \) and \( (1 - \alpha_k) \left( \frac{1}{n_k} \sum_{l=1}^{n_k} (1 - \alpha_l) \right)^{-1} \). for \( \alpha_k \)

Simulations

Figure: Capacity of point-to-point MIMO in two-cell downlink, single-user (left) and MMSE (right) decoding, \( n_k = 16, n_k \in \{8, 16\}, \) interferer strength 0.25.

Conclusion

- deterministic equivalent of the capacity of multi-user correlated MIMO systems: SU decoding with power allocation and MMSE decoder.
- asymptotic independence of channel realization.