Second Generation Growth Model with Trade

Oct 2009

Abstract

I take a new look at both long-run and short run effect of trade on economic growth purely through comparative advantate, without through the other channels, i.e. technology spillover, and scale effect. I propose a second generation growth model with asymmetric industries. The comparative advantage is decided endogeously, and the trade pattern could be either complete or incomplete specialization. Incomplete specialization could evolve into a complete specialization, or could stay incomplete forever; while a complete specialization is stable.

Trade will never reduce output levels at the moment of openning for both partners, whether it increases output depends on if it's full specialization or not; in long run, however, trade could either increase or decrease the balanced growth rate, depended on the type of good imported but not the type exported. And trade always guarentees a world balance growth, hense a stable world income distribution.

1 Introduction

There are varieties of theoretical models which try to link the cross-country differences in long run growth rate to international trade mainly through 3 ways: Enhanced R&D, i.e. Grossman and Helpman's (1990, 1991, 1995), Young(1991), Galor and Mountford (2006); Scale Effect, i.e. Rivera-Batiz and Romer(1991); Technology Transfer, i.e. Rivera-Batiz and Romer(1991), Howitt (2000), Peretto(2003). Although all these studies consider important channels through which trade may affect growth, most do not provide a strong link between comparative advantage itself and economy growth rates.

Seater(2007), Arabshahi(2008), and Yenokyan(2009) however provides a mechanism to study the effect of trade on economic growth purely through comparative advantage. They consider two-sector two-factor model introduced in Barro and Sala-I-Martin(2004, chapter 5), where two goods are produced by two types of reproducible factors and Cobb-Douglas production functions. One sector produces the good(Y) that could be used as consumption(C) or investment good (K) and the other sector produces another type of capital good (H) which augments labor. Seater(2007) extend this model to an open economy, and assumes both factors of production are tradeable. The resulting growth rates are those that would emerge from technology transfer, even though no technology transfer actually occurs, leading to a technology equalization theorem. Trade, which is guaranteed by comparative advantage, never reduces growth. If each country has an absolute advantage then trade raises the growth rates of both countries and balance growth path exists for individual country as well as the world. If one country has absolute advantage in both sectors, then its growth rate is unaffected and the other country's growth rate is increased but still lower. Trade therefore does not necessarily guarantee a stable world income distribution.

The main critique for Seater(2007) is the other type of capital (H), which is usually called "human capital" in traditional two sector model is not tradable. And that model is also a first generation growth model with scale effect hiding inside.

It would be nice if we set up a second-generation growth model to discuss the effect of trade purely through comparative advantage.

In order to do that, we need to find another tradeable factor that augments labor, but not human capital (H). Peretto(2007) presents a model in which technical progress (Z) augments labor but is embodied in intermediate goods (G). There are three sectors in Peretto's model– final good sector, intermediate good sector and R&D sector – together with a mixture of perfect and imperfect competition, endogenous entry and variety expansion. The main idea of this paper is to incorporate Peretto's setting of technology into a trade model, in order to show free trade could affect the growth rates of countries through the channel of comparative advantage. In addition although there is no technology spillover (transfer), by trading intermediate goods, which embody technology improvements, and also factors of final good, trade could affect both countries' growth rates as if countries exchanged technologies, even though no technology transfer (spillover) actually happens.

In order to talk about trade, the model needs to have two countries producing two kinds of intermediate goods (G_i) with different qualities (Z_i) inside to augment labor. Production technologies of G_i and Z_i differ across countries. Both intermediate goods and their qualities are essential for the production of final good. The factors of production are produced rather than simply endowed. The intermediate goods are tradeable, but the qualities embodied cannot be traded directly, unless together with the intermediate good.

These aspects of the analysis make the model as a dynamic Ricardian-type model. It's not a traditional Ricardian since the factors are produced not endowed. It's a Ricardian-type model, since like Ricardian model, there's only one factor – the resource from final good sector to produce tradeable goods – intermediates and their qualities. And the production technologies differ across countries.

The trade pattern is decided endogeously, and it could be either complete or incomplete specialization. Incomplete specialization has two possible equilibriums; while a complete specialization is stable.

Trade will never reduce output levels at the moment of openning for both partners, whether it increases output depends on if it's full specialization or not; in long run, however, trade could either increase or decrease the balanced growth rate, depended on the type of good imported but not the type exported. And trade always guarentees a world balance growth hense a stable world income distribution.

2 The model - closed economy

Let's see the closed economy first, which is a modified version of Peretto (1996, 1998, 1999, 2007).

Assume there are two asymmetric industries, Y_1 and Y_2 in the economy. These two industries are connected by a Cobb-Douglas production function into a final good, Y. The final good can be consumed, used to prodcue intermediate goods, invested in R&D that rises the quality of existing intermediate goods, or invested in the creation of new intermediate firms[1].

In each industry, one competitively Representative firm produces industry goods Y_i, $i = 1, 2$ (i.e. services) with a variety of intermediate goods G_{ij} and labor L_i. As long as he uses the intermediate good G_{ij}, he directly get the quality $Z_{ij}$of it. So Z_{ij}augments the bunch of labors who are using $G_{ij}$to produce the final good. Besides that, there is spillover from the other industry, $Z_{kj,k\neq i}$, which also augments labor L_i in industry i.

Competitive monopolistic firms produce intermediate goods G_{ij}(i.e. manufactures) with the resource from final goods. These firms undertake in-house research and development (R&D) to improve the qualities Z_{ij} of their products, in order to grab more market size demanded by the final good producer. Entrepreneurs can start a new firm by running R&D and developed a new

[1]But here I assume a zero cost free entry

products; the entry cost is assumed to be zero.

The growth is driven by the quality improvements of each intermediate good.

2.1 Final Good and Industry Goods

One Representative firm produces and sells a homogeneous final good, Y, in a competitive market. And the final good is produced by two industry goods:

$$Y = Y_1^{\epsilon} Y_2^{1-\epsilon} \tag{1}$$

where $Y_1$and $Y_2$are the industry goods, which are produced by intermediate goods and labor in each industry. Set the price of final good as numeraire, $P_Y = 1$.

One Representative firm in each industry produces its own industry production Y_i, $i = 1, 2$ and sells a homogeneous industry good, Y_i, in a competitive market to the final good producer. The production functions are defined as below:

$$Y_1 = \int_0^{N_1} G_{1j}^{\lambda} [Z_1^{\delta} Z_2^{1-\delta} l_{1j}]^{1-\lambda} dj, \quad 0 < \lambda, \delta < 1 \tag{2}$$

$$Y_2 = \int_0^{N_2} G_{2j}^{\lambda} [Z_2^{\delta} Z_1^{1-\delta} l_{2j}]^{1-\lambda} dj, \quad 0 < \lambda, \delta < 1 \tag{3}$$

where $N_{i,\,i=1,2}$ is the number of varieties of non-durable intermediate goods in each industry. These goods are vertically differential according to their quality, while general equilibrium requires them to be symmetric[2]. The productivity of l_{ij} amount of workers using G_{ij}unites of intermediate goods depends on the the average quality of this industry,$Z_i = \frac{1}{N_i}\int_0^{N_i} Z_{ij}dj$[3] and the average quality of the other industry $Z_{k,\,k\neq J} = \frac{1}{N_k}\int_0^{N_k} Z_{kj}dj$ (spillover). Notice that the quality Z_{ij} is embodied in G_{ij} but augments labor in final good production.

Combine the industry good fuctions, eq.(2) and eq.(3) into Cobb-Douglas structure, eq.(1), it's easy to see the final good producers pay compensation $\epsilon\lambda Y$ to intermediate producers of industry 1[4]; $(1-\epsilon)\lambda Y$ to intermediate producers of industry 2; $\epsilon(1-\lambda)Y$ to labors of industry 1 and $(1-\epsilon)(1-\lambda)Y$ to intermediate producers of industry 2. Sum them up we see the final good producer compensates λY and $(1-\lambda)$ to intermediate producers and labors respectively, i.e. $\int_0^{N_1}(G_1P_{G_1} + G_1P_{G_1})\,di = \lambda Y$,$w(N_1l_1 + N_2l_2) = (1-\lambda)Y$.[5]

Define $P_{G_{ij}}$ as the price of intermediate good j of industry i; and W as the wage. Based on value marginal benefit equals the marginal cost, the

[2]See Peretto(1998, 1999) for a discussion of the conditions under which it is reasonable to work with symmetric equilibria in models of this class. There conditions essentially reduce to the two requirements that: (a) the firm=specific return to quality innovation is decreasing in Z_i; (b) extrants enter at the average level of quality Z. The first implies that if one holds constant the mass of firms and starts the model from an asymmetric distribution of firm sizes, then the model converges to a symmetric distribution. The second requirement simply ensures that entrants do not perturb such symmetric distribution.

[3]which is also individual quality level due to symmetry

[4]Note that $P_Y \equiv 1$

[5]Note that quality $Z_i$dosent get paid directly from final good sector. It's the quality inside G_i. By increasing quality Z_i,we can see the demand for $G_i$increases from equation(4).

demand functions for intermediate goods are

$$G_{1i} = [\frac{\lambda\epsilon(\frac{Y_1}{Y_2})^{\epsilon-1}}{P_{G_1}}]^{\frac{1}{1-\lambda}} Z_1^{\delta} Z_2^{1-\delta} \frac{L_1}{N_1} \quad (4)$$

$$G_{2i} = [\frac{\lambda(1-\epsilon)(\frac{Y_1}{Y_2})^{\epsilon}}{P_{G_2}}]^{\frac{1}{1-\lambda}} Z_2^{\delta} Z_1^{1-\delta} \frac{L_2}{N_2} \quad (5)$$

where $\frac{Y_1}{Y_2} = \frac{\epsilon}{1-\epsilon}(\frac{P_{G_1}}{P_{G_2}})^{-\lambda}(\frac{Z_1}{Z_2})^{-(1-\delta)(1-\lambda)}$[6]. Later we will see $\frac{P_{G_1}}{P_{G_2}}$ is always constant, and $\frac{Z_1}{Z_2}$is constant on balance growth path.

Assume labors flow freely across industries, so the value marginal returns of labor must be the same, from which we get the allocation of labor[7].

$$\frac{L_1}{L_2} = \frac{\epsilon}{1-\epsilon} \quad (6)$$

2.2 Intermediate goods

Intermediate good producers of both industries behave non-cooperatively. The task of this section is to construct an equilibrium with free entry and free exit for the intermediate good sectors in both industries.

Assume it's asymmetric across industries, and symmetric inside industry by equilibrium. Across industries, the production, R&D, fixed operation cost and demand functions are different. And the coresponsible parameters are

[6] Detail derivation see Appendix (1)

[7] See Appendix (2) for detail.

Industry	Industry 1	Industry 2
Production func for intermediate good	$G_{1j} = A \bullet Y$	$G_{2j} = B \bullet Y$
R&D production	$\dot{Z}_{1j} = \alpha R_{1j}$	$\dot{Z}_{2j} = \beta R_{2j}$
fixed operating cost	$\theta_1 \frac{Z_1+Z_2}{2}$	$\theta_2 \frac{Z_1+Z_2}{2}$
entry/exit cost	0	0
Demand Func	eq. (4)	eq.(5)

In each industry, all firms face identical production, R&D production and demand function. I restrict the analysis to symmetric case in the same industry in order to simplify the analysis and focus on the properties of the model.

In each industry, I consider a Nash Equilibrium (NE) in open loop strategies. To simplify the analysis, I assume the entry and exit involve zero costs[8]. Thus the number of firms is free to jump to its equilibrium level. I construct an equilibrium where at time t firms commit to time-path strategies, while simultaneously free entry and exit determine the number of firms in the market. So among the same industry, there's only one decision point in time, where time-path of market structure and R&D decision are simultaneously determined, and the model is indeed a one-shot game in each industry.

I construct the equilibrium inside one industry in three steps. First, I focus on the determination of the strategies (price and investment in R&D)

[8] This is the assumption in Peretto(1996) and (1999). In his current papers, he usu sally assume there's a sunk cost for entry, which gives a richer transitional dynamic. However, this complicates the calculation a lot, and cannot generate a explicit solution for my two-industry model, hence cannot further the discussion about trade issue. So in this paper, I will focus on the simpler case, where the entry and exit costs are zero.

of the firms that are already active in the market (incumbent) in the same industry. Next, I focus on the free entry and exist decisions and the determination of the number of firms in the market. Finally, I combine two sets of results to describe the equilibrium of this industry.

It's similar to the case with only one industry in Peretto (1996), except that in this model with asymmetric industries, there is competition for assets between different industries. If the return of R&D in one industry is higher, then all investment will go to that industry, and the other industry will not have resource to do quality improvement.

In the following sections, please notice that r_i, which is the return in R&D in the i's industry is not necessarily equal to r, which is the return to assets. If they are equal, then no-arbitrage condition is satisfied, and both industries do R&D so the economy goes on balance growth path; if they are not equal, then all investments of R&D go to the industry with higher return. I will discuss it further in section (2.5)

2.2.1 Intermediate good producers in industry 1 – incumbent

Now let's discuss the intermediate good producers in industry 1 first. The case in industry 2 is similar to industry 1.

The typical intermediate firm produces its differential good with a technology that require A unit of final output per unit of intermediate good, and a fixed operating cost $\theta_1 \frac{Z_1+Z_2}{2}$, where $Z_1$and $Z_2$are the average quality level of industry 1 and 2 respectively, which are taken as given by individual firms.

The firm can invest units of final output to increase quality according to the technology

$$\dot{Z}_{1j} = \alpha R_{1j} \tag{7}$$

where $R_{1j}$is the firm's R&D investment in units of final output.

The firm's gross cash flow, which is revenues minus production costs is

$$F_{1j} = G_{1j}(P_{G_{1j}} - A) - \theta_1 \frac{Z_1 + Z_2}{2} \tag{8}$$

So the firm's profit is

$$\Pi_{1j} = F_{1j} - R_{1j} \tag{9}$$

the firm takes average quality $Z_1$and Z_2 as given.

The typical incumbent maximizes the present discounted value of net cash flow,

$$\begin{aligned} V_{1j}(t) &= \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} \Pi_{1j}\, d\tau \\ &= \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} [G_{1j}(P_{G_{1j}} - A) - \theta_1 \frac{Z_1 + Z_2}{2} - R_{1j}]\, d\tau \end{aligned} \tag{10}$$

where $V_{1j}(t)$ is the present discounted value of net cash flow for each firm in industry 1.

The firm chooses the time path of its product's price and R&D expenditure in order to maximize eq.(10) subject to the demand function (4) and

R&D production function 7. The firm take average quality, Z_1 and Z_2 as given. So the Current Value Hamiltonian is[9]

$$CVH_{1j} = G_{1j}(P_{G_{1j}} - A) - \theta_1 \frac{Z_1 + Z_2}{2} - R_{1j} + q_{1j}(\alpha R_{1j}) \tag{11}$$

where the co-state variable q_{1j},measures the value of the marginal unit of quality, and the state variable is Z_{1j}. The firm has power to set up its own optimal price $P_{G_{1j}}$and decide how much devoted into research, R_{1j}.

The transversality condition $lim_{t\to\infty} e^{-\int_t^{\tau} r(s)ds} q_{1j}(t) Z_{1j}(t) = 0$.

Taking the first order derivative subject to $P_{G_{1j}}$, the optimal price of this firm is:

$$P_{G_{1j}} = A \frac{-\lambda\epsilon + \lambda - 1}{-\lambda\epsilon} \tag{12}$$

where $\frac{-\lambda\epsilon+\lambda-1}{-\lambda\epsilon} > 1$[10], which means firms have monopolistic power to set a price higher than its unit cost.

The Hamiltonian is linear in R&D investment. The optimal investment policy is

$$\begin{cases} R_{1j} = \infty & \text{if } \frac{1}{\alpha} > q_{1j} \\ R_{1j} > 0 & \text{if } \frac{1}{\alpha} = q_{1j} \\ R_{1j} = 0 & \text{if } \frac{1}{\alpha} < q_{1j} \end{cases}$$

[9]See the whole derivation in Appendix (3)
[10]See Appendix (4.1) for proof

The former case violates the general equilibrium condition and is ruled out. The first order conditions for the interior solution $R_{1j} > 0$ are given by the equality between the marginal revenue from R&D (q_{1j} units of final good) and its marginal cost ($\frac{1}{\alpha}$ unit of final good). So for interior solution, we have

$$\frac{1}{\alpha} = q_{1j} \tag{13}$$

.

Differentiating the co-state variable q_{1j} will get

$$r_1 = \frac{\partial F_{1j}}{\partial Z_{1j}} \frac{1}{q_{1j}} + \frac{\dot{q_{1j}}}{q_{1j}} \tag{14}$$

which defines the rate of return to R&D as the ratio between revenue from one R&D project and the shadow price of the project, plus the change rate of the shadow price. Since $\frac{1}{\alpha} = q_{1j}$, the transversality condition is satisfied in the steady state with constant growth if quality does not grow at a rate higher than the interest rate. Combine equation (4),(19),(8) and (13)into (14), and symmetric equilibrium, we get the rate of return for quail innovation.

$$\begin{aligned} r_1 &= \frac{\partial F_{1j}}{\partial Z_{1j}} \\ &= [\delta - (1-2\delta)(\epsilon-1)]\alpha A \frac{1-\lambda}{\lambda\epsilon}[\frac{\lambda\epsilon}{A}\frac{-\lambda\epsilon}{-\lambda\epsilon+\lambda-1}]^{\frac{1}{1-\lambda}}(\frac{\epsilon}{1-\epsilon})^{\frac{\epsilon-1}{1-\lambda}}(\frac{P_{G_1}}{P_{G_2}})^{\frac{-\lambda(\epsilon-1)}{1-\lambda}}\frac{L_1}{N_1}(\frac{Z_{1j}}{Z_{2j}})^{[\delta-(1-2\delta)(\epsilon-1)]-1} \end{aligned} \tag{15}$$

where $\frac{L_1}{N_1}$is firm size of industry 1. $0 < [\delta - (1-2\delta)(\epsilon - 1)] < 1$[11], and it's the fraction of Z_{1j} in cash flow F_{1j}. $[\delta - (1-2\delta)(\epsilon-1)] - 1 < 0$ means the return of Z_{1j} is diminishing, and increasing in Z_{2j} can increase the return in Z_{1j} to eliminate the diminishing return. In the other word, increase of the ratio $\frac{Z_{1j}}{Z_{2j}}$ will also decrease the return to Z_{1j}.

A perfect-foresight, no-arbitrage condition requires that the rate of return to R&D r_1 must be equal to the cost of the R&D project financed by borrowing at the rate r (direct cost of R&D); and this must be equal to the return from a riskless loan at rate r of the resources required for the R&D project (opportunity cost of R&D). When no-arbitrage condition is violated, there will be a bang-bang situation. We will discuss it in detail in section(2.5).

We can also use another expression for r_1, which is $r_1 = \lambda \frac{G_{1j}(P_{1j}-A)}{Z_{1j}}$. This says the incentives to do incumbent innovation are driven by the incumbent cash flow (revenue minus variable costs). [12]

2.2.2 Intermediate good producers in industry 1 – entry and exit

The value of the firm V_{1j} defined by equation (10) and the optimal price and investment strategies, described by (12) and (13) give the value of incumbent. To determin the entry and exit of the firm, this value V_{1j} has to be compared with the cost of entry and exit. Assume the cost of entry and exit is zero[13],

[11] See the proof in Appendix(4.2)

[12] Note that the cash flow here is not gross cash flow which expressed by F_{1j}, in equation (8). The expression of $r_1$is irrelevant with fixed operating cost.

[13] This is a strong assumption and requires that workers hired away from the other firms can blend together in a new firm at zero cost. Costly entry has been analyzed by other

firms entry when $V_{1j} > 0$; firms exit for $V_{1j} < 0$. Since the entry and exit cost is zero, the number of firms N_1 is a jumping variable, and, at all the time, free entry and exit makes $V_{1j} = 0$.

Differentiating with respect to time of Eq.(10), get the firm's rate of return to equity is

$$r = \frac{\Pi_{1j}}{V_{1j}} + \frac{\dot{V}_{1j}}{V_{1j}} \tag{16}$$

Where V_{1j} is the price of firm $1j$'s shares. A perfect-forsight, no-arbitrage condition for the equilibrium of the capital market requires the return to firm ownership be equal to the rate of return to a riskless loan (r) of size V_{1j}. The return to firm ownership is given by the ratio between profit $(\prod_{1j})$ and the firm's stock market value (V_{1j}), plus the capital gain (loss) from the stock appreciation (depreciation).

Eq. (16) can also be written as $rV_{1j} = \prod_{1j} + \dot{V}_{1j}$. Since zero cost entry/exit makes $V_{1j} = 0$ all the time, it means for all value of interest rate r, the LHS is 0; and the RHS should also be zero. This implies z*ero profit condition*, $\Pi_{1j} = 0$. Since $\Pi_{1j} = F_{1j} - R_{1j}$, the level of R&D expenditure can be obtain by

$$R_{1j} = F_{1j} \tag{17}$$

papers of Peretto. I tried to combined a costly entry in this model too, but it complicates the model a lot without more insides. So I focus on zero entry condition here.

So the growth rate of quality innovation should be

$$\begin{aligned}\frac{\dot{Z}_{1j}}{Z_{1j}} &= \frac{R_{1j}}{Z_{1j}} = \frac{F_{1j}}{Z_{1j}} \\ &= \alpha\{A\frac{1-\lambda}{\lambda\epsilon}[\frac{\lambda\epsilon}{A}\frac{-\lambda\epsilon}{-\lambda\epsilon+\lambda-1}]^{\frac{1}{1-\lambda}}(\frac{\epsilon}{1-\epsilon})^{\frac{\epsilon-1}{1-\lambda}}(\frac{P_{G_1}}{P_{G_2}})^{\frac{-\lambda(\epsilon-1)}{1-\lambda}}\frac{L_1}{N_1}(\frac{Z_{1j}}{Z_{2j}})^{[\delta-(1-2\delta)(\epsilon-1)]-1} - \theta_1\frac{1}{2}(1+\frac{Z_{2j}}{Z_{1j}})\}\end{aligned} \tag{18}$$

Please notice that the entry stops as long as the profit becomes zero. In this model, entry is not the engine for long run growth, which is one of the key difference from variety expansion model.

2.2.3 Intermediate good producers in industry 2

Following the same steps as in industry 1, we get the solutions for industry 2.

The optimal price of this firm is:

$$P_{G_{2j}} = B\frac{\lambda\epsilon-1}{\lambda(\epsilon-1)} \tag{19}$$

The return to R&D in industry 2 is:

$$\begin{aligned}r_2 &= \frac{\partial F_{2j}}{\partial Z_{2j}} \\ &= [\delta+(1-2\delta)\epsilon]\beta B\frac{1-\lambda}{\lambda(1-\epsilon)}[\frac{\lambda(1-\epsilon)}{B}\frac{1-\epsilon}{1-\lambda\epsilon}]^{\frac{1}{1-\lambda}}(\frac{\epsilon}{1-\epsilon})^{\frac{\epsilon}{1-\lambda}}(\frac{P_{G_1}}{P_{G_2}})^{\frac{-\lambda\epsilon}{1-\lambda}}\frac{L_2}{N_2}(\frac{Z_{2j}}{Z_{1j}})^{[\delta+(1-2\delta)\epsilon]-1}\end{aligned} \tag{20}$$

where $\frac{L_2}{N_2}$is firm size of industry 2. $0 < [\delta+(1-2\delta)\epsilon] < 1$, and it's the fraction of Z_{2j}in cash flow F_{2j}. $[\delta+(1-2\delta)\epsilon]-1 < 0$ means the return of

$Z_{2j}$is diminishing, which could be eliminated by the accumulation of Z_{1j}.

By the free entry assumption, profit is driven to zero through entry, so the growth rate of quality innovation should be

$$\begin{aligned}\frac{\dot{Z}_{2j}}{Z_{2j}} &= \frac{R_{2j}}{Z_{2j}} = \frac{F_{2j}}{Z_{2j}} \\ &= \beta\{B\frac{1-\lambda}{\lambda(1-\epsilon)}[\frac{\lambda(1-\epsilon)}{B}\frac{1-\epsilon}{1-\lambda\epsilon}]^{\frac{1}{1-\lambda}}(\frac{\epsilon}{1-\epsilon})^{\frac{\epsilon}{1-\lambda}}(\frac{P_{G_1}}{P_{G_2}})^{\frac{-\lambda\epsilon}{1-\lambda}}\frac{L_2}{N_2}(\frac{Z_{2j}}{Z_{1j}})^{[\delta+(1-2\delta)\epsilon]-1} - \theta_2\frac{1}{2}(1+\frac{Z_{1j}}{Z_{2j}})\}\end{aligned} \tag{21}$$

2.3 Households

The economy is populated by Representative households who supply labor inelastically in perfect competitive market, and purchase assets (corporate equity). Assume there's no population growth. The utility function of the Representative household is

$$U(t) = \int_t^\infty log(c)\, e^{-\rho t} \tag{22}$$

where $c = \frac{C}{L}$. c is the consumption per capita, C is the aggregate consumption for the economy, and ρ is the individual time preference discount rate.

The households faces the flow budget constraint

$$\dot{S} = rS + wL - C \tag{23}$$

where S is assets holding and r is the rate of return on assets. The intertemporal consumption and saving plan that maximize discounted utility (22) is given by Euler equation

$$r = \rho + \frac{\dot{C}}{C} \tag{24}$$

2.4 General Equilibrium

In this section, I am going to construct the general equilibrium of the economy. By imposing symmetry of the intermediate good firms in each sector, combined the demand functions of intermediate goods into the final good production to eliminate G_i, we get

$$Y = \kappa Z_1^{1-\epsilon} Z_2^{\epsilon} L, \tag{25}$$

where $\kappa = \lambda^{\frac{2\lambda}{1-\lambda}}(1-\epsilon)^{\frac{1}{1-\lambda}}\epsilon^{\frac{\lambda}{1-\lambda}}(\frac{\epsilon}{1-\epsilon})^{\frac{2\lambda\epsilon-\lambda+\epsilon}{1-\lambda}}P_{G_1}^{\frac{-\lambda\epsilon}{1-\lambda}}P_{G_2}^{\frac{-\lambda(1-\epsilon)}{1-\lambda}}$

This is essentially a two-factor one-sector model, which generates a balance growth path, where the growth rate of output equals the growth rates of both qualities.

From household budget constraint (23)combing with zero entry and exit, $V_i = 0$, we get $wL = C$. We have already known $wL = (1-\lambda)Y$ from the Cobb-Douglas final good production function, so the ratio between consumption and output is constant like a Slow model, i.e. $\frac{C}{Y} = 1-\lambda$. So on balanced growth path, we have the growth rates all equal as

$$g^\star = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = \frac{\dot{Z}_1}{Z_1} = \frac{\dot{Z}_2}{Z_2} = \frac{\dot{w}}{w} \tag{26}$$

We can easily get the quality ratio value on balanced growth path, $(\frac{Z_1}{Z_2})^\star$ from $\frac{\dot{Z}_1}{Z_1} = \frac{\dot{Z}_2}{Z_2}$, $r_1 = r_2$ and Euler Equation, which are Eq(18) = Eq(21); Eq(15) = Eq(20); and Euler Eq (24).

$$\mathbb{Q}(\frac{Z_1}{Z_2})^2 + \mathbb{R}(\frac{Z_1}{Z_2}) + \mathbb{H} = 0 \tag{27}$$

where $\mathbb{Q} = -\frac{1}{2}\frac{1-\Gamma}{\Gamma}\beta\theta_2 < 0$; $\mathbb{R} = \frac{\Gamma}{1-\Gamma}\frac{1}{2}\alpha\theta_1 - \frac{1-\Gamma}{\Gamma}\frac{1}{2}\beta\theta_2 - \frac{\Gamma}{1-\Gamma}\rho + \frac{1-\Gamma}{\Gamma}\rho$; $\mathbb{H} = \frac{\Gamma}{1-\Gamma}\frac{1}{2}\alpha\theta_1$; $\Gamma = \delta - (1-2\delta)(\epsilon - 1) \in [0,1]$

The solutions for the quadratic function are:

$$\begin{aligned} (\frac{Z_1}{Z_2})^\star &= \frac{-\mathbb{R} - \sqrt{\mathbb{R}^2 - 4\mathbb{Q}\mathbb{H}}}{2\mathbb{Q}} > 0 \\ (\frac{Z_1}{Z_2})^\star &= \frac{-\mathbb{R} - \sqrt{\mathbb{R}^2 - 4\mathbb{Q}\mathbb{H}}}{2\mathbb{Q}} < 0 \end{aligned}$$

And we cancel out the negative solution to get the growth rate on balanced growth path is:

$$g^\star = \frac{\Gamma}{1-\Gamma}\frac{1}{2}\alpha\theta_1[1 + [(\frac{Z_1}{Z_2})^\star]^{-1}] - \frac{1}{1-\Gamma}\rho = \frac{1-\Gamma}{\Gamma}\frac{1}{2}\beta\theta_2[1 + (\frac{Z_1}{Z_2})^\star] - \frac{1}{\Gamma}\rho \tag{28}$$

$$where \qquad \Gamma = \delta - (1-2\delta)(\epsilon - 1) \in [0,1] \tag{29}$$

The growth rate positively relates with the R&D productivities, α, β;

and the fixed operating cost parameters, θ_1 and θ_2. The higher productivity of R&D, the higher return to R&D, and the higher incentive to accumulate more qualities to drive up the growth rate. The higher fixed operating cost, the lower profit for incumbents and less amounts of firms in the market which drives up the market size for each incumbent. From Eq(15) and (20) we see that the larger market size, $\frac{L_i}{N_i}$ is, the higher return in R&D, while others fixed. So the growth rate is positively related with fixed operating cost. Define $\alpha\theta_1$ and $\beta\theta_2$ as R&D ability for industry 1 and 2 respectively, so the growth rate is positively related with R&D abilities of both industries.

The growth rate is unrelated with unit costs, A and B at all. Any changes in the unit costs cause the changes in profits, hens the market share for each incumbent. There two effects are in opposite directions and cancel out with each other, so the unit costs do not affect the growth rate. Later we will see this is the reason why trade doesn't guarantee a higher growth rate. The trade pattern is determined by the quality-adjusted price ratio which includes the unit cost, but the growth rate only depends on R&D ability but not unit costs. So a country could come up with importing a good with a very low unit cost, but also low in R&D ability, which decreases the growth rate of this country. We will see the detail in section(3).

2.5 Dynamic System

The model is essentially a two-factor one-sector model and similar to Barro and Sala-i-Martin(2003, section 5.1). Both qualities are irreversible. While the return to $Z_1$is higher than that of Z_2, all resource goes to accumulate Z_1. Since there's diminishing return to each quality, so the return to Z_1 keeps decreasing until equals to the return to Z_2, then both qualities grow together at the same rate. Similar if the return of $Z_2$is higher.

From the return to Z_1, Eq (15); growth rate of Z_1, Eq(18) and Euler Equation, we see the growth rate of Z_1only depends on the ratio of qualities, $\frac{Z_1}{Z_2}$. Same thing happens to the growth rate of Z_2. On balance growth path, growth rate of both qualities are the same, and the change of the quality ratio is zero. While one return is higher, only that quality is accumulated to change the quality ratio back to the steady state. And the expression is as follow:

$$\frac{(Z_1 \dot{/} Z_2)}{Z_1/Z_2} = \frac{\dot{Z}_1}{Z_1} - \frac{\dot{Z}_2}{Z_2} = \begin{cases} \frac{\dot{Z}_1}{Z_1} = \frac{\Gamma}{1-\Gamma}\frac{1}{2}\alpha\theta_1[1+(\widetilde{\frac{Z_1}{Z_2}})^{-1}] - \frac{1}{1-\Gamma}\rho & if \ \frac{Z_1}{Z_2} < (\frac{Z_1}{Z_2})^{\star}, r_1 > r_2 \\ 0 & if \ \frac{Z_1}{Z_2} = (\frac{Z_1}{Z_2})^{\star}, r_1 = r_2 \\ -\frac{\dot{Z}_2}{Z_2} = \frac{1-\Gamma}{\Gamma}\frac{1}{2}\beta\theta_2(1+\widetilde{\frac{Z_1}{Z_2}}) - \frac{1}{\Gamma}\rho & if \ \frac{Z_1}{Z_2} > (\frac{Z_1}{Z_2})^{\star}, r_1 < r_2 \end{cases} \quad (30)$$

where $\Gamma = \delta - (1-2\delta)(\epsilon - 1) \in [0,1]$

3 The model - Open Economy

We now introduce international trade. There're two countries, home country and foreign country. They have the same production functions for final good and industry goods, and also the same utility. They are producing the same type of intermediate goods, but have different abilities in unit costs, R&D productivities and fixed operating costs of the intermediate goods. Assume only trade in intermediate goods, So the consumers will choose the goods with a lower quality-adjusted price.

Assume both countries are large economies. And no international capital flow, no direct technology transfer across borders. The only way to get the quality of the intermediate good to augment labor, is to obtain that good.

3.1 Trade Patterns

While home country is open, it has more options for the intermediate goods. It could choose produce them itself, or import them from foreign countries. So the production function for industry good 1 becomes:

$$\begin{aligned} Y_{H1} &= \int_0^{N_{H1}} (G_{H1j} - X_{H1j})^{\lambda} [Z_{H1}^{\delta} (\overbrace{Z_{H2}})^{1-\delta} l_{H1H}]^{1-\lambda}\, dj]^{1-\lambda} \\ &\quad \int_0^{N_{F1}} (M_{F1j})^{\lambda} [Z_{F1}^{\delta} (\overbrace{Z_{H2}})^{1-\delta} l_{H1F}]^{1-\lambda}\, dj \end{aligned}$$

where Y_{H1} is the output of industry good 1 in home country; G_{H1j} is the quantity each firm produces in industry 1 of home. X_{H1j} is the amount of export per firm from industry 1 in home. So $(G_{H1j} - X_{H1j})$ is the amount of intermediate good 1 that home is keeping for itself. M_{F1j} is the amount of intermediate good 1 that home imports from foreign country. Z_{H1} is the quality of domestic intermediate good 1 and Z_{F1} is the quality of foreign intermediate good 1. And $\overbrace{Z_{H2}}$ is some combination of qualities of intermediate good 2 that home country gets, which depends on whether home country is using domestic or foreign intermediate good 2, or both of them. If home uses domestic production, then it will only get the quality of domestic good Z_{H1} to augment the labors who are using this good, l_{H1H}. Home could choose intermediate good 1 from domestic firms, and foreign firms. Which one it chooses depends on which good provides a higher marginal product to labor.

$$\begin{aligned} MPl_{H1H} &= \Upsilon(\frac{Z_{H1}^{\frac{\delta(1-\lambda)}{\lambda}}}{P_{G_{H1}}})^{\frac{\lambda}{1-\lambda}}; \\ MPl_{H1F} &= \Upsilon(\frac{Z_{F1}^{\frac{\delta(1-\lambda)}{\lambda}}}{P_{G_{F1}}})^{\frac{\lambda}{1-\lambda}} \end{aligned}$$

where $\Upsilon = (1-\epsilon)Y_{H1}^{\epsilon}Y_{H2}^{1-\epsilon}(1-\lambda)[\lambda(1-\epsilon)(\frac{Y_{H1}}{Y_{H2}})^{\epsilon}]^{\frac{\lambda}{1-\lambda}}(\overbrace{Z_{H2}})^{1-\delta}$; MPl_{H1H} is the marginal product of labor who uses domestic intermediate good 1; MPl_{H1F} is the marginal product of labor who uses foreign good. Which product provides a higher marginal product to the labors depends on which one has a lower quality adjusted price, or a higher quality per dollar. At the moment of opening,

- $If\ \frac{P_{G_{H1}}}{Z_{H1}^{\frac{\delta(1-\lambda)}{\lambda}}} < \frac{P_{G_{F1}}}{Z_{F2}^{\frac{\delta(1-\lambda)}{\lambda}}}, then\ MPl_{H1H} > MPl_{H1F}$; In this case, the quality adjusted price of foreign good is too high. So home country produces industry 1 by itself. Whether it exports depends on the demand from foreign country.

- $If \frac{P_{G_{H1}}}{Z_{H1}^{\frac{\delta(1-\lambda)}{\lambda}}} = \frac{P_{G_{F1}}}{Z_{F2}^{\frac{\delta(1-\lambda)}{\lambda}}}, then\, MPl_{H1H} = MPl_{H1F}$; in this case, home country is indifferent with both goods.

- $If \frac{P_{G_{H1}}}{Z_{H1}^{\frac{\delta(1-\lambda)}{\lambda}}} > \frac{P_{G_{F1}}}{Z_{F2}^{\frac{\delta(1-\lambda)}{\lambda}}}, then\, MPl_{H1H} < MPl_{H1F}$; in this case, the quality adjusted price of domestic good is too high. home country only imports industry 1, but not produces by itself.

International trade happens if each country has a lower (or equal) quality-adjusted price at one intermediate good, i.e.

$$\mathbf{\frac{P_{G_{H1}}}{Z_{H1}^{\frac{\delta(1-\lambda)}{\lambda}}} \leq \frac{P_{G_{F1}}}{Z_{F1}^{\frac{\delta(1-\lambda)}{\lambda}}}; and \frac{P_{G_{H2}}}{Z_{H2}^{\frac{\delta(1-\lambda)}{\lambda}}} \geq \frac{P_{G_{F2}}}{Z_{F2}^{\frac{\delta(1-\lambda)}{\lambda}}}} \tag{31}$$

Home specializes in intermediate good 1, and foreign country specializes in good 2. It's full specialization while the strict inequality applies. If the ranking switches, then then trade pattern switches. Here we focus on the case while home specializes in intermediate good 1, and foreign country specializes in good 2.

Set the price of final good in home country as numeraire, $P_Y \equiv 1$; then the price of final good in foreign country is P_{Y_F}. Recall that price of intermediate good equals to monopolistic markup times unit cost, Above equation is equivalent to

$$\frac{B_H}{B_F}(\frac{Z_{F2}}{Z_{H2}})^{\frac{\delta(1-\lambda)}{\lambda}} \geq P_{Y_F} \geq \frac{A_H}{A_F}(\frac{Z_{F1}}{Z_{H1}})^{\frac{\delta(1-\lambda)}{\lambda}} \tag{32}$$

This is a typical trade condition for Ricardian-type model. In basic static Ricardian model, labor is the only production factor for trading goods, and the relative wage across countries need to be inside a interval of productivity ratios. In this model, what we trade is intermediate goods, which only uses final good as production factors. So the factor price ratio (final good price ratio) needs to be inside the interval of productivity ratios. The difference is the productivity ratios in this model not only include unit cost ratios, but also quality ratios, and the qualities are state variables and keep changing over time. While complete specialization, where strict inequality applies, the change in qualities only enhance the trade pattern; however, while incomplete specialization, where P_{Y_F} hits the conner, the change in qualities could possibly change the incomplete specialization into complete specialization. We will discuss this in section (3.2) and section (3.3).

According to the ranking in Eq (32), we can arrange it into a typical version to show comparative advantage:

$$(\frac{A_H}{Z_{H1}^{\frac{\delta(1-\lambda)}{\lambda}}})/(\frac{B_H}{Z_{H2}^{\frac{\delta(1-\lambda)}{\lambda}}}) \leq (\frac{A_F}{Z_{F1}^{\frac{\delta(1-\lambda)}{\lambda}}})/(\frac{B_F}{Z_{F2}^{\frac{\delta(1-\lambda)}{\lambda}}}) \tag{33}$$

Home country has lower quality-adjusted price ratio (intermediate good 1 over good 2), so home specializes in intermediate good 1. Whether it's complete specialization or not depends on if the final good price ratio is inside the interval or hits the conner in Eq(32).

3.2 Results under complete specialization

When $\frac{B_H}{B_F}(\frac{Z_{F2}}{Z_{H2}})^{\frac{\delta(1-\lambda)}{\lambda}} > P_{Y_F} > \frac{A_H}{A_F}(\frac{Z_{F1}}{Z_{H1}})^{\frac{\delta(1-\lambda)}{\lambda}}$, home country has a strictly lower quality-adjusted price in intermediate good 1, and foreign country has a strictly lower price in good 2. So both of them completely specialize and import the good with a lower quality-adjusted price from each other. In this case, for home, the value of imports (the whole bunch of intermediate good 2 from foreign country) equals the value of exports (the whole bunch of intermediate good 1), which means $1 \cdot Y_H(1-\epsilon)\lambda = P_{Y_F} \cdot Y_F\epsilon\lambda$, according to the property of Cobb-Douglas function. it's easy to get $P_{Y_F} = \frac{(1-\epsilon)L_H}{\epsilon L_F}$, which indicates the relative value of all intermediate goods imported over the value of those exported. So the condition for complete specialization is $\frac{B_H}{B_F}(\frac{Z_{F2}}{Z_{H2}})^{\frac{\delta(1-\lambda)}{\lambda}} > \frac{(1-\epsilon)L_H}{\epsilon L_F} > \frac{A_H}{A_F}(\frac{Z_{F1}}{Z_{H1}})^{\frac{\delta(1-\lambda)}{\lambda}}$. This means the relative population size must be inside a certain interval, which depends on the initial quality ratio at the moment of opening to trade, and the unit costs. If the relative population size is too high or too low, then it's impossible for complete specialization. We will discuss the incomplete specialization in the next section.

Now we discuss under complete specialization, the level and growth effect of trade on economies. After completely specializes, home abandons industry 2, and foreign country abandons industry 1, which makes Z_{H2} and Z_{F1} stop growing. This enhances the trade pattern, according to $\frac{B_H}{B_F}(\frac{Z_{F2}}{Z_{H2}})^{\frac{\delta(1-\lambda)}{\lambda}} > \frac{(1-\epsilon)L_H}{\epsilon L_F} > \frac{A_H}{A_F}(\frac{Z_{F1}}{Z_{H1}})^{\frac{\delta(1-\lambda)}{\lambda}}$. So as long as the the world economy starts complete specialization, it lasts forever.

Complete specialization increases output levels at the moment of opening for both trade partners. Eq (25) shows the output level is negatively related with quality-adjusted prices, i.e. $Y = cons \cdot [(\frac{P_{G_1}}{Z_1^{\frac{\delta(1-\lambda)}{\lambda}}})^{\frac{-\lambda\epsilon}{1-\lambda}}(\frac{P_{G_2}}{Z_2^{\frac{\delta(1-\lambda)}{\lambda}}})^{-\frac{\lambda(1-\epsilon)}{1-\lambda}}]^{\frac{1}{\delta}}$. According to comparative advantage, each country imports the good with lower quality-adjusted price from each other, such that increases the output level at the moment of opening to trade.

The growth effect depends on the R&D ability of the imports. Recall that the growth rate is positively related with R&D abilities, $\alpha\theta_1$ and $\beta\theta_2$. After complete specialization, home abandons the whole bunch of domestic intermediate good 2 and imports them from foreign country. This means it gives up the domestic R&D abilities, $\beta_H\theta_{H2}$, and gets that from foreign country, $\beta_F\theta_{F2}$. The effect of trade on a home country's growth rate is the same as if that country had learned the R&D technology used by its trading partner to produce the good that the home country imports. However, the comparative advantage, which is decided by quality-adjusted price ratio, doesn't guarantee home imports the good with higher R&D ability. The trade pattern condition is only decided by unit costs and the initial value of qualities, which are unrelated with growth rate. It is possible that home imports the good with a lower quality-adjusted price at the moment of opening, but also with a lower R&D ability. This situation increases output level at the moment of trade, but decreases growth rate, which means a decrease in the future output.

One thing needs to be clear here. A country might come up to import a good with worse quality than domestic product, but also with a much lower unit cost, so the quality-adjusted price still beats domestic product. However, it doesn't necessarily decrease balanced growth rate, because the "worse quality" is only the initial quality level at the moment of opening. As long as the country is importing a good with a higher R&D ability, it gets a higher balanced growth rate. And the R&D ability only depends on R&D productivity and fixed operating cost coefficient, but not the initial level of quality.

We need to notice that the growth rate effect doesn't depend on technology spillover. A country cannot learn the technology from each other even with trade. How to produce and improve the quality of an intermediate good is a "secrete" that protected by each trade partner. A country can only get the "quality" but not "how-to-do" while importing a good. However, growth rate is affected by trade in the same way as by learning the trading partner's R&D technology for producing the good imported from the trading partner.

Scale effect doesn't play a role in growth rate either. Any increase in the market size faced by individual firms will increase profit and induce a zero-cost entry instantaneously to drive profit back to zero. For example, in Eq (14), when home opens to trade, L_{H1}increases from from ϵL_H to L_H, and zero-cost entry causes N_{H1} to increase proportionally to make the firm size unchanged, and so does the effect on the return to R&D.

The dynamic system of open economy is driven by the interaction of the qualities of two trading goods. It's equivalent to an intergrated economy with intermediate good 1 totally from home, and intermediate good 2 totally from foreign. Assume no international investment, both qualities are accumulated even their returns are different. Since these two qualites are complements, on balance growth path, their returns are driven to be equal even without international investment. So it generates a stable world income distribution.

Similar to the closed model, the difference on the growth rate of qualities of the world is expressed as:

$$\frac{(Z_{H1}\dot{/}Z_{F2})}{Z_{H1}/Z_{F2}} = \frac{\dot{Z_{H1}}}{Z_{H1}} - \frac{\dot{Z_{F2}}}{Z_{F2}} = \tilde{\mathbb{Q}}(\frac{Z_1}{Z_2})^2 + \tilde{\mathbb{R}}(\frac{Z_1}{Z_2}) + \tilde{\mathbb{H}} \tag{34}$$

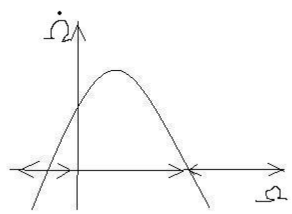
where $\tilde{\mathbb{Q}} = -\frac{1}{2}\frac{1-\Gamma}{\Gamma}\beta_F\theta_{F2} < 0; \tilde{\mathbb{R}} = \frac{\Gamma}{1-\Gamma}\frac{1}{2}\alpha_H\theta_{H1} - \frac{1-\Gamma}{\Gamma}\frac{1}{2}\beta_F\theta_{F2} - \frac{\Gamma}{1-\Gamma}\rho + \frac{1-\Gamma}{\Gamma}\rho; \tilde{\mathbb{H}} = \frac{\Gamma}{1-\Gamma}\frac{1}{2}\alpha_H\theta_{H1}$

The difference between above equation and Eq (27) shows that, trading in goods is equivalent with trading in R&D abilities. The coefficients now are combinations of $\alpha_H\theta_{H1}$ and $\beta_F\theta_{F2}$, instead of only from domestic parameters under autarky.

Since there's no international investment, so each trade partner accumulate their own qualities in different returns during transition, until the returns equalize on balance growth path. The diagram is in Figure 1.

$$\begin{aligned}(\frac{Z_{H1}}{Z_{F2}})^\star &= \frac{-\tilde{R} - \sqrt{\tilde{\mathbb{R}}^2 - 4\tilde{\mathbb{Q}}\tilde{\mathbb{H}}}}{2\tilde{\mathbb{Q}}} > 0 \\ (\frac{Z_{H1}}{Z_{F2}})^\star &= \frac{-\tilde{\mathbb{R}} + \sqrt{\tilde{\mathbb{R}}^2 - 4\tilde{\mathbb{Q}}\tilde{H}}}{2\tilde{\mathbb{Q}}} < 0\end{aligned}$$

Figure 1: Dynamic System for Open Economy under Complete Specialization



$\Omega \equiv \frac{Z_{H1}}{Z_{F2}}$

The second steady state is negative which is meaningless for quality ratio, and it's also unstable. So the only positive and stable steady state for world quality ratio is $\frac{-\tilde{R}-\sqrt{\tilde{\mathbb{R}}^2-4\tilde{\mathbb{Q}}\tilde{\mathbb{H}}}}{2\tilde{\mathbb{Q}}}$.

So we've seen a trade under complete specialization always increase output level at the moment of opening, but the growth effect depends on the R&D ability of the imports. And complete specialization always generate a balanced world income distribution even without international investment.

3.3 Results under incomplete specialization

If $\frac{(1-\epsilon)L_H}{\epsilon L_F}$ is out side the interval $(\frac{B_H}{B_F}(\frac{Z_{F2}}{Z_{H2}})^{\frac{\delta(1-\lambda)}{\lambda}}, \frac{A_H}{A_F}(\frac{Z_{F1}}{Z_{H1}})^{\frac{\delta(1-\lambda)}{\lambda}})$, then the world is incomplete specialization, and P_{Y_F}hits a boundary, i.e. $\frac{(1-\epsilon)L_H}{\epsilon L_F} \geq \frac{B_H}{B_F}(\frac{Z_{F2}}{Z_{H2}})^{\frac{\delta(1-\lambda)}{\lambda}} = P_{Y_F} > \frac{A_H}{A_F}(\frac{Z_{F1}}{Z_{H1}})^{\frac{\delta(1-\lambda)}{\lambda}}$. This means at the moment of trade, home can produce good 1 cheaper, and the quality-adjusted prices of good 2 are the same across countries. So foreign country would like to import good 1 from home; while home is indifferent in domestic and foreign good 2. Trade happens when foreign country completely specializes in good 2, while home country produces both goods but also imports good 2 to keep the trade balanced. In effect, foreign country is not "big" enough to satisfy home's requirement for good 1, i.e. $\frac{(1-\epsilon)L_H}{\epsilon L_F}$ is too high.

At the moment of opening to trade, the output level doesn't change for home, and increases for foreign country. Because home imports intermediate good 2, which has the same quality-adjusted price as domestic product, so it doesn't change its output level. While foreign country imports a cheaper good from home, so it increases its output level.

Incomplete specialization could evolve to complete specialization. For example, if at the moment of trade, $\frac{(1-\epsilon)L_H}{\epsilon L_F} = \frac{B_H}{B_F}(\frac{Z_{F2}}{Z_{H2}})^{\frac{\delta(1-\lambda)}{\lambda}}$, and Z_{F2} grows faster than Z_{H2}, then instantaneously $\frac{B_H}{B_F}(\frac{Z_{F2}}{Z_{H2}})^{\frac{\delta(1-\lambda)}{\lambda}} > \frac{(1-\epsilon)L_H}{\epsilon L_F} > \frac{A_H}{A_F}(\frac{Z_{F1}}{Z_{H1}})^{\frac{\delta(1-\lambda)}{\lambda}}$ and both countries completely specialize and follow the dynamic system to balance growth path, like section (3.2).

$\frac{B_H}{B_F}(\frac{Z_{F2}}{Z_{H2}})^{\frac{\delta(1-\lambda)}{\lambda}}$ might never reach $\frac{(1-\epsilon)L_H}{\epsilon L_F}$, and home stays in incomplete specialization. In this case, home accumulates $Z_{H1}$and Z_{H2} at the same time. The dynamic depends on not only the return of both qualities of home country, but also the quality level from foreign country, Z_{F2}, since home imports intermediate good 2. The system is similar to Eq (30) but with the involve of Z_{F2}.

$$\frac{(Z_{H1}\dot{/}Z_{H2})}{Z_{H1}/Z_{H2}} = \frac{\dot{Z_{H1}}}{Z_{H1}} - \frac{\dot{Z_{H2}}}{Z_{H2}} = \begin{cases} \frac{\dot{Z_{H1}}}{Z_{H1}} = \frac{\Gamma}{1-\Gamma}\frac{1}{2}\alpha_H\theta_{H1}[1+(\frac{Z_{H1}}{\frac{1}{2}\widetilde{Z_{H2}+\frac{1}{2}}Z_{F2}})^{-1}] - \frac{1}{1-\Gamma}\rho & if\ \frac{Z_{H1}}{\frac{1}{2}Z_{H2}+\frac{1}{2}Z_{F2}} < (\frac{Z_{H1}}{\frac{1}{2}Z_{H2}+\frac{1}{2}Z_{F2}})^{\star}, r_{H1} > r_{H2} \\ 0 & if\ \frac{Z_{H1}}{\frac{1}{2}Z_{H2}+\frac{1}{2}Z_{F2}} = (\frac{Z_{H1}}{\frac{1}{2}Z_{H2}+\frac{1}{2}Z_{F2}})^{\star}, r_{H1} = r_{H2} \\ -\frac{\dot{Z_{H2}}}{Z_{H2}} = \frac{1-\Gamma}{\Gamma}\frac{1}{2}\beta_H\theta_{H2}(1+\frac{\widetilde{Z_{H1}}}{\frac{1}{2}Z_{H2}+\frac{1}{2}Z_{F2}}) - \frac{1}{\Gamma}\rho & if\ \frac{Z_{H1}}{\frac{1}{2}Z_{H2}+\frac{1}{2}Z_{F2}} > (\frac{Z_{H1}}{\frac{1}{2}Z_{H2}+\frac{1}{2}Z_{F2}})^{\star}, r_{H1} < r_{H2} \end{cases}$$

where $\Gamma = \delta - (1-2\delta)(\epsilon - 1) \in [0,1]$

In this system, Z_{F2} is accumulated by foreign country. Since foreign country imports good 1 from home, and completely specializes in good 2, so the dynamic of Z_{F2} also depends on the accumulation of Z_{H1}.

$\frac{\dot{Z_{F2}}}{Z_{F2}} = \frac{1-\Gamma}{\Gamma}\frac{1}{2}\beta_F\theta_{F2}(1+\frac{Z_{H1}}{Z_{F2}}) - \frac{1}{\Gamma}\rho$

Combine above equation with the dynamic system of home country, we get the dynamic system for the world. On balance growth path, $\frac{\dot{Z_{H1}}}{Z_{H1}} = \frac{\dot{Z_{H2}}}{Z_{H2}} = \frac{\dot{Z_{F2}}}{Z_{F2}}$, and $\frac{Z_{H1}}{\frac{1}{2}Z_{H2}+\frac{1}{2}Z_{F2}} = (\frac{Z_{H1}}{\frac{1}{2}Z_{H2}+\frac{1}{2}Z_{F2}})^{\star}$, which is a function of the R&D abilities of home, i.e.α_H, β_H, θ_{H1} and θ_{H2}; and it's the same as autarky. So trade doesn't change the balanced growth rate for home. And the effect on foreign country depends on the magnitude of $\alpha_H\theta_{H1}$ and $\alpha_F\theta_{F1}$. After trade, both countries grow at the same rate.

4 Conclusion

We used a second-generation growth model with asymmetric industries to discuss the effect of trade on both output level and growth, without involving any scale effect or technology spillover. However, by trading Intermediate good, country also gets the quality inside to augment labor. It's equivalent to direct technology transfer.

The comparative advantage is determined endogenously, and it guarantees trade never reduces output level at the moment of opening. Whether it increases output also depends on whether it's complete specialization or not. However, comparative advantage doesn't guarantee a higher growth rate. Trade could either increase or decrease growth rates, depended on the absolute advantage in the R&D ability of the imports. Young(1991) also gets a similar result, but the growth rate effect depends on the good exported.

The change in unit costs and market size here dosent show up in growth rate, and totally absorbed by free entry of firms.

Since the qualities of goods are complements, world balance growth is reached even without international investment.

References

[1]