Instructions: Please, write your name and section number both on your blue book and on the question-sheet, and submit the question-sheet with your work. You may use left pages of the blue book for scratches and right pages for clean versions. You don’t have to write solutions to the problems in the given order, but a solution to each problem should start on its own page and the problem number should be clearly marked. Write all solutions clearly and show all your work. No calculator, no notes, no book and no other aid will be allowed. Each problem is out of 20 points.

Problem 1. Radium has a half-life of 1600 years. How many years does it take for 90% of a given amount of radium to decay? (Assume exponential decay.)

Problem 2. Consider a non-homogeneous differential equation

\[ y'' - 4y' + 4y = 2. \]

(a) Does this equation have constant solutions? If yes find them, if not, explain why.
(b) Write down the complimentary homogeneous equation and find its general solution.
(c) Find the general solution of the given non-homogeneous differential equation.

Problem 3. A spring has a 5-kg mass and its spring constant is 50 N/m. At \( t = 0 \) the spring begins to stretch from its natural position \( (x = 0) \) after being pushed with the velocity 2 m/s. Find the position of the mass after \( t \) seconds. (Assume simple harmonic motion).

Problem 4. A spring has a 1-kg mass and its spring constant is 10 N/m. For each of the three values of the damping constant \( c \): 10, 20, 30 decide what type of damping occurs: overdamping, underdamping, or critical damping. Justify your answers fully.

Problem 5. Write first four terms of each of the sequences listed below. Determine whether each sequence converges or diverges. If it converges, find the limit. If it diverges, indicate whether the limit is infinite or does not exist. Provide a brief justification for your answers.
(a) \( a_n = \frac{1}{10^n} \);
(b) \( a_n = \frac{n^2+1}{5n^2+n} \);
(c) \( a_n = \frac{2^n}{2n} \);
(d) \( a_n = \frac{3^n}{2^n} \).

Problem 6. For each of the sequences listed in Problem 5, determine whether the corresponding series \( \sum_{n=1}^{\infty} a_n \) converges or diverges. If it converges, find the sum. If it diverges state so. Provide a brief justification for your answers.